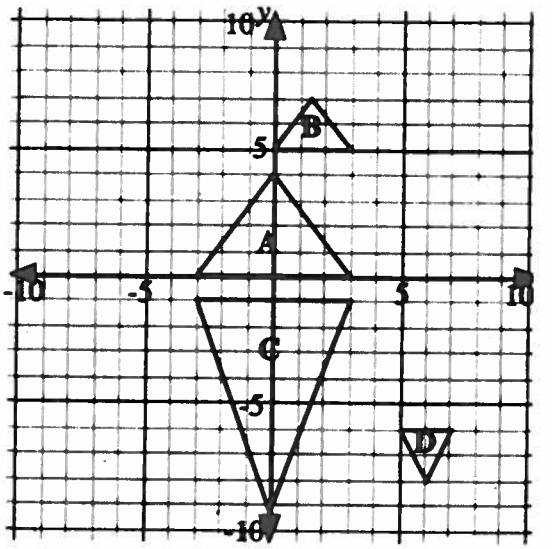
Assignment

1. Describe a series of transformations required to transform



a) graph A to graph B

- vertical stretch by a factor of $\frac{1}{2}$ about x-axis
- horizontal stretch by a factor of $\frac{1}{2}$ about the y-axis
- then a translation 1.5 units right + 5 units up

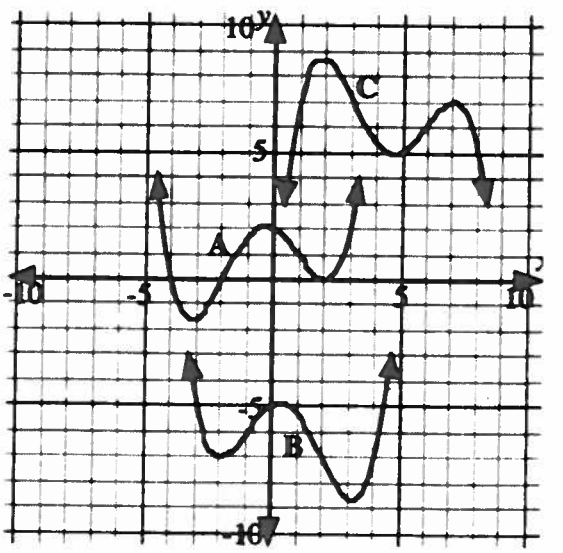
b) graph A to graph C

- vertical stretch by a factor of 2 about x-axis
- reflection in the x-axis
- vertical translation 1 unit down.

c) graph A to graph D

- vertical stretch by a factor of $\frac{1}{2}$ about the x-axis
- horizontal stretch by a factor of $\frac{1}{3}$ about the y axis
- reflection in the x-axis
- translation 6 units (R) + 6 units down.

2. Describe a series of transformations required to transform



a) graph A to graph B

- reflection in y-axis, translation 7 units down.

b) graph A to graph C

- reflection in x-axis + translation 5 units right + 7 units up.

3. In each of the following, transformations are applied to the graph of $y = f(x)$. In each case
   - describe which transformations are applied to the graph when the indicated replacements are made
   - determine the equation of the final graph if the replacements are made in the order giv

a) Replace x with $x + 2$ and y with $-y$.

- horizontal translation 2 units left
- reflection in the x-axis

$y = f(x)$ $y = f(x+2)$
$-y = f(x+2)$ $y = -f(x+2)$

b) Replace x with $4x$ and y with $y - 7$.

- horizontal stretch about y-axis by a factor of $\frac{1}{4}$
- vertical translation 7 units up

$y = f(x)$ $y = f(4x)$
$y - 7 = f(4x)$ $y = f(4x) + 7$

c) Replace x with $\frac{1}{3}x$, y with $-2y$, and y with $y + 2$.

- horizontal stretch by a factor of 3 about y-axis
- vertical stretch about x-axis by a factor of $\frac{1}{2}$
- reflection in x-axis
- vertical translation 2 units down.

$y = f(x)$ $y = f(\frac{1}{3}x)$
$-2y = f(\frac{1}{3}x)$
$y = -\frac{1}{2} f(\frac{1}{3}x)$
$y + 2 = -\frac{1}{2} f(\frac{1}{3}x)$
$y = -\frac{1}{2} f(\frac{1}{3}x) - 2$

d) Replace x with $2x$, y with $\frac{1}{4}y$, x with $-x$, and y with $y + 10$.

