Polynomial Functions and Equations Lesson #11:
Practice Test

Section A *No calculator may be used for this section of the test.*

1. Which of the following is an integral polynomial of degree 3?

A. $3x^3 - \frac{1}{3}x^2 + 3$ **B.** $x + 2x^2 + 3x^3$

C. $\frac{x^3 + 2x}{x}$ **D.** $3x^6 + 3x^4 + 3x$

Numerical Response 1. Consider the following partially completed synthetic division where the divisor is $x - 3$.

3	3	−4	3	−b
		9	15	54
	3	c 5	a 18	−25

$-b+54=-25$

$79=b$

The value of b is ________.

(Record your answer in the numerical response box from left to right.)

79

2. Consider two polynomial functions $f(x)$ and $g(x)$. 7 is a zero of f and when g is divided by $x - 7$, the remainder is 2. Which of the following statements must be true?

A. $f(0) + g(7) = 9$

B. $f(7) + g(-7) = 2$

C. $f(7) + g(2) = 7$

D. $f(7) + g(7) = 2$

$f(7)=0 \quad g(7)=2$

$f(7)+g(7)=0+2=2$

3. The x-intercept(s) of the graph of the function $f(x) = x^3 + 3x^2 + 3x + 1$ is/are

A. −1 only

B. 1 only

C. −1 and 1 only

D. −1, 0 and 1

−1	1	3	3	1
		−1	−2	−1
	1	2	1	0

$f(x)=(x+1)(x^2+2x+1)$

$=(x+1)(x+1)(x+1)$

4. Which of the following best defines an integral polynomial function?

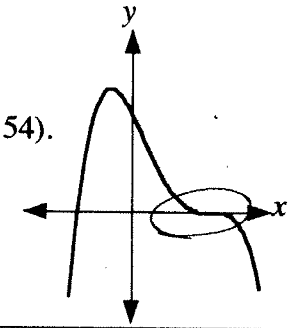
A. an integral y-intercept **B.** integral zeros

C. integral coefficients **D.** both integral zeros and integral coefficients

Use the following information to answer the next question.

The partial graph of a fourth degree polynomial function $y = P(x)$ is shown.

The graph passes through the points $(-2, 0)$, $(3, 0)$, and $(0, 54)$.



5. The equation of the polynomial function is

A. $P(x) = (x + 2)(x - 3)^3$

B. $P(x) = (x - 2)(x + 3)^3$

C. $P(x) = -(x + 2)(x - 3)^3$

D. $P(x) = -(x - 2)(x + 3)^3$

leading coefficient is $(-)$

$P(x) = c(x+2)(x-3)^3$

$54 = c(0+2)(0-3)^3$

$54 = -54c$

$-1 = c$

$P(x) = -1(x+2)(x-3)^3$.

A graphing calculator may be used for the remainder of the test.

Section B

6. When a polynomial $P(x)$ is divided by $3x - 4$, the quotient is $x^2 - x - 4$ and the remainder is -9. The polynomial $P(x)$ is

A. $3x^3 - 7x^2 - 8x + 7$

B. $3x^3 - 7x^2 - 8x + 25$

C. $3x^3 - 7x^2 - 8x - 25$

D. $x^2 - 4x - 17$

$P(x) = D(x)Q(\cancel{x}) + R$

$= (3x-4)(x^2 - x - 4) - 9$

$= 3x^3 - 3x^2 - 12x - 4x^2 + 4x + 16 - 9$

$= 3x^3 - 7x^2 - 8x + 7$.

Numerical Response

2. When the polynomial $2x^3 - 5x^2 + ax - 5$ is divided by $x - 3$, the remainder is 67. The value of a is ______ .

(Record your answer in the numerical response box from left to right.)

