

Assignment

1. Describe how the graph of the second function compares to the graph of the first function

a) $y = x^3$

$y = x^3 - 1$
 $x + 1 = x^3$
 $y \rightarrow y + 1$
 vertical translation
 1 unit down

b) $y = 7x - 1$

$y = 7(x - 3) - 1$
 $x \rightarrow x - 3$
 horizontal translation
 3 units right

c) $y = \cos x^\circ$

$y = \cos(x + 45)^\circ$
 $x \rightarrow x + 45$
 horizontal translation
 45° left.

d) $y = |x|$

$y + 3 = |x + 6|$
 $x \rightarrow x + 6$
 $y \rightarrow y + 3$
 translation 6 units left
 and 3 units down.

e) $y = \frac{1}{x^2 + 1}$
 $y - 2 = \frac{1}{(x - 3)^2 + 1}$

$x \rightarrow x - 3$,
 $y \rightarrow y - 2$
 translation 3 units right
 & 2 units up

f) $y = a^x$

$y - 1 = a^{x+1}$
 $x \rightarrow x + 1$
 $y \rightarrow y - 1$
 translation 1 unit
 left & 1 unit up.

2. Write the equation of the image of:

a) $y = x^4$ after a horizontal translation of 2 units to the left.

$x \rightarrow x + 2$ $y = (x + 2)^4$

b) $y = 2|x|$ after a translation of 3 units down and 1 unit left.

$x \rightarrow x + 1$, $y \rightarrow y + 3$ $y + 3 = 2|x + 1|$ or $y = 2|x + 1| - 3$

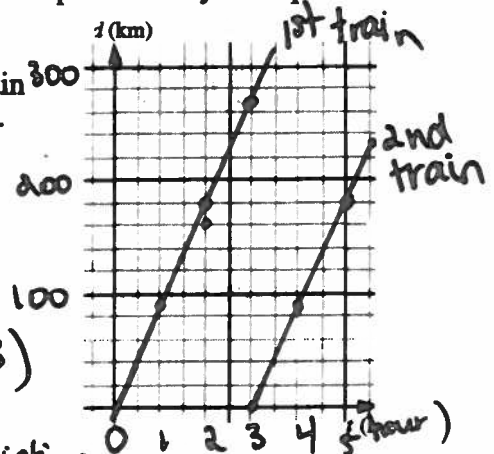
c) $y = \frac{1}{\sqrt{x}}$ after a horizontal translation of 3 units to the right
 and a vertical translation of 2 units up.

$x \rightarrow x - 3$ $y - 2 = \frac{1}{\sqrt{x - 3}}$ or $y = \frac{1}{\sqrt{x - 3}} + 2$

3. On a certain route, trains travel at an average speed of 90 km/h. The distance, d kilometres they travel can be described as a function of time, t hours, and represented by the equation $d = f(t) = 90t$.

A train leaves the station at 12:00 p.m. ($t = 0$). A second train travels with the same average speed, but leaves 3 hours later.

a) Sketch a distance time graph for each train on the grid.



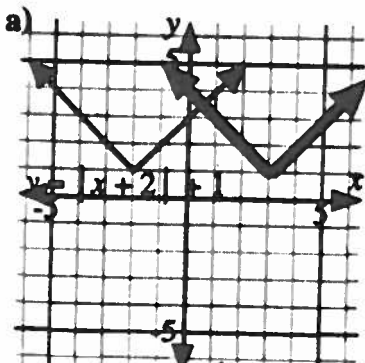
b) Explain why the equation that describes the distance travelled by the second train is $d = 90(t - 3)$, $t \geq 3$.

- graph for 2nd train is a horizontal translation 3 units to right ($t \rightarrow t - 3$) of the graph of 1st train so

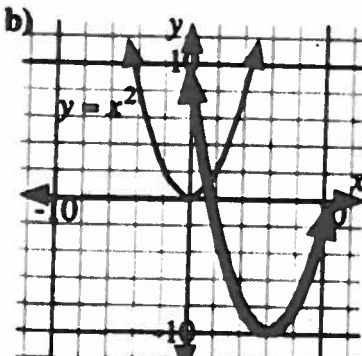
$d = 90(t - 3)$. Domain restriction

$t \geq 3$ is given because 2nd train does not leave until 3pm when $t = 3$.

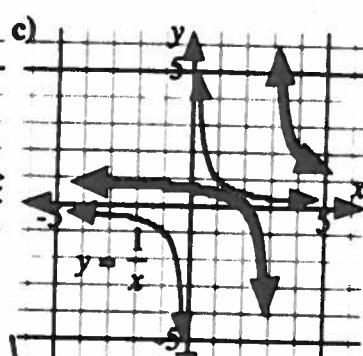
4. The function represented by the thick line is a transformation of the function represented by the thin line. Write an equation for each function represented by the thick line.



