Class Ex. #4 A polynomial function has the equation $P(x) = (x - 4)(x + 3)(x + 6)$.
Determine the zeros and the y-intercept if the following transformations are applied.

a) $y = -3P(x)$

$y \to -y$ $\Big\}$ $y \to -\frac{1}{3}y$
$y \to \frac{1}{3}y$

zeros of $P(x) = 4, -3, -6$
y-int of $P(x) = (0-4)(0+3)(0+6)$
$= -72$

vert stretch by factor of 3 about x axis $\Big\}$
reflection
-zeros stay same, y-int multiplied by -3

$(-72)(-3) = \boxed{216}$

zeros: $-6, -3, 4$
yint = 216

b) $y = P\left(-\frac{1}{2}x\right)$

$x \to \frac{1}{2}x$ $\Big\{$ $x \to -\frac{1}{2}x$ hor stretch by factor
$x \to -x$ $\Big\}$ of 2 + reflection in
     y-axis

- zeros are multiplied by -2, no change to y-int

zeros: $4(-2), -3(-2), -6(-2)$
zeros. $-8, 6, 12$
yint. -72

Complete Assignment Questions #1 - #6

Assignment

1. Write the equation of the image of

   a) $y = |x + 1|$ after a vertical stretch about the x-axis by a factor of $\frac{7}{9}$

   $y \to \frac{9}{7}y$ $\frac{9}{7}y = |x+1|$ · $y = \frac{7}{9}|x+1|$

   b) $y = 2^x$ after a horizontal stretch by a factor of 3 about the y-axis

   $x \to \frac{1}{3}x$ $y = 2^{\frac{1}{3}x}$

   c) $y = \sqrt{x - 2}$ after a vertical stretch about the x-axis by a factor of 4 and a reflection in the x-axis

   $y \to \frac{1}{4}y$ $\Big\{$ $y \to -\frac{1}{4}y$ $y = -4\sqrt{+x-2}$ $y = -4\sqrt{x-2}$
   $y \to -y$

   d) $y = \sqrt{x - 2}$ after a horizontal stretch about the y-axis by a factor of 4 and a reflection the y-axis

   $x \to \frac{1}{4}x$ $\Big\{$ $x \to -\frac{1}{4}x$ $y = \sqrt{-\frac{1}{4}x - 2}$
   $x \to -x$

   e) $y = \sin x°$ after a horizontal stretch about the y-axis by a factor of $\frac{3}{4}$ and

   a vertical stretch about the x-axis by a factor of $\frac{1}{2}$

   $x \to \frac{4}{3}x$

   $y \to 2y$ $2y = \sin \frac{4}{3}x°$

   $y = \frac{1}{2}\sin \frac{4}{3}x°$

f) $y = 2x - 11$ after a horizontal stretch about the y-axis by a factor of $\frac{1}{3}$ and a reflection in the y-axis

$x \to 3x$ ⎫ $x \to -3x$
$x \to -x$ ⎭

$y = 2(-3x) - 11$
$y = -6x - 11$

g) $y = \frac{1}{x+3}$ after a horizontal stretch about the y-axis by a factor of 3, a <u>vertical stretch</u> about the x-axis by a factor of 2, and a reflection in the <u>y-axis</u>

$x \to \frac{1}{3}x$ ⎫ $x \to -\frac{1}{3}x$
$x \to -x$ ⎭

$\frac{1}{2}y = \frac{1}{-\frac{1}{3}x + 3}$

$y \to \frac{1}{2}y$

$y = \dfrac{2 \times 3}{-\frac{1}{3}x + 3 \times 3}$ or $y = \dfrac{-6}{x-9}$

h) $y = \frac{1}{x} + 3$ after a <u>vertical stretch</u> about the <u>x-axis</u> by a factor of $\frac{1}{2}$, a <u>horizontal stretch</u> about the y-axis by a factor of $\frac{1}{4}$, and a <u>reflection</u> in both the x-axis and y-axis

$y \to \frac{1}{2}y$ $y \to -\frac{1}{2}y$
$x \to 4x$ $x \to -4x$
$y \to -y$
$x \to -x$

$-2y = \dfrac{1}{-4x} + 3$

$y = -\frac{1}{2}\left(\dfrac{1}{-4x} + 3\right)$

$y = \dfrac{1}{8x} - \dfrac{3}{2}$

2. Describe how the graph of the second function compares to the graph of the first function.

a) $y = \sqrt{x}$, $y = \sqrt{\frac{1}{3}x}$

$x \to \frac{1}{3}x$

horizontal stretch about y-axis by a factor of 3

b) $y = x^4$, $\frac{1}{4}y = x^4$

$y \to 4y$

vertical stretch by a factor of 4 about x-axis

c) $y = 5x + 10$, $y = 5\left(-\frac{1}{4}x\right) + 10$

$x \to -\frac{1}{4}x$

horizontal stretch by a factor of 4 about y-axis, reflection in y-axis.

