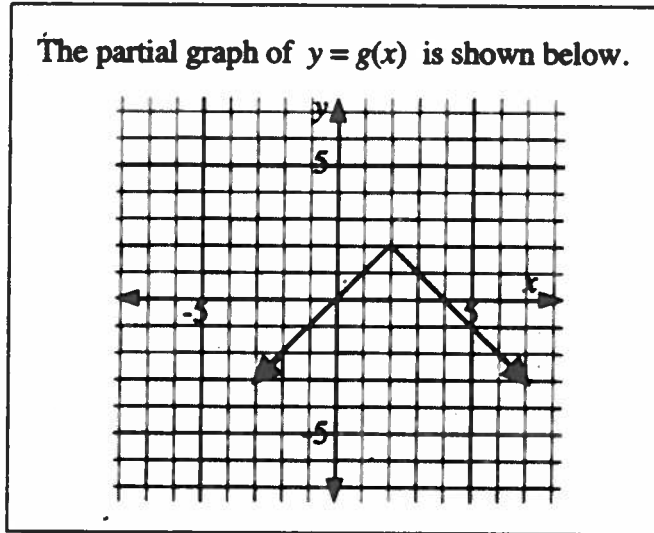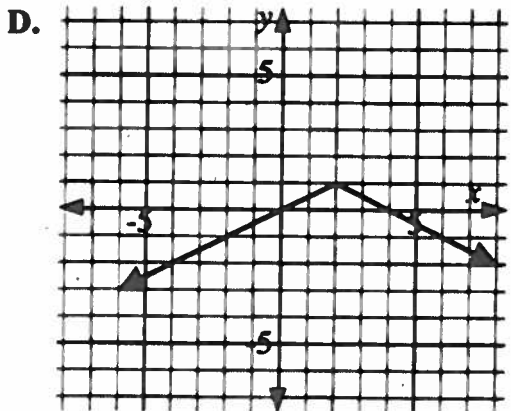# Transformations Lesson #9:
Practice Test

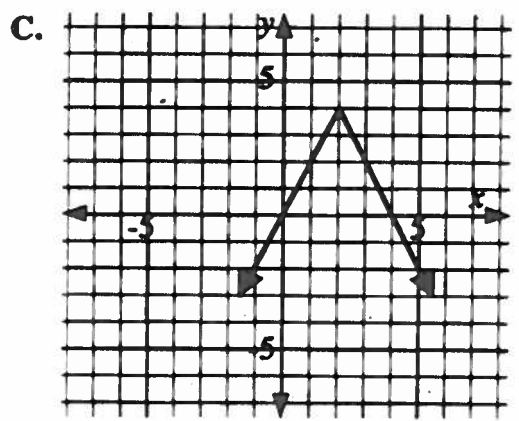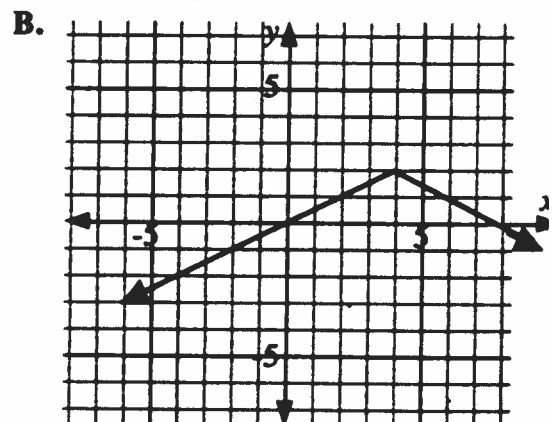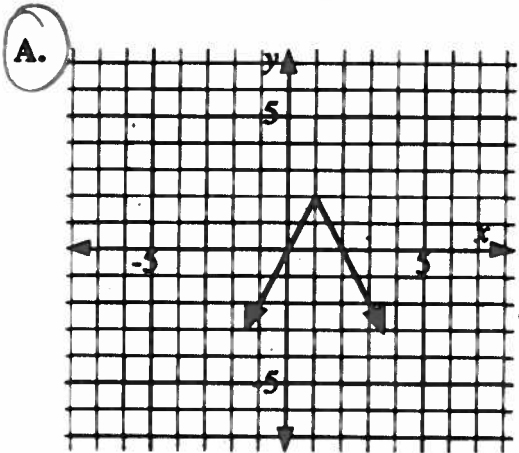*No calculator may be used for this section of the test.*

Use the following information to answer the next question.