- hor. stretch about y-axis by factor $\frac{1}{2}$
- vertical stretch about x axis by factor of 4
- reflection in y-axis
- vertical translation 10 units down

$y = f(x)$ $y = f(2x)$
$\frac{1}{4}y = f(2x)$ $y = 4f(2x)$
$y = 4f(-2x)$
$y + 10 = 4f(-2x) \rightarrow y = 4f(-2x) - 10$

4. **a)** In which of the parts of question 3 does the order in which the transformations are performed affect the final graph? 3c + 3d.

b) In each of the cases where the order matters in a)
   - determine the equation of the final graph if the order is changed
   - describe the relationship between the two graphs

3c. reverse the order

$y = f(x)$ $y + 2 = f(x)$ $y = f(x-2)$
$-2y = f(x) - 2$ $y = \frac{-1}{2} f(x) + 1$
$y = -\frac{1}{2} f(\frac{1}{3}x) + 1$

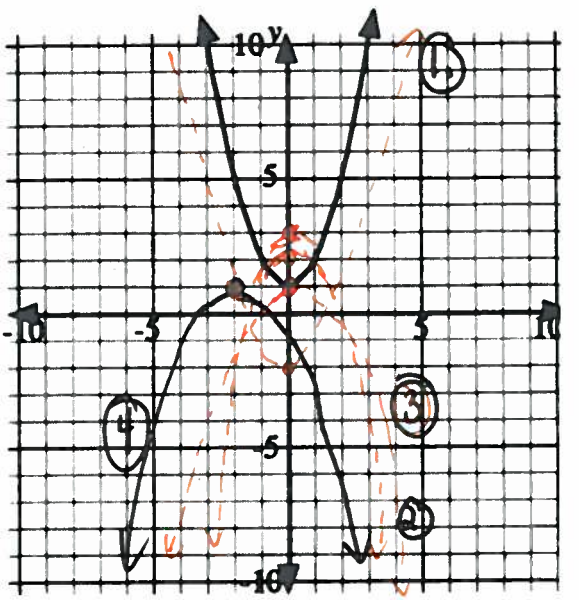
- the 2 graphs are separated vertically by 3 units

3d. reverse order

$y = f(x)$ $y + 10 = f(x)$ $y = f(x) - 10$
$y = f(-x) - 10$ $\frac{1}{4}y = f(-x) - 10$
$y = 4f(-x) - 40$ $y = 4f(-2x) - 40$

- 2 graphs are separated vertically by 30 units.

5. A graph of the parabola $y = x^2 + 1$ is shown. The following transformations are applied to $y = x^2 + 1$ **in the order shown:**



1. • a vertical translation 3 units down
2. • a reflection in the x-axis
3. • a vertical stretch about the x-axis by a factor of 0.5
4. • a horizontal translation 2 units left

a) For each transformation
- graph the image on the grid
- write the replacement for x or y and the current equation in the table

Transformation	*Replacement*	*Current Equation*
1. a vertical translation 3 units down	$y \to y+3$	$y+3 = x^2+1$ $y = x^2-2$
2. a reflection in the x-axis	$y \to -y$	$-y = x^2-2$ $y = -x^2+2$
3. a vertical stretch about the x-axis by a factor of 0.5	$y \to 2y$	$2y = -x^2+2$ $y = -\frac{1}{2}x^2+1$
4. a horizontal translation 2 units left	$x \to x+2$	$y = -\frac{1}{2}(x+2)^2+1$

b) Write the equation which represents the final position of the graph.

$$y = -\frac{1}{2}(x+2)^2+1$$

c) Verify using a graphing calculator.

d) Explain why the equation in this example is different from the equation in Class Ex. #4

- order of transformations are different
- changing order of vertical stretch + translation will result in a different equation.

6. The graph of $y = f(x)$ is reflected in the x-axis, then vertically stretched by a factor of 3 ab the x-axis, and then translated 4 units to the right and 1 unit up. Determine the equation of the final graph.

