

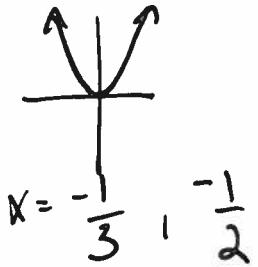
Math 20-1

Quadratic Functions and Equations

Assignment 6: The Quadratic Formula & The Discriminant

1. Find the exact roots of the equation $6x^2 + 5x + 1 = 0$ by using:

a. Graphing



b. The quadratic formula $a=6, b=5, c=1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{25 - 4(6)(1)}}{2(6)}$$

$$x = \frac{-5 \pm \sqrt{1}}{12} = \frac{-5 \pm 1}{12}$$

2. Find the roots of the following quadratic equations using the quadratic formula. Your answers should be both exact and rounded to the nearest tenth.

a. $2x^2 - 3x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{41}}{4}, x = -0.9, 2.4$$

c. $x^2 - 10x - 15 = 0$

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(-15)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{160}}{2}$$

$$= \frac{10 \pm 4\sqrt{10}}{2}$$

$$= 5 \pm 2\sqrt{10}$$

$$= 11, 3, -1, 3$$

b. $10t^2 = 7t + 1$ $10t^2 - 7t - 1$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(10)(-1)}}{2(10)}$$

$$x = \frac{7 \pm \sqrt{89}}{20}, x = -0.1, 0.8$$

$$= \frac{7 + \sqrt{89}}{20}, \frac{7 - \sqrt{89}}{20}$$

d. $3x^2 - 12x + 11 = 0$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(3)(11)}}{2(3)}$$

$$x = \frac{12 \pm \sqrt{12}}{6}$$

$$x = \frac{12 \pm 2\sqrt{3}}{6} = \frac{6 \pm \sqrt{3}}{3}$$

$$= 2.6, 1.4$$

3. Find the zeros of the following quadratic functions. Give answers as exact values in simplest form and to the nearest hundredth.

a. $f(x) = x^2 + 20x + 15$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-20 \pm \sqrt{(20)^2 - 4(1)(15)}}{2(1)} \\ &= \frac{-20 \pm \sqrt{340}}{2} = \frac{-20 \pm 2\sqrt{85}}{2} \\ x &= -10 \pm \sqrt{85} \\ &= -19.22, -0.78 \end{aligned}$$

b. $f(x) = 5x^2 + 12 - 5$

$$\begin{aligned} x &= \frac{-12 \pm \sqrt{12^2 - 4(5)(-5)}}{2(5)} \\ &= \frac{-12 \pm \sqrt{144}}{10} = \frac{-12 \pm 2\sqrt{61}}{10} \\ x &= \frac{-6 \pm \sqrt{61}}{5} \\ &= -2.76, 0.36 \end{aligned}$$

4. Form a quadratic equation and solve. Give answers as exact values in simplest form.

a. $(2x - 1)(3x + 2) = (x + 3)(2x + 1)$

$$\begin{aligned} 6x^2 + 4x - 3x - 2 &= 2x^2 + x + 6x + 3 \\ 4x^2 - 6x - 5 &= 0 \end{aligned}$$

$$x = \frac{6 \pm \sqrt{(6)^2 - 4(4)(-5)}}{2(4)} = \frac{6 \pm \sqrt{116}}{8}$$

$$x = -0.6, 2.1$$

b. $3(x - 1)(x + 2) - (x^2 + 3) = 0$

$$\begin{aligned} 3(x^2 + 2x - x - 2) - x^2 - 3 &= 0 \\ 3x^2 + 6x - 3x - 6 - x^2 - 3 &= 0 \end{aligned}$$

$$2x^2 + 3x - 9 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4(2)(-9)}}{2(2)} = \frac{-3 \pm \sqrt{81}}{4}$$

$$x = -3, \frac{3}{2}$$

5. Find a quadratic equation in simplest form which is equivalent to the given equation, but has integral coefficients. Find the roots of the given equation to the nearest tenth.

a. $(1.4x^2 - 2.8x - 1.8) \stackrel{10}{=} 0$ Hint: X 10

$$14x^2 - 28x - 18 = 0$$

$$x = \frac{28 \pm \sqrt{28^2 - 4(14)(-18)}}{2(14)}$$

$$x = \frac{28 \pm \sqrt{1792}}{28}$$

$$= -0.5, 2.5$$

b. $\left(\frac{x^2}{2} - \frac{x}{1} - \frac{5}{4}\right) = 0$

$$2x^2 - 4x - 5 = 0$$

$$x = \frac{4 \pm \sqrt{4^2 - 4(2)(-5)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{56}}{4} = \frac{4 \pm 2\sqrt{14}}{4} = \frac{2 \pm \sqrt{14}}{2}$$

$$= -0.9, 2.9$$

6. Find the value of the discriminant in each of the following equations.

a. $x^2 + x + 9 = 0$

$$b^2 - 4ac$$

$$1^2 - 4(1)(9)$$

$$1 - 36$$

$$= -35$$

b. $3x^2 - 18x + 27 = 0$

$$b^2 - 4ac$$

$$(-18)^2 - 4(3)(27)$$

$$= 0$$

7. Determine the nature of the roots of the following equations without solving or graphing.

a. $2x^2 + 4x + 8 = 0$

$$b^2 - 4ac$$

$$4^2 - 4(2)(8)$$

$$-48$$

non real

b. $9x^2 - 24x + 16 = 0$

$$(-24)^2 - 4(9)(16)$$

$$= 0$$

real & equal

c. $-2x^2 - x + 3 = 0$

$$(-1)^2 - 4(-2)(3)$$

$$= 25$$

real & unequal

d. $-2(x+3)^2 + 40 = 0$

$$-2(x+3)(x+3) + 40 = 0$$

$$-2(x^2 + 6x + 9) + 40 = 0$$

$$-2x^2 - 12x - 18 + 40 = 0$$

$$12^2 - 4(-2)(22)$$

$$= 96 \text{ real & unequal}$$

e. $x^2 + 10 + 3x = 0$

$$3^2 - 4(1)(10)$$

$$= -31$$

nonreal

f. $4x^2 + 4x + 1 = 0$

$$4^2 - 4(4)(1)$$

$$= 0$$

real & equal

8. For what values of n does each equation have real roots? ≥ 0

a. $nx^2 - 2x + 1 = 0$

$$b^2 - 4ac \geq 0$$

$$(-2)^2 - 4n(1) \geq 0$$

$$4 - 4n \geq 0$$

$$-4n \geq -4$$

$$n \leq 1, \epsilon R$$

b. $2x^2 + 20x + n = 0$

$$b^2 - 4ac \geq 0$$

$$20^2 - 4(2)n \geq 0$$

$$400 - 8n \geq 0$$

$$-8n \geq -400$$

$$n \leq 50, \epsilon R$$

For what values of a does the equation $ax^2 + (2a - 3)x + a = 0$ have non-real roots?

$$b^2 - 4ac < 0$$

$$(2a-3)^2 - 4aa < 0$$

$$4a^2 - 12a + 9 - 4a^2 < 0$$

$$-12a + 9 < 0$$

$$-12a < -9$$

$$\rightarrow a > \frac{9}{12}$$

$$a > \frac{3}{4}, \epsilon R$$

