# Assignment

1.  Write the replacement for $x$ or $y$ and write the equation of the image of $y = f(x)$ after each transformation.

    a)  a horizontal stretch by a factor of 3 about the y-axis

    $$x \to \tfrac{1}{3}x \qquad y = f\left(\tfrac{1}{3}x\right)$$

    b)  a vertical stretch by a factor of 6 about the x-axis

    $$y \to \tfrac{1}{6}y \qquad \tfrac{1}{6}y = f(x) \; , \; y = 6f(x)$$

    c)  a horizontal stretch about the y-axis by a factor of $\textcircled{\tfrac{5}{7}}$  $\tfrac{7}{5}$

    $$x \to \tfrac{7}{5}x \qquad y = f\left(\tfrac{7}{5}x\right)$$

    d)  a vertical stretch about the x-axis by a factor of $\tfrac{2}{3}$

    $$y \to \tfrac{3}{2}y \qquad \tfrac{3}{2}y = f(x) \qquad y = \tfrac{2}{3}f(x)$$

    e)  a reflection in the y-axis and a horizontal stretch by a factor of 3 about the y-axis

    $$\left. \begin{array}{l} x \to -x \\ x \to \tfrac{1}{3}x \end{array} \right\} \quad x \to -\tfrac{1}{3}x \qquad y = f\left(-\tfrac{1}{3}x\right)$$

    f)  a reflection in the x-axis and a vertical stretch by a factor of $\tfrac{3}{4}$ about the x-axis

    $$\left. \begin{array}{l} y \to -y \\ y \to \tfrac{4}{3}y \end{array} \right\} \, y \to -\tfrac{4}{3}y \qquad -\tfrac{4}{3}y = f(x) \qquad y = -\tfrac{3}{4}f(x)$$

    g)  a reflection in the y-axis and a horizontal stretch about the y-axis by a factor of $\tfrac{3}{4}$

    $$\left. \begin{array}{l} x \to -x \\ x \to \tfrac{4}{3}x \end{array} \right\} \, x \to -\tfrac{4}{3}x \qquad y = f\left(-\tfrac{4}{3}x\right)$$

    h)  a horizontal stretch about the y-axis by a factor of 4 and a vertical stretch about the x-axis by a factor of 4

    $$x \to \tfrac{1}{4}x \qquad \tfrac{1}{4}y = f\left(\tfrac{1}{4}x\right) \qquad y = 4f\left(\tfrac{1}{4}x\right)$$
    $$y \to \tfrac{1}{4}y$$

    i)  a horizontal stretch about the y-axis by a factor of 0.5, a vertical stretch by a factor of 2 about the x-axis and a reflection in the x-axis

    $$x \to 2x$$
    $$\left. \begin{array}{l} y \to \tfrac{1}{2}y \\ y \to -y \end{array} \right\} y \to -\tfrac{1}{2}y \qquad -\tfrac{1}{2}y = f(2x)$$
    $$y = -2f(2x)$$

2. The function $y = f(x)$ is transformed to $y = af(bx)$. Determine the values of $a$ and $b$ for:

   a) a horizontal stretch by a factor of $\frac{2}{3}$ about the y-axis

   $x \to \frac{3}{2}x \quad y = f\left(\frac{3}{2}x\right) \quad a = 1, b = \frac{3}{2}$

   b) a vertical stretch about the x-axis by a factor of 5

   $y \to \frac{1}{5}y \quad \frac{1}{5}y = f(x) \quad y = 5f(x) \quad a = 5, b = 1$

   c) a horizontal stretch about the y-axis by a factor of $\frac{5}{2}$ and a reflection in the y-axis

   $x \to \frac{2}{5}x \left. \begin{cases} \\ \end{cases} \right\} x \to -\frac{2}{5}x \quad y = f\left(\frac{-2}{5}x\right) \quad a = 1, b = -\frac{2}{5}$
   $x \to -x$

   d) a vertical stretch about the x-axis by a factor of $\frac{1}{3}$, a horizontal stretch about the y-ax

   by a factor of $\frac{1}{10}$ and a reflection in the y-axis

   $y \to 3y$
   $x \to 10x \left. \begin{cases} \\ \end{cases} \right\} x \to -10x \quad 3y = f(-10x) \quad a = \frac{1}{3}$
   $x \to -x \qquad\qquad\qquad y = \frac{1}{3}f(-10x) \quad b = -10$

3. Consider the function $f(x) = x^2$

   a) Determine the equation of the image of the function if it is stretched <u>vertically</u> by a factor of 4 about the x-axis.

   $y \to \frac{1}{4}y \quad \frac{1}{4}y = x^2 \quad y = 4x^2$

   b) Determine the equation of the image of the function if it is stretched <u>horizontally</u> by a factor of $\frac{1}{2}$ about the y-axis.

   $x \to 2x \quad y = (2x)^2 \quad y = 4x^2$

   c) What do you notice?

