Name: Key

Math 20-1
Quadratic Functions and Equations
Assignment 5: Completing the Square

1. What number must be added to each to make a perfect square?

a. $x^2 + 8x$ $+ 16$ $\frac{8}{2} = 4$	b. $x^2 - 24x$ $+ 144$	c. $x^2 + 40x$ $+ 400$
d. $x^2 - x$ $+ \frac{1}{4}$	e. $x^2 + \frac{1}{2}x$ $+ \frac{1}{16}$	f. $x^2 - \frac{2}{3}x$ $+ \frac{1}{9}$

2. Complete the square in each part.

a. $x^2 + 6x + 9 = (x + 3)^2$	b. $x^2 - 20x + 100 = (x - 10)^2$
c. $x^2 + 5x + \frac{25}{4} = (x + \frac{5}{2})^2$	d. $x^2 - 9x + \frac{81}{4} = (x - \frac{9}{2})^2$
e. $x^2 + 0.6x + 0.09 = (x + 0.3)^2$	f. $x^2 - \frac{3}{4}x + \frac{9}{64} = (x - 3/8)^2$

3. Express the following in competed square form.

a. $y = x^2 + 10x + 3$ $y = (x^2 + 10x + 25) - 25 + 3$ $y = (x+5)^2 - 22$	b. $y = x^2 - 4x - 21$ $y = (x^2 - 4x + 4) - 4 - 21$ $y = (x-2)^2 - 25$
c. $y = x^2 + 14x - 2$ $y = (x^2 + 14x + 49) - 49 - 2$ $y = (x+7)^2 - 51$	d. $f(x) = x^2 + 9x + 22$ $y = (x^2 + 9x + \frac{81}{4}) - \frac{81}{4} + 22$ $y = (x + 9/2)^2 + 7/4$
e. $g(x) = x^2 - x + 1$ $= (x^2 - x + \frac{1}{4}) - \frac{1}{4} + 1$ $g(x) = (x - \frac{1}{2})^2 + 3/4$	f. $h(x) = x^2 + bx + c$ $h(x) = (x^2 + bx + (\frac{b}{2})^2) - \frac{b^2}{4} + c$ $h(x) = (x + \frac{b}{2})^2 + c - \frac{b^2}{4}$

4. Express $f(x) = x^2 - 14x - 40$ in completed square form. State the coordinated of the vertex and the equation of axis of symmetry of the graph of the function.

$f(x) = (x^2 - 14x + 49) - 49 - 40$

$f(x) = (x - 7)^2 - 89$

vertex: $(7, -89)$

axis of symm $x = 7$

5. Express the following in completed square form.

a. $f(x) = 2x^2 + 12x + 5$ $f(x) = 2(x^2 + 6x + 9 - 9) + 5$ $f(x) = 2(x^2 + 6x + 9) - 18 + 5$ $f(x) = 2(x+3)^2 - 13$	b. $p(x) = 2x^2 + 14x - 11$ $p(x) = 2\left(x^2 + 7x + \frac{49}{4} - \frac{49}{4}\right) - 11$ $p(x) = 2\left(x^2 + 7x + \frac{49}{4}\right) - 24.5 - 11$ $p(x) = 2(x+7)^2 - \frac{71}{2}$.
c. $y = -x^2 + 10x + 20$ $y = -(x^2 - 10x + 25 - 25) + 20$ $y = -(x^2 - 10x + 25) + 25 + 20$ $y = -(x-5)^2 + 45$	d. $f(x) = -4x^2 - 8x + 7$ $f(x) = -4(x^2 + 2x + 1 - 1) + 7$ $f(x) = -4(x^2 + 2x + 1) + 4 + 7$ $f(x) = -4(x^2 + 2x + 1) + 11$ $f(x) = -4(x+1)^2 + 11$

$5x^2 - 20x + m$

e. $g(x) = 11x - x^2$	f. $y = 5x^2 20x + m$
$g(x) = -x^2 + 11x$ $g(x) = -\left(x^2 - 11x + \frac{121}{4} - \frac{121}{4}\right)$ $g(x) = -\left(x^2 - 11x + \frac{121}{4}\right) + \frac{121}{4}$ $g(x) = -\left(x - \frac{11}{2}\right)^2 + \frac{121}{4}$	$y = 5(x^2 - 4x + 4 - 4) + m$ $y = 5(x^2 - 4x + 4) - 4(5) + m$ $y = 5(x^2 - 4x + 2) - 20 + m$ $y = 5(x-2)^2 + m - 20$

6. Write the function $f(x) = ax^2 + bx + c$ in completed square form.

$$f(x) = \underbrace{ax^2 + bx}_{} + c$$

$$\left(\frac{bx}{a} \div 2\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$f(x) = a\left(x^2 + \frac{bx}{a}\right) + c$$

$$f(x) = a\left(x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$$

$$f(x) = a\left(x^2 + \frac{bx}{a} + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a^2} \cdot a + c$$

$$f(x) = a\left(x^2 + \frac{bx}{a} + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c$$

$$c - \frac{b^2}{4a} = \frac{4ac - b^2}{4a}$$

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \underbrace{c - \frac{b^2}{4a}}$$

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$