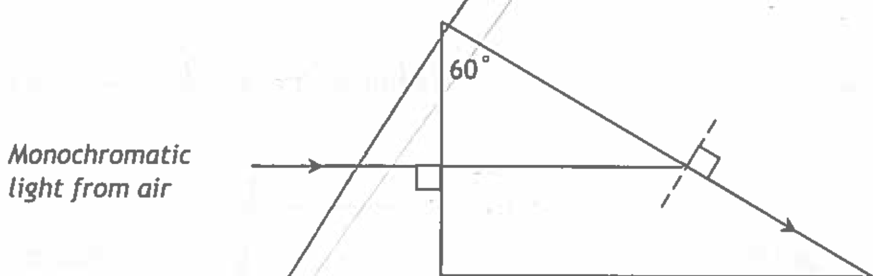
Use the information below to answer question 26.

Honey that contains too much water will easily spoil. The moisture content may be determined using the honey's refractive index. The refractive index is proportional to the moisture content as shown in the chart.

Refractive index at 20 °C	Moisture content (%)
1.5044	13.0
1.5028	13.6
1.5018	14.0
1.5002	14.6
1.4992	15.0
1.4976	15.6
1.4966	16.0
1.4951	16.6
1.4940	17.0

26. Determine the percent moisture in honey which shows a critical angle of 41.819° with air. Use a refractive index of 1.0003 for air. [Appendix A]

27. A beam of monochromatic light strikes a prism at a right angle as shown in the diagram below. A critical angle is obtained as the light travels from the prism to the air. Determine the refractive index of the prism. [1.15]



5.4 Summary Notes - Lenses

Notes:

Characteristics of an image: An object placed in front of a lens may produce an image that has different characteristics than the object. The characteristics are:

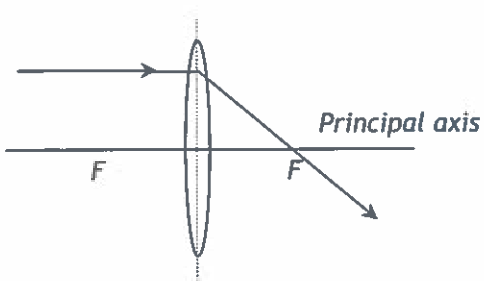
- **Size:** Larger, smaller, or same size as the original object (magnification is an expression of relative size)
- **Orientation:** inverted or upright (erect)
- **Image type:** real or virtual.

A **real image** is an image that can be projected onto a screen (light rays converge at one point). Real images are always inverted.

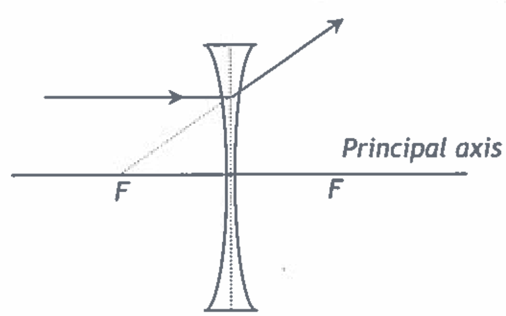
A **virtual image** cannot be projected onto a screen. You must look through the lens, or in the mirror to see a virtual image. Virtual images are always upright.

optical centre (*O*): The centre of the lens.
principal axis: A line drawn through the vertex and the optical centre.
focal point (*F*): The point where all the light rays meet.
focal length (*f*): The distance from the lens to the focal point. It is dependent on the refractive index of the material and the shape of the lens.

Convex Lens (a.k.a., converging lens)



Concave Lens: (a.k.a., diverging lens)



- All concave lenses produce images that are smaller, upright and virtual.
- Concave lenses have a negative focal length.

To draw ray diagrams:

> i. draw the lens showing the principal axis and the principal focal points.
> ii. place a vertical arrow on the principal axis to indicate the position, size and attitude of the object.
> iii. draw two rays from the tip of the arrow to the lens. Where these two rays meet (real image) or appear to diverge from (virtual image) is the position of the image. Any two of the following three rays may be used:
>   - A ray coming in parallel to the principal axis emerges towards the focus.
>   - A ray coming through the focus emerges parallel to the principal axis.
>   - A ray passing through the optical centre is left unchanged.

The characteristics of an image in a lens may be determined algebraically:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Where:

d_o is the distance from the object to the lens

d_i is the distance from the image to the lens

f is the focal length of the lens

Notes:

Sign Convention:

Distance (d)		Height (h)	
real focal points or images	+	upright images	+
virtual focal points or images	-	inverted images	-

Magnification

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

N.B.,

- All real images are inverted and all virtual images are upright.
- Any units may be used as long as they are consistent.
- The negative sign is needed due to sign convention.
- A concave lens has a negative focal length.