d) $y = \frac{1}{x+1}$, $3y = \frac{1}{x+1}$

$y \to 3y$

vertical stretch about x-axis by a factor of $\frac{1}{3}$

e) $y = 5^x$, $y = 5^{0.5x}$

$x \to \frac{1}{2}x$

hor. stretch about y-axis by a factor of 2.

f) $y = \frac{1}{x+1}$, $y = \frac{②}{x+1}$

$y \to \frac{1}{2}y$

vertical stretch by a factor of 2 about x-axis.

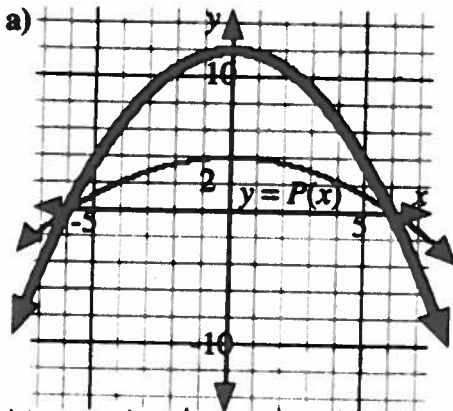g) $y = \cos x°$, $y = 3\cos 2x°$

$y \to \frac{1}{3}y$

$x \to 2x$

vertical stretch about x-axis by factor of 3, horizontal stretch about y-axis by factor of $\frac{1}{2}$
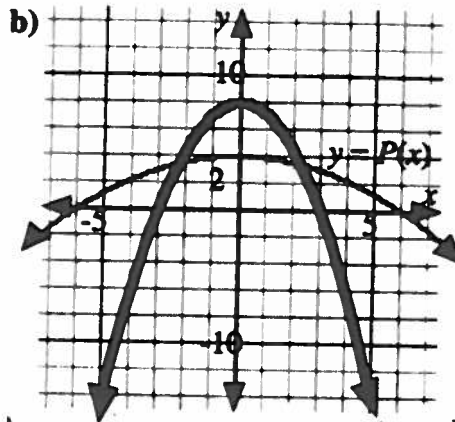
h) $y = \dfrac{1}{x+1}$, $y = -\dfrac{4}{2x+1}$

$y \to -\frac{1}{4}y$, $x \to 2x$

hor. stretch about y-axis by factor of $\frac{1}{2}$, vertical stretch about x-axis by factor of 4, reflection in x-axis.
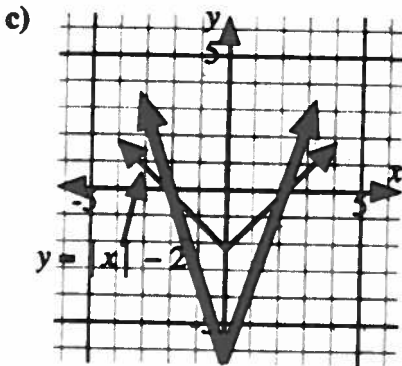
3. The function represented by the thick line is a stretch of the function represented by the thin line. Write an equation for each function represented by the thick line.

a)

$y = P(x)$

vertical stretch about x-axis by a factor of 3 $y \to \frac{1}{3}y$
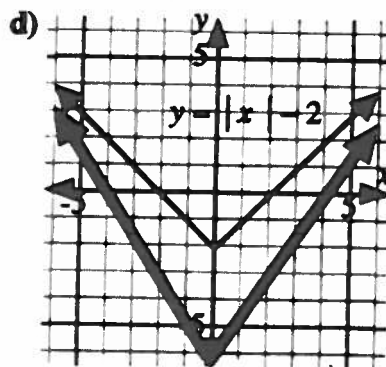
$\frac{1}{3}y = P(x)$ $y = 3P(x)$

b)

$y = P(x)$

horizontal stretch about y-axis by a factor of $\frac{1}{2}$ + a vertical stretch about x-axis by factor of 2
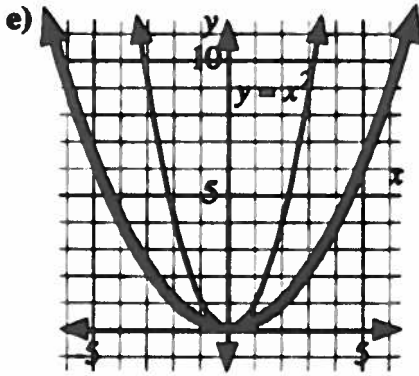
$x \to 2x$, $y \to \frac{1}{2}y$