   $\to$ transformations both result in same equation

   d) Give an example of a function where the stretches in a) and b) would not result in the same image.

   many possible. $f(x) = x^3 \quad$ a) $\frac{1}{4}y = x^3 \quad y = 4x^3$
   $\qquad\qquad\qquad\qquad\qquad$ b) $y = (2x)^3 \quad y = 8x^3$

4. a) What information about the graph of $y = f(kx)$ does $k$ provide?

   horizontal stretch about y-axis by a factor of $\frac{1}{k}$

   b) What information about the graph of $ky = f(x)$ does $k$ provide?

   vertical stretch about x-axis by a factor of $\frac{1}{k}$

   c) What information about the graph of $y - k = f(x)$ does $k$ provide?

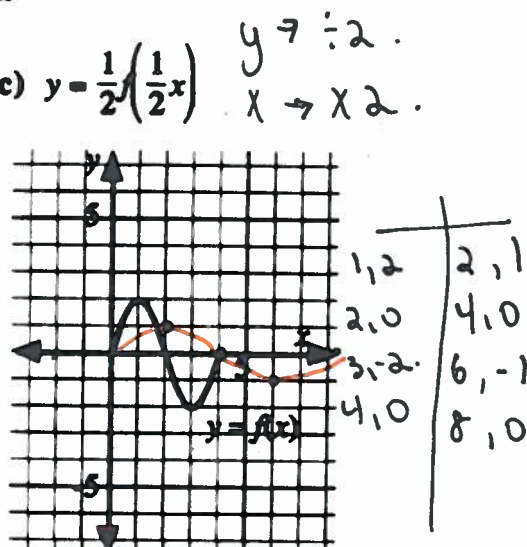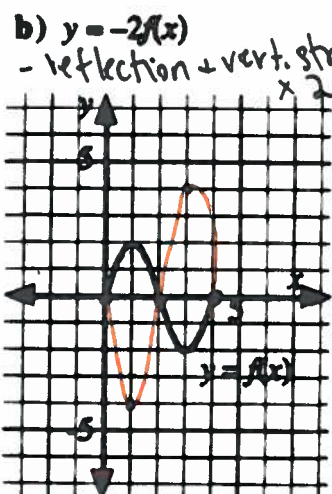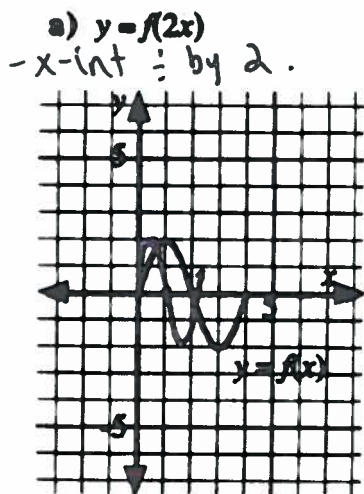   vertical translation of $k$ units:

   d) What information about the graph of $y = f(x - k)$ does $k$ provide?

   horizontal translation of $k$ units

   e) What information about the graph of $y = kf(x)$ does $k$ provide?

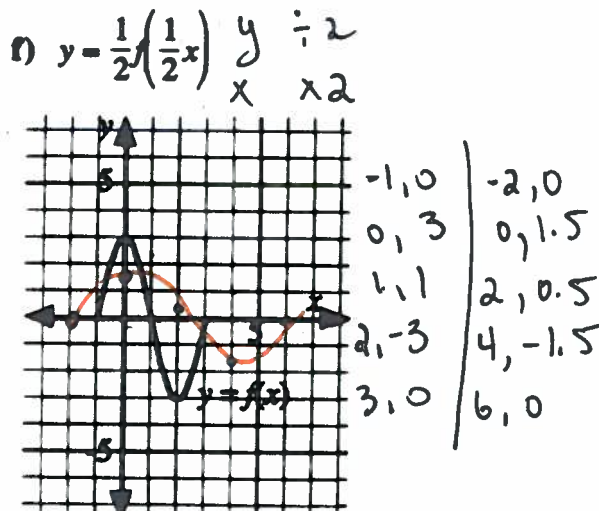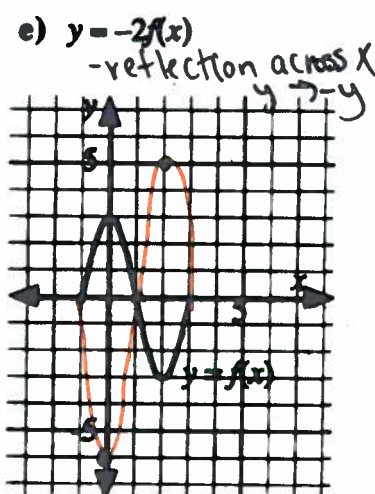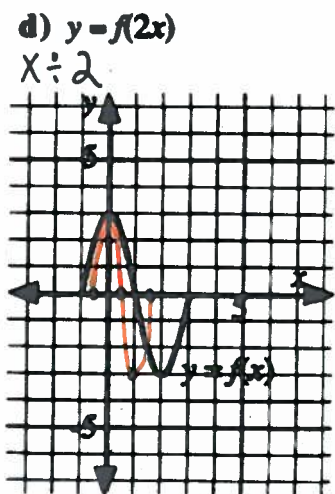   vertical stretch about x-axis by a factor of $k$.