Examples

real image on opposite side
(+) distance to image

1. An object 3.0 cm high is 15 cm from a converging lens of 6.0 cm focal length. Complete the chart.

1 div. = 1.0 cm

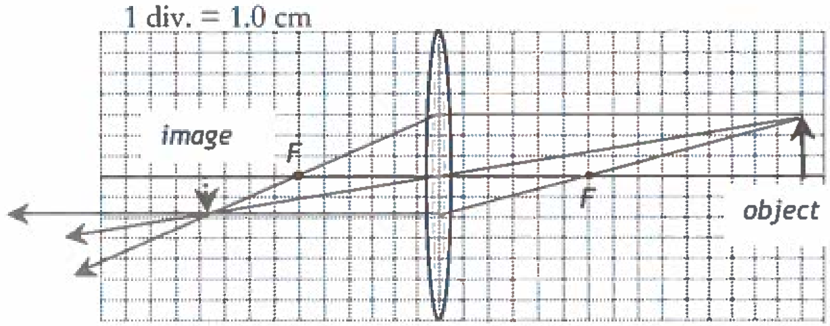


Image Dimensions

	Graphically	Algebraically
d_i	9.6 cm	10 cm
h_i	-1.8 cm	-2.0 cm
M	0.60 X	0.67 X

Characteristics
inverted
smaller
real

$h_o = 3.0\text{cm}$
$d_o = 15\text{cm}$
$f = 6\text{cm}$
$d_i = ?$
$h_i = ?$

$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$
$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$
$\frac{1}{d_i} = \frac{1}{6\text{cm}} - \frac{1}{15\text{cm}}$
$d_i = 10\text{cm}$

$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$
$h_i = -\frac{d_i h_o}{d_o}$
$h_i = \frac{-10\text{cm} * 3\text{cm}}{15\text{cm}}$
$h_i = -2\text{cm}$

2. A 6.0 cm tall object is 11.0 cm from a diverging lens of 4.0 cm focal length. Complete the chart.

image on same side d_i is −

1 div. = 1.0 cm

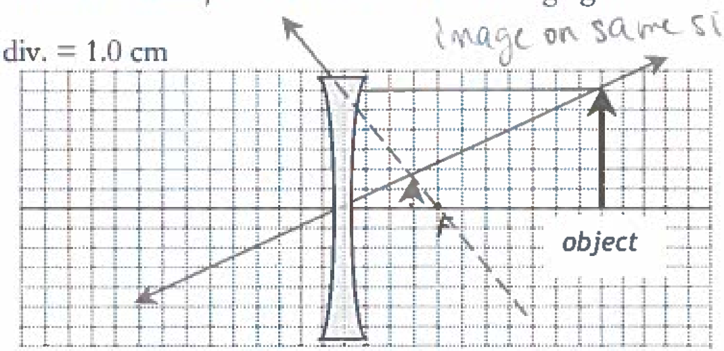


Image Dimensions

	Graphically	Algebraically
d_i	2.9 cm	2.9 cm
h_i	1.4 cm	1.6 cm

Characteristics
upright
smaller
virtual

$h_o = 6\text{cm}$
$d_o = 11\text{cm}$
$f = 4\text{cm}$
$d_i = ?$
$h_i = ?$

$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$
$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$
$\frac{1}{d_i} = \frac{1}{-4\text{cm}} - \frac{1}{11\text{cm}}$
$d_i = -2.933\text{cm}$

$\frac{h_i}{h_o} = -\frac{d_i}{d_o}$
$h_i = -\frac{d_i h_o}{d_o}$
$h_i = \frac{2.93\text{cm} * 6\text{cm}}{11\text{cm}}$
$h_i = 1.6\text{cm}$

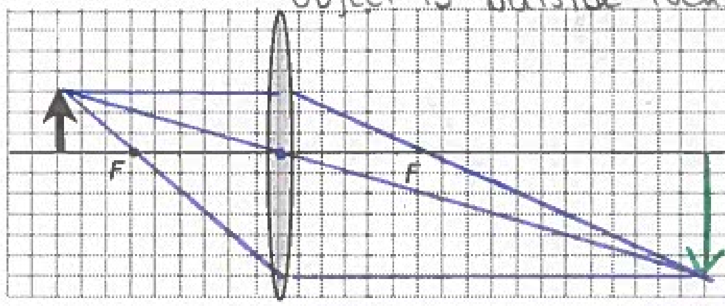
$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$ $m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$