The partial graph of $y = g(x)$ is shown below.

$X \to 2X$

nor stretch by

factor $\frac{1}{2}$

1. Which of the following partial graphs represents the function $y = g(2x)$?

A.

B.

C.

D.

2.   The graph of $y = P(x)$ is translated 4 units up and 7 units right. The equation of the transformed graph is

   A. $y + 4 = P(x - 7)$

   B. $y - 4 = P(x + 7)$

   C. $y - 4 = P(x - 7)$

   D. $y + 4 = P(x + 7)$

   (handwritten) $y \to y - 4$
 $x \to x - 7$
   $y - 4 = P(x - 7)$

3. How is the graph of $y = \left| \dfrac{x}{2} \right|$ related to the graph of $y = |x|$? *(handwritten)* $x \to \dfrac{1}{2} x$

   A. The graph of $y = |x|$ has been stretched vertically by a factor of $\dfrac{1}{2}$ about the x-axis.

   B. The graph of $y = |x|$ has been stretched vertically by a factor of 2 about the x-axis.

   C. The graph of $y = |x|$ has been stretched horizontally by a factor of $\dfrac{1}{2}$ about the y-axis

   D. The graph of $y = |x|$ has been stretched horizontally by a factor of 2 about the y-axis.

4.   The function $y = g(x)$ is shown in diagram 1. The equation of the function shown in diagram 2 could be

   A. $y = g(-x)$

   B. $y = -g(x)$

   C. $y = g^{-1}(x)$

   D. $y = -g(-x)$

Diagram 1   Diagram 2
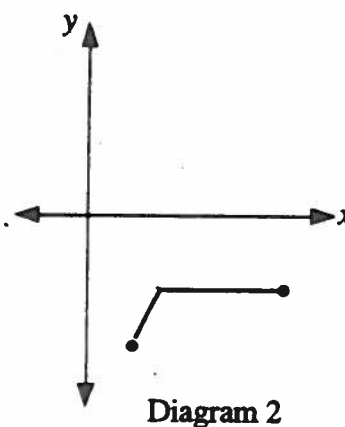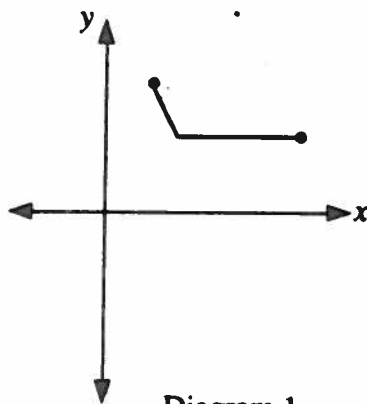
(handwritten) reflection in x-axis
$y \to -y$ $y = g(x)$ $y = -g(x)$

Numerical Response 1.   Consider the following equations of transformations of $y = P(x)$.

    1. $y = P(x) + 9$ 2. $y = P(x) - 9$ $y \rightarrow y + 9$ $9↓$

    $y \rightarrow y - 9$ $9↑$

    3. $y = P(x + 9)$ 4. $y = P(x - 9)$ $x \rightarrow x - 9$ $9R$

    $x \rightarrow x + 9$ $9L$

In the first box write the equation number for the translation 9 units left. **3**

In the second box write the equation number for the translation 9 units right. **4**

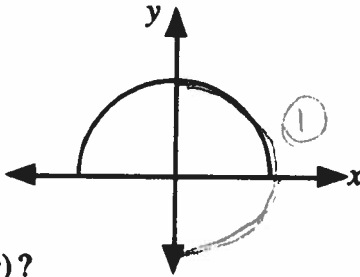In the third box write the equation number for the translation 9 units up. **1**

In the fourth box write the equation number for the translation 9 units down. **2**