2	1		

$P(3) = 67$

$67 = 2(3)^3 - 5(3)^2 + a(3) - 5$

$67 = 54 - 45 + 3a - 5$

$63 = 3a$

$a = 21$

7. Nina is determining the zeros of $P(x) = 3x^2 - 2x^3 + 7x^4 - 4$. A rational number she should consider is

$P(x) = 7x^4 - 2x^3 + 3x^2 - 4$

A. $\frac{4}{3}$ **B.** $\frac{3}{4}$

$\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 7}$

C. $\frac{7}{4}$ **D.** $\frac{4}{7}$

8. If $3x^4 - px^3 + x^2 + qx - 9$ is divided by $x - 2$, the remainder is -4. The equation relating p and q is

$P(2) = -4$

A. $8p - 2q - 39 = 0$

$-4 = 3(2)^4 - p(2)^3 + 2^2 + q(2) - 9$

B. $8p - 2q + 39 = 0$

$-4 = 48 - 8p + 4 + 2q - 9$

C. $8p - 2q - 47 = 0$

D. $8p - 2q + 47 = 0$

$8p - 2q - 47 = 0$

9. One factor of $6x^3 + 23x^2 - 6x - 8$ is $x + 4$. The other two factors are

A. $2x - 1$ and $3x + 2$

B. $2x + 1$ and $3x - 2$

C. $6x + 1$ and $x - 2$

D. $6x - 1$ and $x + 2$

-4	6	23	-6	8
		-24	4	8
	6	-1	-2	0

$= (x+4)(6x^2 - x - 2)$

$= (x+4)(3x-2)(2x+1)$

$\to 6x^2 - x - 2$

$= 6x^2 - 4x + 3x - 2$

$= 2x(3x-2) + 1(3x-2)$

$= (2x+1)(3x-2)$

10. If $-2, 0,$ and 4 are the only zeros of a fourth degree polynomial function, $P(x)$, which one of the following is a possible factored form of $P(x)$?

A. $P(x) = x(x + 2)(x - 4)$ – degree 3 $x(x+2)(x-4)$

B. $P(x) = 6x^2(x + 2)(x - 4)$

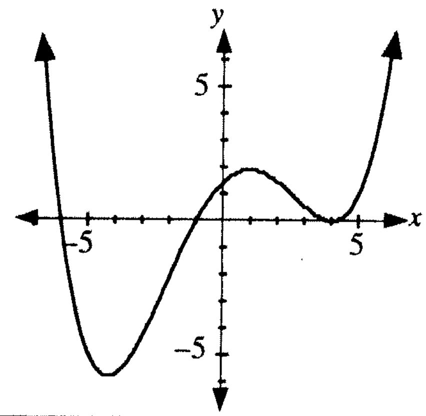
C. $P(x) = x(x + 2)^2(x - 4)^2$ – degree 5

D. $P(x) = 4x(x + 2)(x - 4)$ degree 3

Use the following information to answer the next question.

The partial graph of a fourth degree polynomial function, $y = P(x)$, is shown.

The x-intercepts are integers.



11. When $P(x)$ is divided by $x + 4$, the remainder is $P(-4) < 0$

A. zero **B.** positive

C. negative **D.** unable to be determined from the given information

Numerical Response

3. The binomial $x^2 - 3x - 4$ is a factor of the polynomial $x^3 - 6x^2 + cx + d$, where c and d are integers. The value of $c + d$ is ______ .

(Record your answer in the numerical response box from left to right.)

1	7		

$x^2 - 3x - 4 = (x-4)(x+1)$

$x^3 - 6x^2 + cx + d \rightarrow P(4) = 0 + P(-1) = 0$

$P(4) = 4^3 - 6(4)^2 + 4c + d$

$0 = 64 - 96 + 4c + d$

$4c + d = 32$

$P(-1) = (-1)^3 - 6(-1)^2 + c(-1) + d$

$0 = -1 - 6 - c + d$

$c - d = -7$

$4c + d = 32$

$c - d = -7$

$5c = 25$

$c = 5$

$c - d = -7$

$5 - d = -7$

$d = 12$

$c + d$

$= 5 + 12 = 17$

12. The only factors of a polynomial $P(x)$ are $(3x - 2)$, $(4x + 3)$ and $(x + 7)$. If the polynomial $Q(x) = -3P(x)$, then the x-intercepts of the graph of $y = Q(x)$ are