$y \to -y \quad -y = f(x) \quad y = -f(x)$

$y \to \frac{1}{3}y \quad y = -3f(x)$

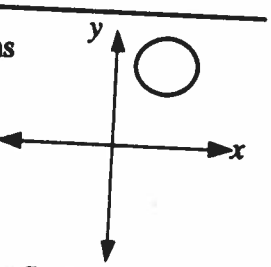
$x \to x-4 \quad y-1 = -3f(x-4)$

$y \to y-1 \quad y = -3f(x-4)+1$

$y = -3f(x-4)+1$

The following information refers to question #7.

Five students were asked to perform a combination of two transformations on the graph shown.



Student I: a reflection in the x-axis and a translation 3 units left

Student II: a reflection in the y-axis and a translation 3 units left

Student III: a horizontal stretch by a factor of 2 about the y-axis and a reflection in the x-axi

Student IV: a translation 2 units up and a vertical stretch by a factor of $\frac{1}{2}$ about the x-axis

Student V: a translation 2 units right and a vertical stretch by a factor of $\frac{1}{2}$ about the x-axis

Multiple Choice

7. For how many of the students does the order in which the transformations are performed affect the final graph?

A. One student

B. Two students

C. Three students

D. Four students

Start with $y = f(x)$

Student 1: order does not matter

$y \to -y \quad -y = f(x) \quad y = -f(x)$

$x \to x+3 \quad y = -f(x+3)$

$x \to x+3 \quad y = f(x+3)$

$y \to -y \quad y = -f(x+3)$

Student 2 order does matter.

$x \to -x \quad y = f(-x)$

$x \to x+3 \quad y = f(-(x+3))$

$y = f(-x-3)$

$x \to x+3 \quad y = f(x+3)$

$x \to -x \quad y = f(-x+3)$

Student 3 - order does not matter

$x \to \frac{1}{2}x \quad y = f(\frac{1}{2}x)$

$y \to -y \quad y = -f(\frac{1}{2}x)$

$y \to -y$

$y = -f(x$

$x \to \frac{1}{2}x$

$y = -f(\frac{1}{2}$

Student 4 order does matter

$y \to y-2 \quad y = f(x)+2$

$y \to 2y \quad y = \frac{1}{2}f(x)+2$

$y \to 2y \quad y = \frac{1}{2}f(x)$

$y \to y-2 \quad y = \frac{1}{2}f(x)+2$

Student 5 - does not matter

$x \to x-2 \quad y = f(x-2)$

$y \to 2y \quad y = \frac{1}{2}f(x-2)$

or

$y \to 2y \quad y = \frac{1}{2}f(x$

$x \to x-2 \quad y = \frac{1}{2}f(x-2)$

8. The graph of $y = f(x)$ is horizontally stretched by a factor of $\frac{1}{4}$ about the y-axis, and then translated 5 units to the left and 3 units up. The equation of the transformed graph is

A. $y = f\left(\frac{1}{4}(x+5)\right) + 3$

B. $y = f\left(\frac{1}{4}x + 5\right) + 3$

C. $y = f(4x + 5) + 3$

D. $y = f[4(x+5)] + 3$

$x \to 4x \quad y = f(4x)$

$x \to x+5 \quad y - 3 = f(4(x+5))$

$y \to y-3$

$y = f(4(x+5)) + 3$

Numerical Response

9. Jack and Jill are working with the graph of the function $f(x) = x^2$.

Jack stretches the graph vertically by a factor of 4 about the x-axis, followed by a translation 2 units up.

Jill takes the graph of $f(x) = x^2$, and translates it 2 units up, followed by a vertical stretch by a factor of 4 about the x-axis. The images of the two graphs are identical except for a vertical separation of k units, $k > 0$.

The value of k, to the nearest tenth, is 6.0.

(Record your answer in the numerical response box from left to right.)

6	.	0	

Jack

$f(x) = x^2 \quad y = x^2$

$y \to \frac{1}{4}y \quad y = 4x^2$

$y \to y-2 \quad y = 4x^2 + 2$

Jill

$y = x^2$

$y \to y-2 \quad y = x^2 + 2$

$y \to \frac{1}{4}y \quad y = 4(x^2+2)$

$y = 4x^2 + 8$

$8 - 2 = 6$