5.  The graph of $y = f(x)$ is shown.  In each case:
    i)    sketch the graph of the transformed function
    ii)   state the domain and range of the transformed function
    iii)  state the coordinates of any invariant points. →

a) $y = f(2x)$
- x-int ÷ by 2.



i) $x | 0 \le x \le 2, x \in R$
ii) $y | -2 \le y \le 2, y \in R$.
iii) invariant points $(0,0)$

b) $y = -2f(x)$
- reflection + vert. stretch
x2.



$x | 0 \le x \le 4, x \in R$
$y | -4 \le y \le 4, y \in R$.
$(0,0)(2,0)(4,0)$

c) $y = \frac{1}{2}f\left(\frac{1}{2}x\right)$   $y \rightarrow \div 2$.
$x \rightarrow x2$.



| 1,2 | 2,1 |
| 2,0 | 4,0 |
| 3,-2 | 6,-1 |
| 4,0 | 8,0 |

$x | 0 \le x \le 8, x \in R$
$y | -1 \le y \le 1, y \in R$.
$(0,0)$

d) $y = f(2x)$
$x \div 2$



$x | -\frac{1}{2} \le x \le \frac{3}{2}, x \in R$
$y | -3 \le y \le 3, y \in R$
$(0,3)$

e) $y = -2f(x)$
- reflection across x
$y \rightarrow -y$



$y \rightarrow -y + x2$
$y2x$
$x | -1 \le x \le 3, x \in R$
$y | -6 \le y \le 6, y \in R$
$(-1,0)(1,0)(3,0)$

f) $y = \frac{1}{2}f\left(\frac{1}{2}x\right)$   $y \div 2$
$x$   $x2$



| -1,0 | -2,0 |
| 0,3 | 0,1.5 |
| 1,1 | 2,0.5 |
| 2,-3 | 4,-1.5 |
| 3,0 | 6,0 |

$x | -2 \le x \le 6, x \in R$
$y | -\frac{3}{2} \le y \le \frac{3}{2}, y \in R$

**6.** What happens to the graph of the function $y = f(x)$ if the following replacements are made

**a)** Replace $x$ with $\frac{1}{2}x$.

horizontal stretch about
y axis by a factor of 2

**b)** Replace $y$ with $4y$.

vertical stretch about x-axis
by a factor of $\frac{1}{4}$

**c)** Replace $y$ with $-2y$ and $x$ with $4x$.

horizontal stretch about y-axis
by factor of $\frac{1}{4}$, vertical
stretch about x-axis by a factor
of $\frac{1}{2}$ + reflection in x-axis.

**d)** Replace $y$ with $y - 4$ and $x$ with $-\frac{1}{4}x$.

horizontal stretch about y-axis
by a factor of 4, reflection in
y-axis + vertical translation
4 units up.

**7.** The graph of $y = f(x)$ is stretched vertically by a factor of $\frac{1}{2}$ about the x-axis, stretched horizontally by a factor of $\frac{1}{4}$ about the y-axis, and reflected in the y-axis. If the equation the image is written in the form $y = af(bx)$, the value of $a - b$, to the nearest tenth, is ____

(Record your answer in the numerical response box from left to right.)

| 4 | . | 5 | |

$y \to 2y$

$\left. \begin{array}{l} x \to 4x \\ x \to -x \end{array} \right\} x \to -4x$

$2y = f(-4x)$

$y = \frac{1}{2}f(-4x)$

$a = \frac{1}{2}, \quad b = -4$

$a - b = \frac{1}{2} - (-4) = \frac{9}{2} = 4.5$

**Answer Key**

**1. a)** $x \to \frac{1}{3}x, \ y = f\left(\frac{1}{3}x\right)$    **b)** $y \to \frac{1}{6}y, \ y = 6f(x)$    **c)** $x \to \frac{7}{5}x, \ y = f\left(\frac{7}{5}x\right)$

**d)** $y \to \frac{3}{2}y, \quad y = \frac{2}{3}f(x)$    **e)** $x \to -\frac{1}{3}x, \ y = f\left(-\frac{1}{3}x\right)$    **f)** $y \to -\frac{4}{3}y, \ y = -\frac{3}{4}f(x)$

**g)** $x \to -\frac{4}{3}x, \ y = f\left(-\frac{4}{3}x\right)$    **h)** $x \to \frac{1}{4}x$ and $y \to \frac{1}{4}y, \ y = 4f\left(\frac{1}{4}x\right)$

**i)** $x \to 2x$ and $y \to -\frac{1}{2}y, \quad y = -2f(2x)$

**2. a)** $a = 1 \quad b = \frac{3}{2}$    **b)** $a = 5 \quad b = 1$    **c)** $a = 1 \quad b = -\frac{2}{5}$    **d)** $a = \frac{1}{3} \quad b = -10$

**3. a)** $y = 4f(x) = 4x^2$    **b)** $y = (2x)^2 = 4x^2$    **c)** Both transformations result in the same image.
**d)** many possible answers including $f(x) = x, f(x) = x^3, f(x) = x^2 + 1$, etc.