A. $\frac{2}{3}, -\frac{3}{4}$ and -7

B. $-\frac{2}{3}, \frac{3}{4}$ and 7

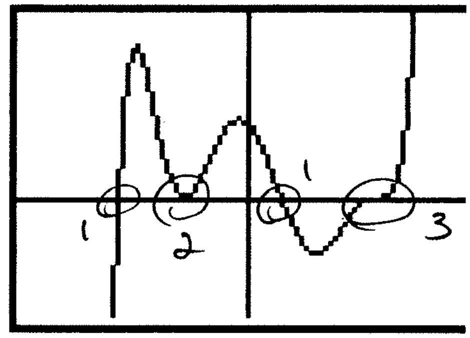
C. $2, -\frac{9}{4}$ and -21

D. $-2, \frac{9}{4}$ and 21

$3x - 2 \rightarrow \frac{2}{3}, -\frac{3}{4}, -7$

vertical stretch + reflection, does not affect x-intercepts

13. A graphing calculator screenshot of the graph of a polynomial function is shown.



$1+2+1+3=7$

If all the x-intercepts are shown, then the minimum degree of the polynomial function is

A. 4 **B.** 5 **C.** 6 **D.** 7

Numerical Response 4. If $P(x) = px^3 + qx + r$, where $P(0) = 2$ and $P(2) = P(-1) = 5$, then the value of $p - 2q$, to the nearest tenth, is ________.

(Record your answer in the numerical response box from left to right.)

1	0	.	5

$P(0) = 2$
$2 = 0 + 0 + r$
$2 = r$
$P(x) = px^3 + qx + 2$

$P(2) = 5$
$5 = p(2)^3 + q2 + 2$
$5 = 8p + 2q + 2$
$8p + 2q = 3$
$P(-1) = p(-1)^3 + q(-1) + 2$
$5 = -p - q + 2$
$p + q = -3$

$8p + 2q = 3$
$(p + q = -3) \cdot 2$

$\rightarrow 8p + 2q = 3$
$-2p + 2q = +6$
$6p = 9$
$p = \frac{3}{2}$

$p + q = -3$
$\frac{3}{2} + q = -3$
$q = -9/2$

$p - 2q$
$\frac{3}{2} - 2\left(-\frac{9}{2}\right) = \frac{21}{2}$

14. All the potential rational zeros of the fourth degree integral polynomial function
$$P(x) = ax^4 + bx^3 + cx^2 + dx + e$$
are ± 1, $\pm\frac{1}{2}$, and $\pm\frac{1}{4}$. The values of a and e, respectively, could be

A. $\frac{1}{4}$ and 1 **B.** 1 and $\frac{1}{4}$

C. 4 and 1 **D.** 1 and 4

$\frac{\text{factors of } e}{\text{factors of } a} \rightarrow \pm 1, \pm\frac{1}{2} \pm \frac{1}{4}$

Numerical Response 5. When the polynomial $2x^3 - 5x^2 + 4x + d$ is divided by $2x + 3$, the remainder is -6. The value of d is _____.

(Record your answer in the numerical response box from left to right.)

1	8		

$P(-6) = 2\left(-\frac{3}{2}\right)^3 - 5\left(-\frac{3}{2}\right)^2 + 4\left(-\frac{3}{2}\right) + d$
$-6 = \frac{-27}{4} - \frac{45}{4} - 6 + d$

$18 = \frac{72}{4} = d$

15. The following values are taken from the graph of a third degree polynomial function.

x	-2	-1	0	1	2
$P(x)$	0	-18	-12	0	0

The equation of the polynomial function is

A. $P(x) = (x-1)(x+2)(x-2)$

B. $P(x) = -3(x-1)(x+2)(x-2)$

C. $P(x) = (x+1)(x+2)(x-2)$

D. $P(x) = 3(x-1)(x+2)(x-2)$

zeros are: $-2, 1, 2$

$P(x) = (x+2)(x-1)(x-2)$

$P(0) = -12$

$-12 = c(0+2)(0-1)(0-2)$

$-12 = 4c$

$-3 = c$

$P(x) = -3(x-1)(x-2)(x+2)$

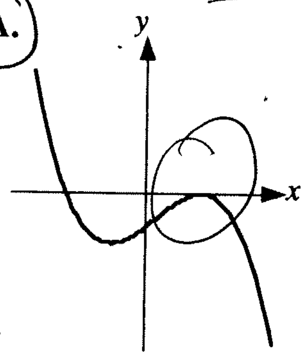
16. Which of the following graphs could be the graph of the polynomial function $P(x) = c(x+a)^2(x+b)$ if $a < 0, b > 0$, and $c < 0$?