Problems

*looking for intersection

1. Determine the dimensions and characteristics by drawing ray diagrams for the following objects placed in front of converging and diverging lenses. Assume 1 division = 1.0 cm. Complete the charts. [Appendix A]

a. - object is outside focus



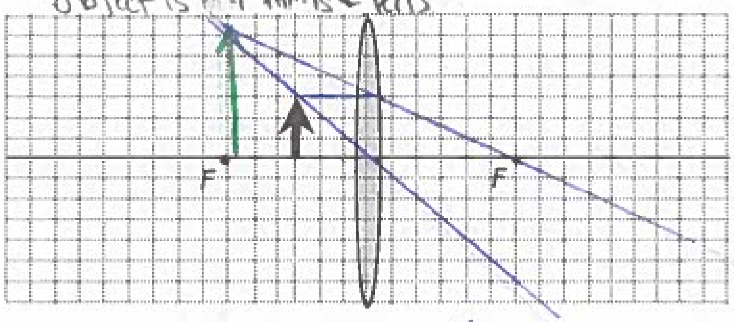
Characteristics
larger
inverted
real

Dimensions
d_i = 18 cm
h_i = -6

$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$ $\frac{1}{d_i} = \frac{1}{6} - \frac{1}{9}$; $d_i = 18$

$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$ $\frac{h_i}{3} = -18/9 = -6$

b. object is b/w focus + lens



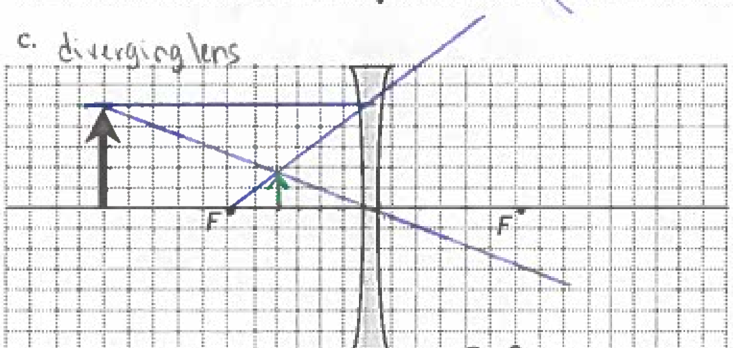
Characteristics
larger
upright
virtual

Dimensions
d_i = -5.8
h_i = 6

$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$ $\frac{1}{d_i} = \frac{1}{6} - \frac{1}{3} = -6$

$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$ $\frac{h_i}{3} = \frac{-(-6)}{3} = 6$

c. diverging lens



*a concave lens has a ⊖ focal length.

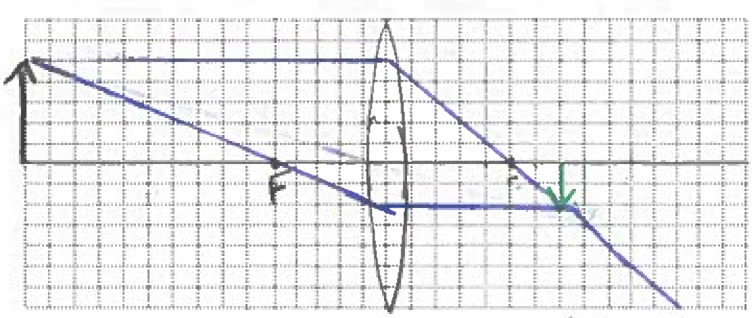
Characteristics
smaller
upright
virtual

Dimensions
d_i = -4
h_i = 1.8

$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$ $= \frac{1}{-6} - \frac{1}{11} = -3.9$

$\frac{h_i}{h_o} = \frac{-d_i}{d_o}$ $\frac{h_i}{5} = \frac{-(-4)}{11.25} = 1.8$

2. Use a ray diagram to determine the characteristics and the size and position of an image projected by a 10.0 cm object, 30.0 cm from a thin positive (converging) lens with a 10.0 cm focal length. [Appendix A]



1 div = 2cm

Characteristics
smaller
inverted
real

Dimensions
d_i = 14 cm
h_i = -4 cm

3. Fill in the blanks. A real image is an image that can be projected onto a screen. A virtual image can only be viewed when looking through a lens. [Appendix A]

4. A 15 cm tall object is 30 cm away from a diverging lens that has a focal length of 5.0 cm. (Hint: a concave lens has a negative focal length.) Determine the
   a. image's distance from the lens. [-4.3 cm]
   b. image size. [2.1 cm]
   c. image magnification. [0.14 X]