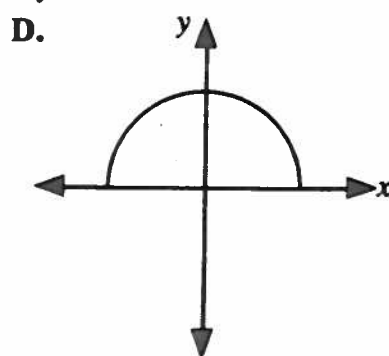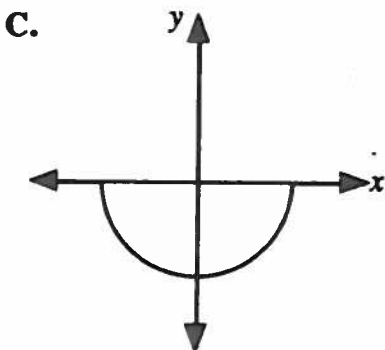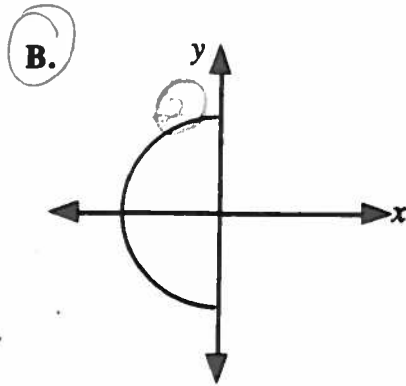(Record your answer in the numerical response box from left to right.) | 3 | 4 | 1 | 2 |

5.   The graph of $y = f(x)$ is shown.

①

1. reflection
    $y = -x$
    reflecting...

Which graph represents $x = -f(y)$?

A.

B.

C.

D.
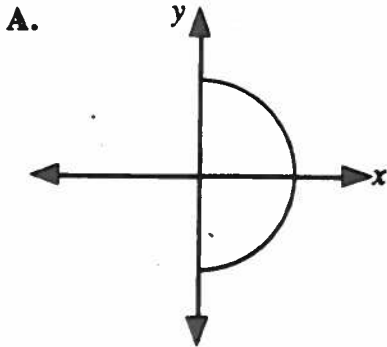
Section B   *A graphing calculator may be used for the remainder of the test.*

6. The point $(2,-4)$ is on the graph of the function $y = f(x)$. The point which must be on the graph of $y = f^{-1}(x)$ is

   A. $(2, -\frac{1}{4})$ B. $(2, 4)$

   C. $(-2, -4)$ **D.** $(-4, 2)$

   (handwritten) nverse transformation
   switch x+y

7. The relation $x = \sqrt{4 - y^2}$ is stretched vertically by a factor of 3 about the x-axis and then translated 2 units to the left. The equation of the transformed relation is

   A. $x = 3\sqrt{4 - (y + 2)^2}$

   B. $x = \sqrt{4 - 9y^2} - 2$

   C. $x = \sqrt{4 - \frac{1}{9}y^2} + 2$

   D. $x = \sqrt{4 - \frac{1}{9}y^2} - 2$

   (handwritten)
   1. $X \rightarrow X + 2$ $x - 2 = \sqrt{4 - y^2}$
   2. $y \rightarrow \frac{1}{3}y$ $x + 2 = \sqrt{4 - (\frac{1}{3}y)^2}$
   $x = \sqrt{4 - \frac{1}{9}y^2} - 2$

Numerical Response 2. The domain of the function $y = f(x)$ is $x \geq 4$. Function f is stretched horizontally by a factor of 5 about the y-axis to form a new function g. The domain of the function $g(x)$ can be written in the form $x \geq k$, where k is a whole number. The value of k is _____

   (Record your answer in the numerical response box from left to right.)   `2 0`

   (handwritten) $4(5) = 20$ $x \geq 20$

8. The function, $f(x)$, whose graph has **two** x-intercepts is shown in the diagram .

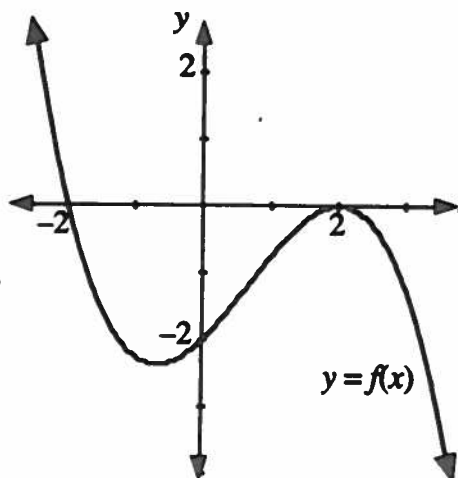   The function whose graph has only **one** x-intercept is