$y = 2P(2x)$

c)

$y = |x| - 2$

vertical stretch about the x-axis by a factor of 3

$y \to \frac{1}{3}y$

$y = 3(|x| - 2)$

$y = 3|x| - 6$

d)

$y = |x| - 2$

horizontal stretch about y-axis by a factor of 2 + vertical stretch about x-axis by a factor of 3

$x \to \frac{1}{2}x$, $y \to \frac{1}{3}y$

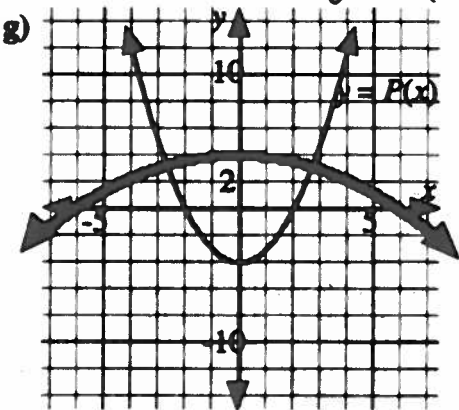$\frac{1}{3}y = |\frac{1}{2}x| - 2$ $y = 3|\frac{1}{2}x| - 6$

e)

horizontal stretch
about the y-axis by a
factor of 2 $x \rightarrow \frac{1}{2}x$

$y = \left(\frac{1}{2}x\right)^2$

or $y = \frac{1}{4}x^2$ or $y \rightarrow 4y$

vertitical stretch about x-axis
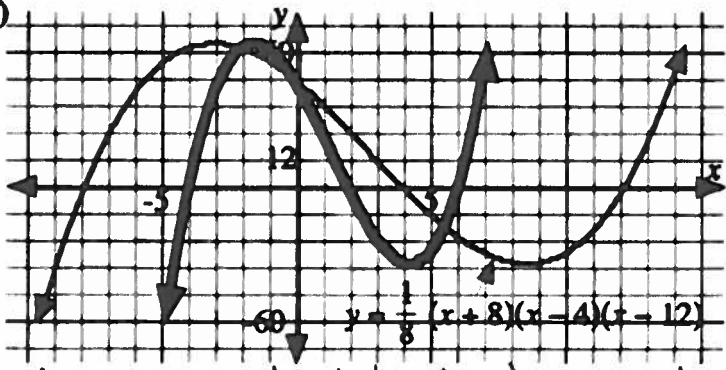by a factor of 4 $y = \frac{1}{4}x^2$

f)

horizontal stretch about y-axis by a factor
of $\frac{1}{2}$, $x \rightarrow 2x$

$y = \frac{1}{8}(2x+8)(2x-4)(2x-12)$

$y = \frac{1}{8} 2(x+4) 2(x-2) 2(x-6)$

$y = (x+4)(x-2)(x-6)$

g)

horizontal stretch about the
y-axis by a factor of 3 +
a reflection in the x-axis

$x \rightarrow \frac{1}{3}x$, $y \rightarrow -y$

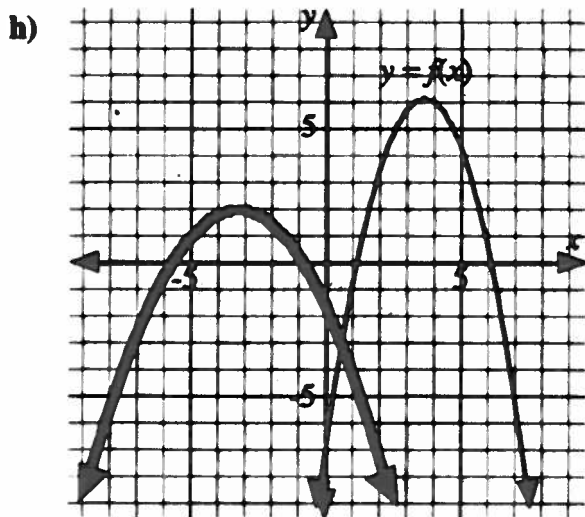$y = P\left(\frac{1}{3}x\right)$

$y = -P\left(\frac{1}{3}x\right)$

h)

vertical stretch about the x-axis
by a factor of $\frac{1}{3}$, a reflection
in the y-axis

$y \rightarrow 3y$, $x \rightarrow -x$

$3y = f(-x)$

$y = \frac{1}{3} f(-x)$

4. A polynomial function has the equation $P(x) = (x-5)(x-2)(x+1)$. Determine the zeros of the polynomial and the y-intercept of the graph of the polynomial if the following transformations are applied.