degree 3

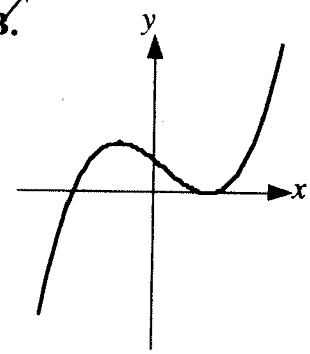
$c \nless 0$ - leading coefficient is $\ominus$

$a < 0 \rightarrow$ zero is $\oplus$ of multiplicity 2.

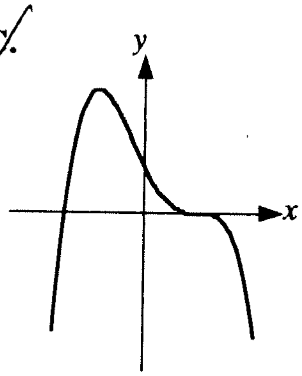
A.



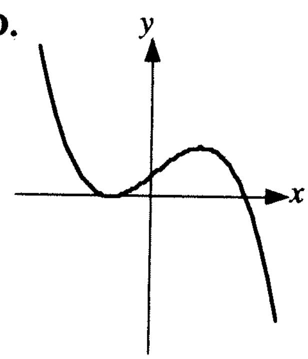
~~B.~~



~~C.~~

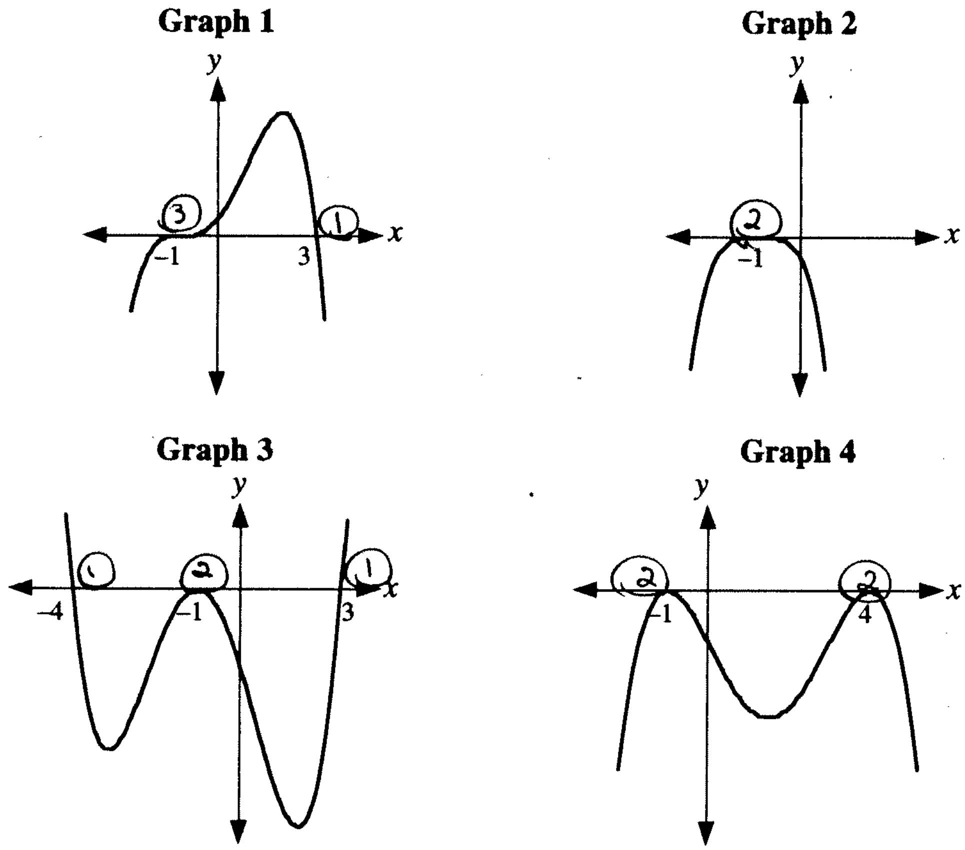


D.