   A. $f(x - 2)$

   B. $f(x + 2)$

   C. $f(x) - 2$

   D. $f(x) + 2$

3. The function $f(x) = -\dfrac{28}{x+2}$ is reflected in the line $y = x$ to form function g.

The ordered pair $(k, -4)$, where $k \in W$, lies on the graph of $y = g(x)$.
The value of k is _____ .

(Record your answer in the numerical response box from left to right.)

$$\boxed{1}\,\boxed{4}\,\boxed{}\,\boxed{}$$

Handwritten: If $k, -4$ is on $y = g(x)$ then $(-4, k)$ is on $y = f(x)$

$$k = \frac{-28}{-4+2} = 14$$

9. The zeros of the function $y = P(x)$ are $-3, -1$ and 5.

The zeros of the transformed function $y = \dfrac{1}{2}P(x+1)$ are

A. $-4, -2, 4$ **B.** $-2, 0, 6$

C. $-2, -1, 2$ **D.** $-\dfrac{5}{2}, -\dfrac{3}{2}, \dfrac{3}{2}$

Handwritten: $2y = P(x+1)$ $x \to x+1$
$y \to 2y$ 1UL
v.s. by   (zeros moved factor 2   1UL)

Use the following information to answer the next question.

The partial graph of $y = f(x)$ is shown.
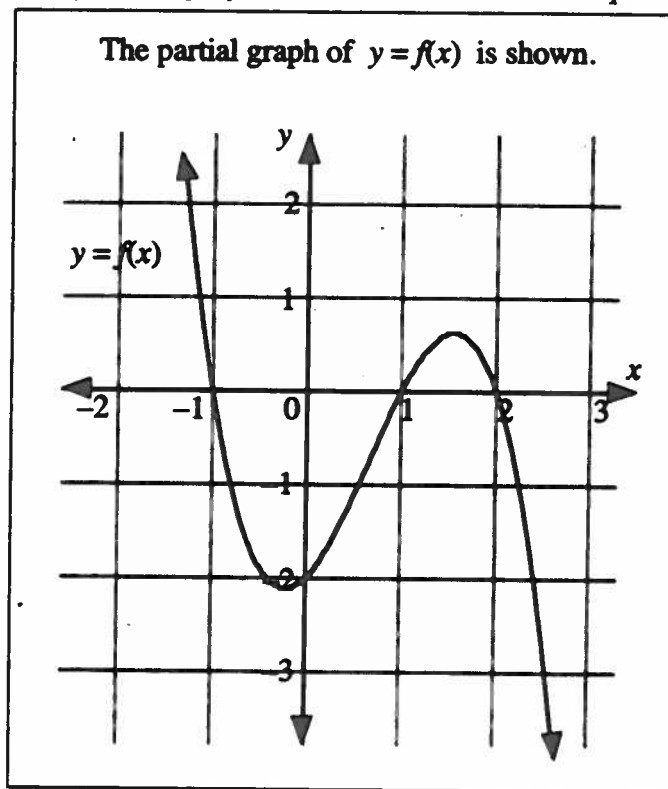
Handwritten: $y = f(x) - 1$
$y - 1 = f(x)$
$y \to y - 1$
1 unit ↓

10. The number of solutions to the equation $f(x) - 1 = 0$ is   *Handwritten:* $f(x) = 1$

A. 1   **B.** 2   **C.** 3   **D.** 4

11. The equation that would cause the graph of $y = g(x)$ to stretch vertically about the x-axis by a factor of $\dfrac{1}{6}$ and then reflect in the y-axis is

(handwritten: $y \to 6y$ $6y = g(x)$; $y = \frac{1}{6}g(x)$)
(handwritten: $x \to -x$ $y = \frac{1}{6}g(-x)$)

A. $y = -6g(x)$

B. $y = -\dfrac{1}{6}g(x)$

C. $y = 6g(-x)$

D. $y = \dfrac{1}{6}g(-x)$ *(circled)*

Use the following information to answer the next question.