zeros of $P(x)$: $5, 2, -1$ y-int $= 10$

a) $y = 4P(x)$

$y \rightarrow \frac{1}{4}y$, $\frac{1}{4}y = P(x)$

-vert. stretch about x-axis by factor of 4, no change in x, y-int multiplied by 4

zeros: $5, 2, -1$
y-int: $10(4) = 40$

c) $y = -\frac{1}{4}P(x)$

$y \rightarrow 4y$ $y \rightarrow -y$

ver. stretch + reflection, y-int multiplied by $-\frac{1}{4}$

zeros (no change): $5, 2, -1$

y-int: $10\left(\frac{-1}{4}\right) = \frac{-5}{2}$

b) $y = P(4x)$

$x \rightarrow 4x$

-hor. stretch by factor of $\frac{1}{4}$, zeros are multiplied by $\frac{1}{4}$.

zeros: $5\left(\frac{1}{4}\right), 2\left(\frac{1}{4}\right), (-1)\left(\frac{1}{4}\right)$

$= \frac{5}{4}, \frac{1}{2}, \frac{-1}{4}$ y-int $= 10$

d) $y = P\left(-\frac{1}{4}x\right)$

$x \rightarrow -\frac{1}{4}x$, no change to y.

-horizontal stretch by factor of 4+ a reflection, zeros multiplied by -4

zeros: $5(4), 2(4), -1(4)$

zeros: $20, 8, -4$
y-int: 10

5. A polynomial function has the equation $P(x) = x(x-a)^2$. Determine the zeros of the function and the y-intercept of the graph of the function under the following transformations.

zeros of $P(x)$ $0, a$ y-int of $P(x) = 0$

a) $y = 7P(x)$

$y \rightarrow \frac{1}{7}y$

vert. stretch by factor of 7 - no change to zeros, y-int multiplied by 7

zeros: $0, a$

y-int: $0(7) = 0$

b) $y = P\left(\frac{4}{3}x\right)$ $x \rightarrow \frac{4}{3}x$

horizontal stretch by factor of $3/4$, zeros multiplied by $\frac{3}{4}$, y-in no change

zeros: $0, \frac{3}{4}a$

y-int: 0

6. How is the graph of $\frac{1}{4}y = x^2$ related to the graph of $y = x^2$?

A. $y = x^2$ has been stretched horizontally about the y-axis by a factor of 4.

B. $y = x^2$ has been stretched horizontally about the y-axis by a factor of $\frac{1}{4}$.

C. $y = x^2$ has been stretched vertically about the x-axis by a factor of 4.

D. $y = x^2$ has been stretched vertically about the x-axis by a factor of $\frac{1}{4}$.

$$y \to \frac{1}{4}y.$$

Answer Key

1. a) $y = \frac{7}{9}|x + 1|$ b) $y = 2^{\frac{1}{3}x}$ c) $y = -4\sqrt{x - 2}$ d) $y = \sqrt{-\frac{1}{4}x - 2}$

   e) $y = \frac{1}{2}\sin\frac{4}{3}x^{\circ}$ f) $y = -6x - 11$ g) $y = \frac{2}{-\frac{1}{3}x + 3}$ or $y = -\frac{6}{x - 9}$ h) $y = \frac{1}{8x} - \frac{3}{2}$

2. a) horizontal stretch about the y-axis by a factor of 3
   b) vertical stretch about the x-axis by a factor of 4
   c) horizontal stretch about the y-axis by a factor of 4 and a reflection in the y-axis
   d) vertical stretch about the x-axis by a factor of $\frac{1}{3}$
   e) horizontal stretch about the y-axis by a factor of 2
   f) vertical stretch about the x-axis by a factor of 2

   g) vertical stretch about the x-axis by a factor of 3 and a horizontal stretch about the y-axis by a factor of $\frac{1}{2}$
   h) vertical stretch about the x-axis by a factor of 4, horizontal stretch about the y-axis by a factor of $\frac{1}{2}$, and a reflection in the x-axis

3. a) $y = 3P(x)$ b) $y = 2P(2x)$ c) $y = 3|x| - 6$ d) $y = 3\left|\frac{1}{2}x\right| - $

   e) $y = \frac{1}{4}x^2$ f) $y = (x + 4)(x - 2)(x - 6)$ g) $y = -P\left(\frac{1}{3}x\right)$ h) $y = \frac{1}{3}f(-x)$

4. a) zeros: $-1, 2, 5$ b) zeros: $-\frac{1}{4}, \frac{1}{2}, \frac{5}{4}$ c) zeros: $-1, 2, 5$ d) zeros: $4, -8, -$

   y-intercept: 40   y-intercept: 10   y-intercept: $-\frac{5}{2}$ y-intercept: 10

5. a) zeros: $0, a$ y-intercept $= 0$ b) zeros: $0, \frac{3}{4}a$ y-intercept $= 0$

6.   C