Name: Key

Math 20-1

Quadratic Functions and Equations

Assignment 7: Applications of Quadratic Functions

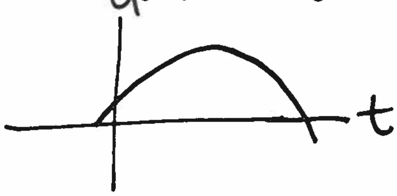
1. A football punted during a high school football game followed a bath of a parabola. The path can be modelled by the function:

$$d(t) = -5t^2 + 15t + 1, \qquad t \geq 0$$

where t is the number of seconds which have elapsed since the football was punted, and $d(t)$ is the number of metres above ground after t seconds.

Answer the questions to the nearest hundredth of a unit (where necessary) using a graphical approach.

a. Sketch the graph of the grid.



X:[0, 10, 1]

y [-10, 15, 1]

b. What was the height of the football above the ground as the punter made contact with the football?

value function $t=0,\ d=1$ height = 1m

c. What was the height of the football above the ground one second after contact?

value function $X=1$? $y = 11$ metres.

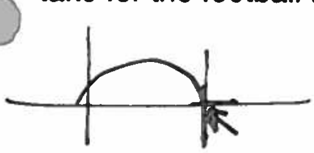
d. What is the maximum height reached by the football? What relation does this have to the vertex of the parabola? y-coor: of vertex (max. function)

max = 12.25m

e. How many seconds had elapsed when the football reached its maximum height? What relation does this have to the vertex?

x-coord. of vertex = 1.5 sec.

f. The punt was not fielded by the opposition and the football hit the ground. How many seconds did it take for the football to hit the ground?

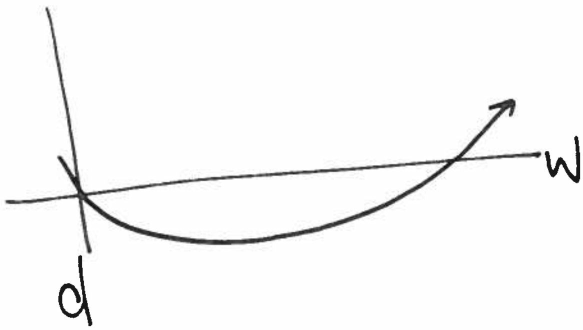


3.07 sec.

zero function

2. The cross-section of a river, from one bank to the other, can be represented by the function $d(w) = \frac{1}{14}w^2 - \frac{5}{7}w$ where $d(w)$ is the depth, in metres, of the river w metres from the left edge of the river bank. Use a graphical approach to answer the following questions.

a. Sketch the graph of the cross-section of the river.



x : [-5, 15, 1]

y : [-10, 5, 1]

b. Determine the depth of the river 3 metres from the left edge.

value function X = 3 , depth = 1.5m

c. What is the maximum depth of the river, to the nearest hundredth of a metre?

min function → y - coor. of vertex

= 1.79 m

d. How far from the left edge or the river, to the nearest tenth of a metre, is the deepest part of the river?

X - coor. of vertex = 5.0m.

e. What is the width of the river to the nearest tenth of a metre?

zero function , X-int = 10.0m.

3. At the local golf course, on the par 3, eight hole, Linda used a 7 iron to reach the green. Her golf ball followed the path of a parabola, approximated by the function $h(t) = -5t^2 + 25t + 0.05$ where t is the number of seconds which have elapsed since Linda hit the ball, and $h(t)$ is the height in metres, of the ball above the ground after t seconds. Answer the following questions algebraically.

a. Write the function in standard form. (HINT: Complete the square…yippee, say Math is fun, say it)

$h(t) = -5(t^2 - 5t\ \ ) + 0.05$

$= -5(t^2 - 5t + 6.25 - 6.25) + 0.05$

$= -5(t - 2.5)^2 + 31.25 + 0.05$

$h(t) = -5(t - 2.5)^2 + 31.3$.

b. Find the height of the golf ball 2 seconds after the ball is hit.

Value function $x = 2$, $h = 30.05\,m$.

or

$= -5(2 - 2.5)^2 + 31.3$

c. Find the maximum height reached by the golf ball.

31.3m (from formula)

d. How many seconds did it take for the gold ball to reach its maximum height?

2.5 sec .

e. How high, in centimetres, did Linda tee up her golf ball before she hit?

$h(t) = -5t^2 + 25t + \underline{0.05}$

0.05m.

f. How long, to the nearest tenth of a second, did it take for the golf ball to hit the ground?

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-25 \pm \sqrt{25^2 - 4(-5)(0.05)}}{2(-5)}$

$= \dfrac{-25 \pm \sqrt{626}}{-10} = -0.00199$ or $\underline{5.00199}$

5.0 sec .

4. The sum of a number x and its reciprocal is $\frac{29}{10}$. Form an equation and find the original number.

$$10x\left(x + \frac{1}{x} = \frac{29}{10}\right) \qquad LCD = 10x$$

$$10x^2 + 10 = 29x$$

$$10x^2 - 29x + 10 = 0$$

$$\begin{array}{c|c} \times & + \\ \hline 100 & -29 \end{array}$$

$(-25, -4)$

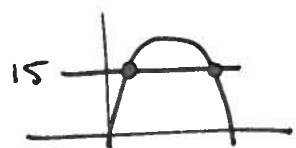
$$10x^2 - 25x - 4x + 10 = 0$$

$$5x(2x-5) - 2(2x-5) = 0$$

$$(5x-2)(2x-5) = 0$$

$$x = \frac{5}{2} \text{ or } \frac{2}{5}$$

5. A stone is thrown vertically upward at a speed of 22m/s. Its height, h metres, after t seconds, is given approximately by the function $h(t) = 22t - 5t^2$. Use this formula to find, to the nearest tenth of a second, when the stone is 15 metres up. Explain why there are two solutions.



2 pts. of intersection, ball goes up + down.

$$15 = 22t - 5t^2 \rightarrow 5t^2 - 22t + 15 = 0.$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{22 \pm \sqrt{22^2 - 4(5)(15)}}{2(5)}$$

$$t = \frac{22 \pm \sqrt{184}}{10} \qquad t = \underline{0.8, 3.6}$$

6. A whole number is multiplied by 5 and added to 3 times its reciprocal to give a sum of 16. Find the number.

$$x\left(5x + \frac{3}{x} = 16\right)$$

$$5x^2 + 3 = 16x$$

$$\begin{array}{c|c} \times & + \\ \hline 15 & -16 \end{array}$$

$$5x^2 - 16x + 3 = 0$$

$$5x^2 - 15x - x + 3 = 0$$

$$5x(x-3) - 1(x-3) = 0$$

$$(5x-1)(x-3) = 0$$

$$x = \cancel{\frac{1}{5}}, 3$$

not whole number

number $= \underline{3}$