The ordered pair (a,b) is on the graph of the function $y = f(x)$. Six ordered pairs, the images of (a,b) derived from transformations of $f(x)$, are shown below.

(handwritten: $x \to \frac{1}{3}x$)   *(handwritten: $x \to \frac{1}{3}x, \; y \to 3y$)*   *(handwritten: 3)*

Ordered pair #1: $(3a, b)$ **Ordered pair #2:** $(3a, 3b)$ **Ordered pair #3:** $\left(\dfrac{a}{3}, \dfrac{b}{3}\right)$

(handwritten: $y \to 3y$)   *(handwritten: $x \to 3x$)*   *(handwritten: $y \to \frac{1}{3}$)*

Ordered pair #4: $\left(a, \dfrac{b}{3}\right)$ **Ordered pair #5:** $\left(\dfrac{a}{3}, b\right)$ **Ordered pair #6:** $(a, 3b)$

Numerical Response 4.

In box 1 write the ordered pair # for the function $3f(x)$. *(handwritten: $y \cdot 3$; $a, 3b$)*

In box 2 write the ordered pair # for the function $f\left(\dfrac{1}{3}x\right)$ *(handwritten: $x \cdot 3$; $3a, b$)*

In box 3 write the ordered pair # for the function $\dfrac{1}{3}f(x)$ *(handwritten: $y \div 3$; $a, \frac{b}{3}$)*

In box 4 write the ordered pair # for the function $f(3x)$ *(handwritten: $x \div 3$; $\frac{a}{3}, b$)*

(Record your answer in the numerical response box from left to right.) | 6 | 1 | 4 | 5 |

---

12. The point $(-9, 3)$ is on the graph of the function $y = f(x)$. The point which must be on the graph of $y = \dfrac{1}{3}f(-x)$ is

(handwritten: $3y = f(-x)$ $-9, 3$)
(handwritten: $y \to 3y$ — V.S. $\frac{1}{3}(\div \text{ by } 3) \; -9, 1$)
(handwritten: $x \to -x$ $9, 1$)

A. $(9, 1)$ *(circled, checked)*

B. $(-3, -3)$

C. $(-9, 1)$

D. $(-9, -1)$

13. The function $y = g(x)$ is graphed to the left below. The equation of the function shown to the right is

(handwritten: is reflection x ... reflecting ny)
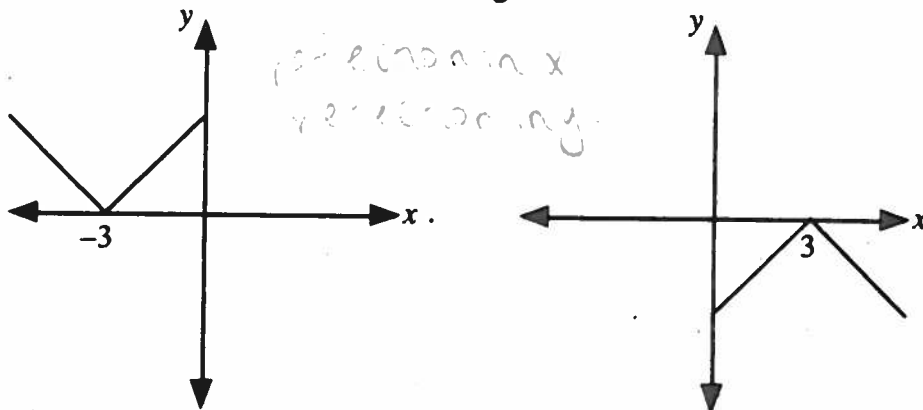
A. $y = g(-x + 6)$ **B.** $y = g(-x - 6)$

C. $y = g^{-1}(x)$ **D.** $y = -g(-x)$

(handwritten: y → -y, x → -x, y = -g(-x))