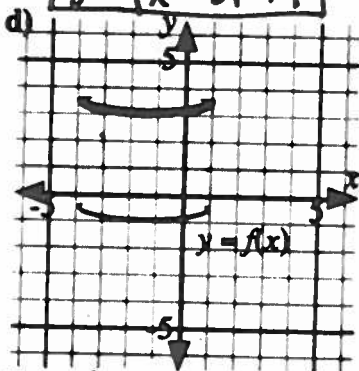
5 units right $x \rightarrow x-5$
 $y = |x-5| + 1$
 $y = |x-3| + 1$



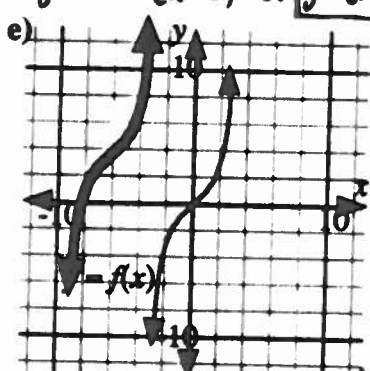
6 units left + 10 units down
 $x \rightarrow x-6$ $y \rightarrow y+10$
 $y+10 = (x-6)^2$ or $y = (x-6)^2 - 10$



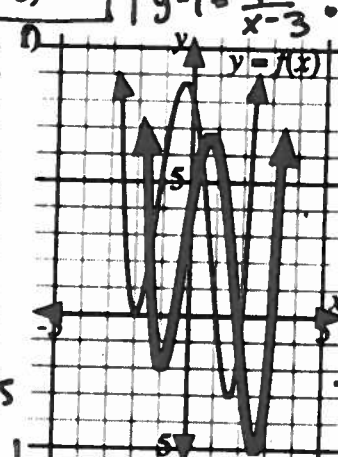
3 units right + 1 unit up.
 $x \rightarrow x-3$, $y \rightarrow y-1$
 $y-1 = \frac{1}{x-3}$ or $y = \frac{1}{x-3} + 1$



4 units up
 $y \rightarrow y-4$
 $y-4 = f(x)$
 $y = f(x) + 4$



6 units left and 4 units up
 $x \rightarrow x+6$
 $y \rightarrow y-4$
 $y-4 = f(x+6)$ or $y = f(x+6) + 4$



1 unit right + 2 units down
 $x \rightarrow x-1$, $y \rightarrow y+2$
 $y+2 = f(x-1)$ or $y = f(x-1) - 2$

5. a) What vertical translation would be applied to $y = x^2$ so that the translation image passes through (3, 5)?

$y = y - k$ $y - k = x^2$
 replace (3, 5) $5 - k = 3^2$
 $5 - k = 9$
 $k = -4$
 $y \rightarrow y + 4$
 vertical translation 4 units down.

b) What horizontal translation would be applied to $y = x^3 + 1$ so that the translation image passes through (5, 28)?

$x \rightarrow x - h$
 $y = (x-h)^3 + 1$
 replace (5, 28) $28 = (5-h)^3 + 1$
 $(5-h)^3 = 27$
 $5-h = 3$
 $h = 2$
 $x \rightarrow x-2$
 horizontal translation 2 units right.

c) What horizontal translation would be applied to $y = \frac{1}{x-3}$ so that the translation image passes through $(1, \frac{1}{2})$?

$x \rightarrow x - h$
 $y = \frac{1}{x-h-3}$
 replace $(1, \frac{1}{2})$
 $\frac{1}{2} = \frac{1}{1-h-3}$
 $1-h-3 = 2$
 $h = -4$
 $x \rightarrow x - -4$
 $x \rightarrow x + 4$
 horizontal translation 4 units left.

6. On a certain route into town, shuttle buses depart every $\frac{15}{60}$ minutes from 06:30 until 07:30. The distance, d kilometres, they travel can be described as a function of time, t hours, and represented by the equation $d = f(t) = 60t$.