Use the following information to answer the next three questions.

The partial graphs of four polynomial functions of **degree four** are shown. There are no x-intercepts other than the ones indicated on the graphs.



17. All the functions which have a zero of multiplicity 2 at $x = -1$, are represented in

A. graph 1 **B.** graphs 2 and 3 **C.** graphs 3 and 4 **D.** graphs 2, 3, and 4

18. All the functions which have three **distinct** zeros are represented in

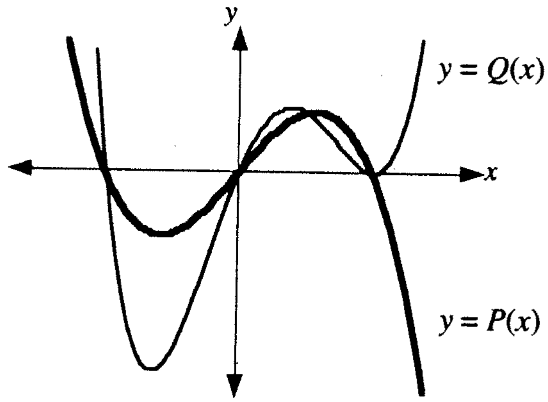
A. graph 1 **B.** graph 3 **C.** graphs 1 and 3 **D.** graphs 3 and 4

19. If the polynomial in Graph 4 is multiplied by $(x + 4)$, a new polynomial, $f(x)$, is formed. Which of the following statements must be true?

A. The x-intercepts of the graph of $y = f(x)$ will be 3 and 8.
B. The x-intercepts of the graph of $y = f(x)$ will be −5 and 0.
C. The y-intercept of the graph of $y = f(x)$ will be 4 units above the y-intercept of Graph 4.
D. The y-intercept of the graph of $y = f(x)$ will be 4 times the y-intercept of Graph 4.

Use the following information to answer the next question.

The graphs of $y = P(x)$ and $y = Q(x)$ have x-intercepts at $-2, 0$, and 2.
The absolute values of the leading coefficients of $P(x)$ and $Q(x)$ are equal.



20. Which of the following describes a relationship between the two polynomial functions?

A. $Q(x) = 2P(x)$
B. $Q(x) = (x + 2)P(x)$
C. $Q(x) = (x - 2)P(x)$
D. $Q(x) = (2 - x)P(x)$

$P(x) = -cx(x+2)(x-2) \quad c > 0$
$Q(x) = cx(x+2)(x-2)^2 \quad c > 0$
$Q(x) = -(x-2)P(x)$
$Q(x) = (2-x)P(x)$.

Numerical Response

6. The graph of the polynomial function $P(x) = px^3 + qx^2 + rx + s$ is tangent to the x-axis at the point $(-3, 0)$ and passes through the point $(2, 0)$. If the graph also passes through $(-1, 24)$, then the value of s is _____. → constant term + y-int.

(Record your answer in the numerical response box from left to right.)

3	6		

$(x+3)^2 \qquad (x-2)$

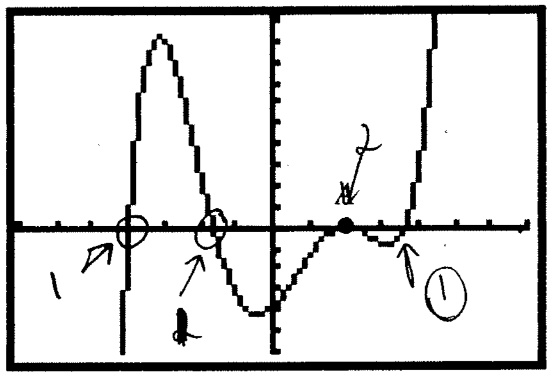
$P(x) = p(x+3)^2(x-2)$
$24 = p(-1+3)^2(-1-2)$
$24 = p(4)(-3)$
$24 = -12p$
$-2 = p$

$P(x) = -2(x+3)^2(x-2)$
$P(0) = -2(0+3)^2(0-2)$
$= -2(9)(-2)$
$= 36$
$s = 36$

Written Response

Use the following information to answer this question.