14. The equation which represents the graph of $y = x^3$ after it is reflected in the line $y = x$ is

A. $x = y^3$
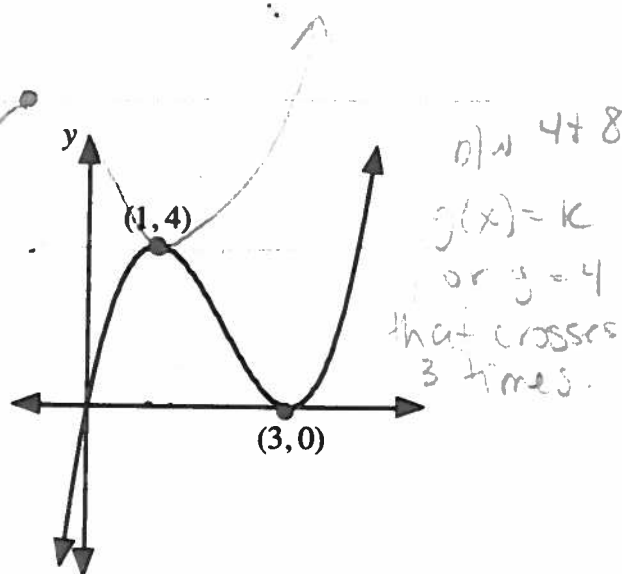
B. $y = -x^3$

C. $y = x^3$

D. $y = \dfrac{1}{x^3}$

(handwritten: x ⟷ y, x = y³)

15. A partial graph of a cubic function with equation $y = f(x)$ is shown.

A function g is defined by $g(x) = f(x + 2) + 4$.

The range of values for which $g(x) = k$ has three distinct real roots is

(handwritten: 2∪L, 4∪↑)

A. $2 < k < 6,\ k \in R$

B. $4 < k < 8,\ k \in R$

C. $-2 < k < 2,\ k \in R$

D. $-4 < k < 0,\ k \in R$

(graph labels: (1,4), (3,0))

(handwritten: 0↓ 4↑8, g(x) = k or y = 4 that crosses 3 times.)

Use the following information to answer the next question.

- The graph of $y = f(x)$ is shown in Diagram 1 .
- Diagram 2 is a transformation of the graph in Diagram 1.
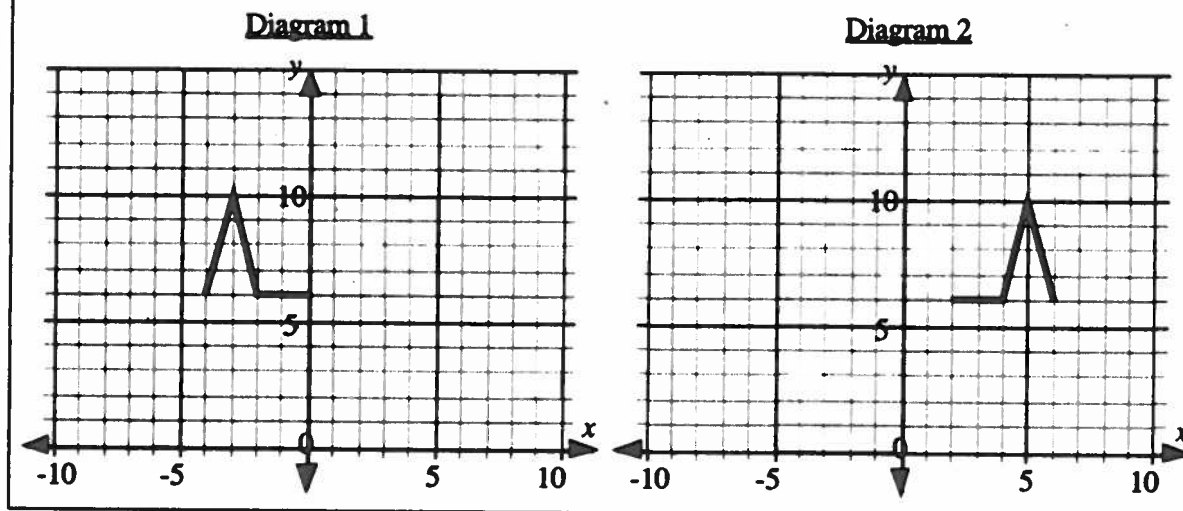
Diagram 1   Diagram 2