If $t = 0$ at 06:30, write an equation which represents the distance travelled by

- a) the second bus
 trans. $\frac{1}{4}$ hr. right
 $t \rightarrow t - \frac{1}{4}$
 $d = 60(t - \frac{1}{4})$
- b) the third bus
 trans. $\frac{1}{2}$ hr. right
 $t \rightarrow t - \frac{1}{2}$
 $d = 60(t - \frac{1}{2})$
- c) the last bus \rightarrow @ 7:30 so 1 hr.
 trans. 1 hr. right
 $t \rightarrow t - 1$
 $d = 60(t - 1)$

Multiple Choice

7. The graph of the function $y = f(x)$ passes through the point $(4, 7)$. Under a transformation, the point $(4, 7)$ is transformed to $(6, 6)$. A possible equation for the transformed function is

- A. $y - 1 = f(x + 2)$
 B. $y - 2 = f(x + 1)$
 C. $y + 1 = f(x - 2)$
 D. $y + 2 = f(x - 1)$

x is increased by 2 \rightarrow translation 2 units right
 y is decreased by 1 \rightarrow translation 1 unit down
 $x \rightarrow x - 2$
 $y \rightarrow y + 1$
 $y + 1 = f(x - 2)$

8. If $f(x) = 4 - x^2$, then the transformation represented by $f(x + 6) - 3$ will change

- A. the domain but not the range of $f(x)$
 B. the range but not the domain of $f(x)$
 C. neither the domain nor the range of $f(x)$
 D. both the domain and the range of $f(x)$

original domain $x \in \mathbb{R}$
 original range $4 - x^2 \leq 4, y \in \mathbb{R}$

original $y = 4 - x^2$
 transformed $y = 4 - (x + 6)^2 - 3$
 or $y + 3 = 4 - (x + 6)^2$
 $x \rightarrow x + 6$ - 6 units left
 $y \rightarrow y + 3$ - 3 unit down.

Numerical Response

9. The function $f(x) = \sqrt{x} + 5$ is transformed by a translation of 2 units down and 4 units to the left. The transformed function passes through the point $(20, y)$. To the nearest tenth, the value of y is _____.

(Record your answer in the numerical response box from left to right.)

7.9

4 left $x \rightarrow x + 4$
 2 down $y \rightarrow y + 2$
 $y + 2 = \sqrt{x + 4} + 5$
 $y + 2 = \sqrt{20 + 4} + 5$
 $y = 7.9$

replace $(20, y)$

10. The function $R(x) = \frac{1}{x+3}$ is transformed by a translation of 3 units up and 5 units to the right. The transformed function passes through the point $(x, 7)$.

The value of x to the nearest hundredth is _____.

(Record your answer in the numerical response box from left to right.)

2	.	2	5
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$y = \frac{1}{x+3}$
 5 units right $x \rightarrow x-5$
 3 units up $y \rightarrow y-3$
 $y-3 = \frac{1}{(x-5)+3}$
 $y-3 = \frac{1}{x-2}$

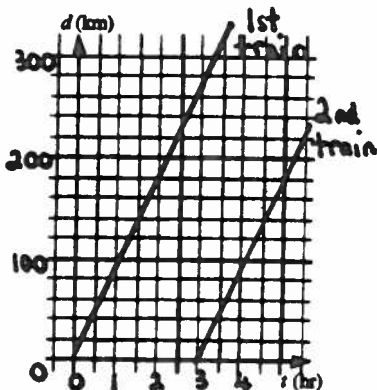
→ replace $(x, 7)$
 $7-3 = \frac{1}{x-2}$
 $4 = \frac{1}{x-2}$
 $4(x-2) = 1$
 $4x-8 = 1$
 $4x = 9$
 $x = 2.25$

Answer Key

1. a) vertical translation 1 unit down
 b) horizontal translation 3 units right
 c) horizontal translation 45° left
 d) translation 6 units left and 3 units down
 e) translation 3 units right and 2 units up
 f) translation 1 unit left and 1 unit up

2. a) $y = (x+2)^4$ b) $y = 2|x+1| - 3$ c) $y = \frac{1}{\sqrt{x-3}} + 2$

3. a) See graph below
 b) Compared to the graph of the first train, the graph for the second train is a horizontal translation three units to the right. t is replaced by $t-3$, to give the equation $d = 90(t-3)$. The domain restriction $t \geq 3$ is given because the second train does not leave until 3:00 pm when $t = 3$.



4. a) $y = |x-3| + 1$ b) $y = (x-6)^2 - 10$ c) $y = \frac{1}{x-3} + 1$
 d) $y = -f(x) + 4$ e) $y = f(x+6) + 4$ f) $y = f(x-1) - 2$

5. a) vertical translation 4 units down
 b) horizontal translation 2 units right
 c) horizontal translation 4 units left

6. a) $d = 60\left(t - \frac{1}{4}\right)$ b) $d = 60\left(t - \frac{1}{2}\right)$ c) $d = 60(t-1)$

7. C

8. B

9.

7	.	9	
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10.

2	.	2	5
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