The illustration shown is a graphing calculator screenshot of the graph of a polynomial function $P(x)$ with window x:[−7, 7, 1] y:[−150, 250, 25]. The graph is tangent to the x-axis at point A.



- Explain how you can use the graph to show that the degree of the polynomial cannot be an even number.

There are 3 zeros of multiplicity = 1 + one zero of even multiplicity ⇒ the sum of all multiplicities will always be odd.

- The equation of the polynomial function is $P(x) = x^5 - 2x^4 - 18x^3 + 40x^2 + 40x - 96$. It appears from the graph that the polynomial function has two integral zeros.

  Algebraically confirm that $P(x)$ has two integral zeros.

The zero 2 has multiplicity 2 + the zero −4 has multiplicity 1

$$\begin{array}{r|rrrrrr} 2 & 1 & -2 & -18 & 40 & 40 & -96 \\ & & 2 & 0 & -36 & 8 & 96 \\ \hline & 1 & 0 & -18 & 4 & 48 & 0 \end{array}$$

$P(x) = (x-2)(x^4 - 18x^2 + 4x + 48)$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -18 & 4 & 48 \\ & & 2 & 4 & -28 & -48 \\ \hline & 1 & 2 & -14 & -24 & 0 \end{array}$$

$P(x) = (x-2)(x-2)(x^3 - 18x^2 + 4x + 48)$

$$\begin{array}{r|rrrr} -4 & 1 & 2 & -14 & -24 \\ & & -4 & 8 & 24 \\ \hline & 1 & -2 & -6 & 0 \end{array}$$

$P(x) = \underbrace{(x-2)(x-2)}_{\text{2 integral zeros}}(x+4)\underbrace{(x^2 - 2x - 6)}_{\text{cannot be factored}}$

- Determine the exact value of the other two zeros.

$P(x) = (x+4)(x-2)^2(x^2-2x-6)$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2-4(1)(-6)}}{2(1)} = \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm 2\sqrt{7}}{2}$

$= 1 \pm \sqrt{7}$

The other 2 zeros are: $1+\sqrt{7}$ + $1-\sqrt{7}$.

- Polynomials $Q(x)$ and $R(x)$ are defined by $Q(x) = -2P(x)$ and $R(x) = P(-2x)$. Determine the integral zeros of $Q(x)$ and $R(x)$.

The Integral zeros of $P(x)$ are -4 + 2. $P(x)$ is stretched vertically by a factor of 2 + reflected in x-axis to form $Q(x)$ → does not affect zeros

$P(x)$ is stretched horizontally by a factor of $1/2$ + reflected in y-axis to form $R(x)$ → the zeros are multiplied by $-\frac{1}{2}$

Answer Key

Integral zeros of $Q(x)$ are -4 + 2, of $R(x)$ → 2 + -1

Multiple Choice

1. B	2. D	3. A	4. C	5. C	6. A	7. D	8. C
9. B	10. B	11. C	12. A	13. D	14. C	15. B	16. A
17. C	18. B	19. D	20. D				

Numerical Response

1. | 7 | 9 | | |

2. | 2 | 1 | | |

3. | 1 | 7 | | |

4. | 1 | 0 | . | 5 |

5. | 1 | 8 | | |

6. | 3 | 6 | | |

Written Response

1. • There are three zeros of multiplicity 1, and one zero of even multiplicity. Therefore the sum of the multiplicities is always odd, and so the degree of the polynomial is always odd and never even.

- The integral zeros are -4 and 2.

- The other two zeros are $1 + \sqrt{7}$ and $1 - \sqrt{7}$.

- The integral zeros of $Q(x)$ are -4 and 2. The integral zeros of $R(x)$ are -1 and 2.