16. The equation of the graph shown in Diagram 2 is

    A. $y = -f(x - 2)$

    B. $y = -f(x + 2)$

    C. $y = f[-(x - 2)]$

    D. $y = f(-x - 2)$

(handwritten: reflection $x \to -x$; translation 2 u right $\to x - 2$; $y = f(-x)$; $y = f[-(x-2)]$ *)*

17. The ordered pair (p, q) is on the function $y = g(x)$. The function is transformed and the resulting function has the equation $y - 6 = g(x + 4)$. The ordered pair which **must** be on the transformed function is

    A. $(p + 4, q + 6)$ **B.** $(p - 4, q + 6)$

    C. $(p + 4, q - 6)$ **D.** $(p - 4, q - 6)$

(handwritten: $x \to x+4$ 4 u L ; $y \to y-6$ 6 u↑ ; $(p,q) \to (p-4, q+6)$ *)*

18. The transformation of $y = g(x)$ to $y = -10g(x)$ is

(handwritten: $-\frac{1}{10} y = g(x)$ $y = -\frac{1}{10} y$ *)*

    A. a vertical stretch by a factor of 10 about the x-axis and a reflection in the x-axis

    B. a vertical stretch by a factor of $\dfrac{1}{10}$ about the x-axis and a reflection in the x-axis

    C. a vertical stretch by a factor of 10 about the x-axis and a reflection in the y-axis

    D. a vertical stretch by a factor of $\dfrac{1}{10}$ about the x-axis and a reflection in the y-axis

Numerical **5.** The range of the function $y = f(x)$ is $y \leq 16$. The range of the function $y = f(x + 6) - 3$
ponse is $y \leq c$ where c is a whole number. The value of c is _____

(Record your answer in the numerical response box from left to right.) | 1 | 3 | | |

$$y + 3 = f(x + 6)$$
$$6 u L, 3 u \downarrow$$

range of $y = f(x + 6) - 3$
16 ; down 3 = 13

19. The point $(8, -4)$ is on the graph of the function $y = f(x)$. The point which must be on
the graph of $4y = -f(-x)$ is

A. $(-2, 4)$

B. $(-32, 4)$

C. $(-8, 1)$

D. $(-8, 16)$

$-4y = f(-x)$ - vert. stretch factor
$$y \cdot \frac{1}{4} \rightarrow$$ $\frac{1}{4}$, reciprocal

reflecting in y

$$(8, -4) \rightarrow (8, -1) \rightarrow (8, 1) \rightarrow (-8, 1)$$
$y \rightarrow -y$ $x \rightarrow -x$

20. The function $f(x) = kx^2$, where $k < 0$, is transformed to $g(x) = k(x + 2)^2 - 6$.
The range of the transformed function is

A. $y \geq -6$

B. $y \leq -6$

C. $y \leq 6$

D. none of the above

range of $f(x)$ $y \leq 0$
with $k < 0$

$y \rightarrow y + 6$ $\rightarrow 6u \downarrow + 2uL$
$x \rightarrow x + 2$

$$y \leq -6$$

Numerical **6.** The point $P(4, 12)$ is on the graph of $y = 2^x - 4$. As a result of the transformation of
Response
the graph $y = 2^x - 4$ into the graph of $y = 2^{\frac{1}{2}x - 8} - 4$, the point P is transformed to
the point $Q(a, 12)$. The value of a is _____ .

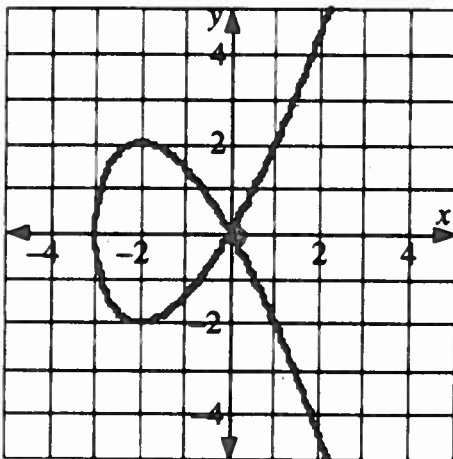(Record your answer in the numerical response box from left to right.) | 2 | 4 | | |

$$y = 2^{\frac{1}{2}(x - 16)} - 4$$ compared to $y = 2^x - 4$
$$x \rightarrow \frac{1}{2}(x - 16)$$ ① $x \rightarrow \frac{1}{2}x$
                                              ② $x \rightarrow x - 16$ R16

$$(4, 12) \rightarrow (8, 12) \rightarrow (24, 12)$$
$x \rightarrow \frac{1}{2}x$ $x \rightarrow x - 16$

Written Response

Use the following information to answer this question.