$h_o = 15cm$
$d_o = 30cm$
$f = -5cm$
$d_i =$
h_i
$m =$

a) $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$

$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$

$\frac{1}{d_i} = \frac{1}{-5} - \frac{1}{30}$

$d_i = -4.29$

$d_i = -4.3cm$

b) $\frac{h_i}{h_o} = \frac{-d_i}{d_o}$

$\frac{h_i}{15} = \frac{-(-4.29)}{30}$

$h_i = 2.14$

$h_i = 2.1cm$

c) $m = \frac{h_i}{h_o}$

$= \frac{2.14}{15}$

$m = 0.14X$

5. An old camera allows light to reflect off of the object it focuses on and then refract through its lens onto sensitive film. Consider a 3.5 m tall object that is placed 6.5 m from the converging lens that allows a 1.1 cm tall real image to be exposed onto the film inside the camera. Determine the
   1.1×10^{-2} → inverted so $-1.1cm$
   a. distance to the image. [2.0 cm]
   b. focal length of the lens. [2.0 cm]

$h_o = 3.5m$
$d_o = 6.5m$
$f =$
$d_i =$
$h_i = -1.1 \times 10^{-2}$

a) $\frac{h_i}{h_o} = \frac{-d_i}{d_o}$

$-d_i = \frac{-h_i d_o}{h_o}$

$= \frac{-(-1.1 \times 10^{-2})(6.5)}{3.5} = 0.02$

$d_i = 2.0cm$

b) $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$

$\frac{1}{f} = \frac{1}{0.02} + \frac{1}{6.5}$

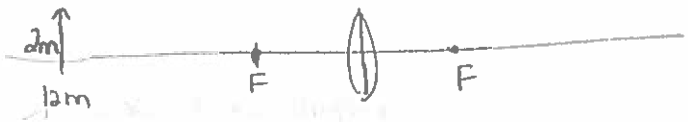
$f = 0.02036 m$

$f = 2.0cm$

6. The cornea of the eye acts as a converging lens. An eye with a radius of curvature of 8.00 mm observes an object from a distance of 45 cm.
   a. Determine the characteristics of the image that forms on the back of the eye. [Appendix A]
   b. What must the brain do to correctly interpret the image? [Appendix A]

a) Since the object is beyond the focal point the image is real, smaller and inverted

b) the brain must invert the image so we see it right side up.



7. A 2.0 m tall object is 12.0 m from a convex lens that has a 4.0 m focal length. Determine the
   a. image's characteristics. [Appendix A]
   b. image distance from the lens. [6.0 m]
   c. image size. [-1.0 m]

a) image characteristics are real, smaller, + inverted.

$h_o = 2m$, $d_o = 12m$, $f = 4m$, $d_i =$, $h_i =$

b) $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$

$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$

$\frac{1}{d_i} = \frac{1}{4} - \frac{1}{12}$

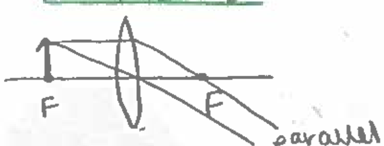
$d_i = 6.0m$

c) $\frac{h_i}{h_o} = \frac{-d_i}{d_o}$

$\frac{h_i}{2} = \frac{-6}{12}$

$h_i = -1.0m$

8. A 25 cm tall object is 50 cm from a convex lens with a focal point that is twice the distance of the object's height. Will there be an image visible? Explain. [Appendix A]



No - the object is located at the lens' focal point. This means the light rays will not converge to form an image.

9. A 6.5 cm high object is 22.0 cm in front of a lens with a -10.5 cm focal length. Determine the image's
   a. characteristics. [Appendix A]
   b. distance from the lens. [-7.1 cm]
   c. size. [2.1 cm]

a) image is virtual, upright + smaller

$h_o = 6.5cm$, $d_o = 22.0cm$, $f = -10.5$, $d_i =$, $h_i =$

b) $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$

$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$

$\frac{1}{d_i} = \frac{1}{-10.5} - \frac{1}{22}$

$d_i = -7.1cm$

c) $\frac{h_i}{h_o} = \frac{-d_i}{d_o}$

$\frac{h_i}{6.5} = \frac{-(-7.1)}{22.0}$

$h_i = 2.0977$

$h_i = 2.1cm$

10. In an experiment a student inserts a concave lens into an old camera. Describe what the student would observe if an object 8.5 m away from the lens is brought into focus considering the lens has a focal length of 4.0 cm. [Appendix A]

- the student would observe a smaller virtual image of the object while looking into the lens
- the student could not take a picture of it as the light diverges rather than converges onto the location of the film.

11. Reading glasses are often sold using a unit called "dioptre". The larger the dioptre number the stronger or more powerful the reading glasses. The dioptre is the inverse of the focal length. Determine the dioptre of reading glasses having a focal length of 0.67 m. [1.5]

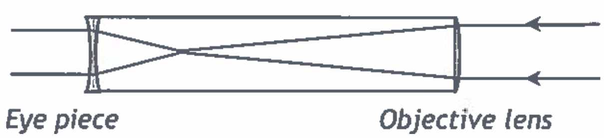
dioptre = $\frac{1}{f}$ = $\frac{1}{0.67}$

Dioptre = 1.5

Use the information below to answer question 12.

While Galileo never invented the telescope he was one of the first to use it to observe the night sky. One of his telescopes used two lenses (a convex and concave) to produce a magnification of 20 X. When using it to observe the Moon he noted that it was "marked here and there with chains of mountains and depths of valleys."

12. Identify the type of lens used as the objective lens. [Appendix A]

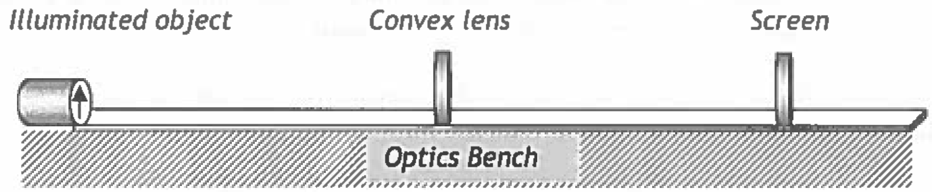


converging

13. The refractive index for water is 1.33 while the refractive index for the cornea in the eye is 1.37. Explain why this makes focusing underwater without the aid of goggles difficult. [Appendix A]

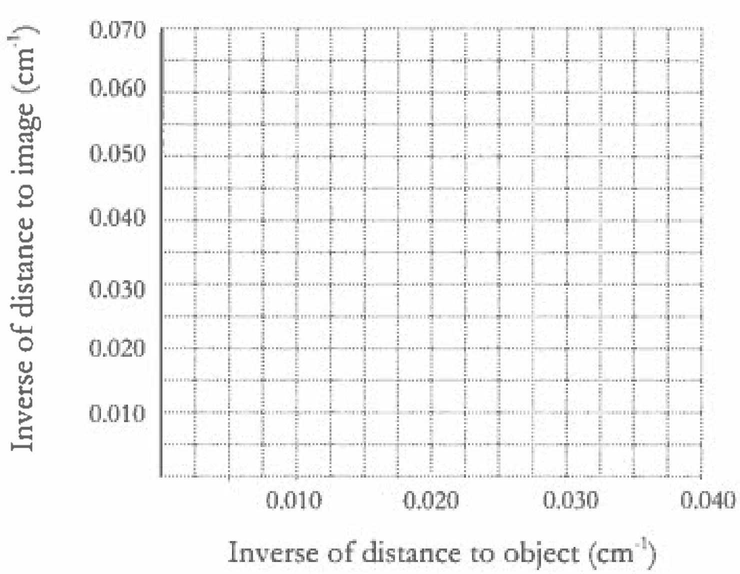
The refractive index of the water and the lens + cornea are so similar that light does not bend enough to focus on the back of the eye.

14. Zoë conducted an experiment to determine the focal length of a convex lens. She used an optics bench to illuminate an object and project a real image of it onto a screen and recorded the data in the chart below:



Object distance (cm)	Inverse of distance to object (cm^{-1})	Image distance (cm)	Inverse of distance to image (cm^{-1})
25.0		100.0	
27.0		77.1	
31.0		56.4	
35.0		46.7	
42.0		38.2	

a. Complete the chart above.
b. Graph the data to produce a straight line.
c. Use the graph to determine the focal length of the convex lens. [~19 cm]



Use the information below to answer question 15.

The Italian, Giambattista della Porta wrote the first systematic work on lenses, *Magia Naturalis* (Natural Magic), in 1589. In it he designs an optical system composed of two lenses. Multi-lens systems, such as Giambattista's, make up telescopes and microscopes. Janssen of Holland, in 1590, is credited with actually building the first compound microscope.

5.4a

15. A certain compound microscope has two convex lenses that are 6.0 cm apart. The objective lens has a focal length of 1.0 cm, and the eyepiece has a focal length of 7.0 cm. The object under study is placed 1.5 cm from the objective lens as shown in the diagram below where 2 division = 1.0 cm. Determine the microscope's magnifying power. [~3.5 X]