A Tschirnhausen Cubic is a curve given by an equation such as $y^2 = x^3 + 3x^2$. A partial graph of $y^2 = x^3 + 3x^2$ is shown below.

The graph passes through the points $(-3, 0)$, $(-2, 2)$, $(-2, -2)$ and $(0, 0)$.

The questions below are about various transformations applied to the Tschirnhausen Cubic with equation $y^2 = x^3 + 3x^2$.

- If the Tschirnhausen Cubic with equation $y^2 = x^3 + 3x^2$ is transformed 3 units to the right, state the domain and range of the transformed relation.

  $x \mid x \geq 0, x \in R$.

  $y \in R$.

- Describe a single transformation applied to the Tschirnhausen Cubic with equation $y^2 = x^3 + 3x^2$ such that the graph of the image coincides with the original curve

  — reflection in x-axis

- Write the replacement for x or y associated with the transformation in the previous bullet and show that the equation of the image is identical to the original equation.

  $y \to -y$

  $(-y)^2 = x^3 + 3x^2$

  $y^2 = x^3 + 3x^2$ (same as original reflection)

- The image of the Tschirnhausen Cubic with equation $y^2 = x^3 + 3x^2$ is given by the equation $(y+2)^2 = (x-1)^3 + 3(x-1)^2$.

  On the grid shown, sketch the transformed image, and mark on the grid the coordinates of the images of the four points given in the information above.
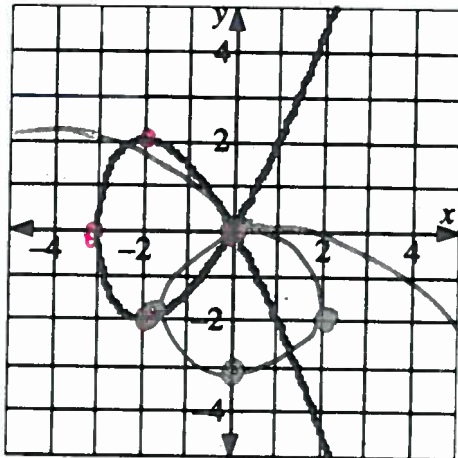

$y - 2$ $2 u \downarrow$
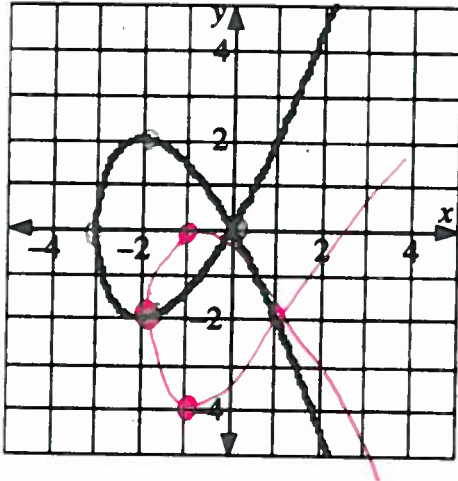
$x - 1$ $1 u R.$

$(-2, -2)$
$(-1, 0)$
$(-1, -4)$
$(1, -2)$

- The Tschirnhausen Cubic with equation $y^2 = x^3 + 3x^2$ is reflected in the line $y = x$.

  Determine the equation of the image of the Tschirnhausen Cubic as a result of this reflection and sketch the transformed relation on the grid below.

$x \Longleftrightarrow y$

$x^2 = y^3 + 3y^2$

$(-2, 2) \rightarrow 2, -2$

$(-3, 0) \rightarrow 0, -3$

$(0, 0) \rightarrow 0, 0$

$(-2, -2) \rightarrow -2, -2$