Assignment

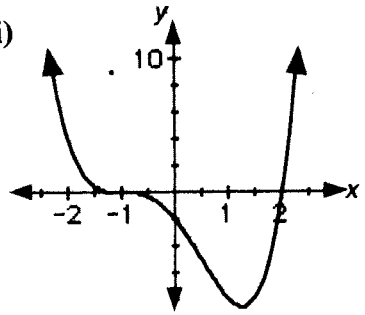
1. How does the concept **number of zeros** differ from the concept **multiplicity of zeros**?

Number of zeros is how many distinct zeros a function has, multiplicity refers to the number of times a zero repeats.

2. Consider the graphs below.

a) In each case state the number of distinct zeros and the possible multiplicities of each zero

i)

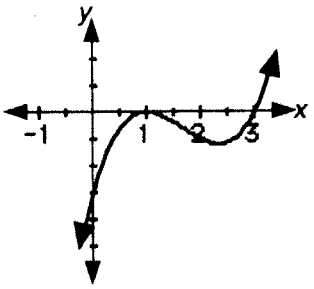


two distinct zeros

-1 → multiplicity of 3, 5 or 7...

2 → multiplicity of 1

ii)

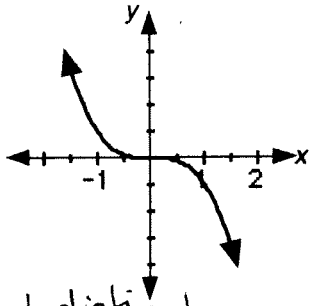


two distinct zeros.

1 → multiplicity of 2, 4, 6 ...

3 → multiplicity of 1.

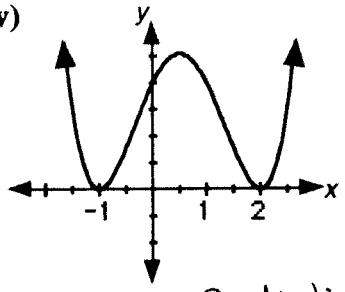
iii)



1 distinct zero

0 → multiplicity of 3, 5 or 7...

iv)



2 distinct zeros

-1 → multiplicity 2, 4 or 6...

2 → " " "

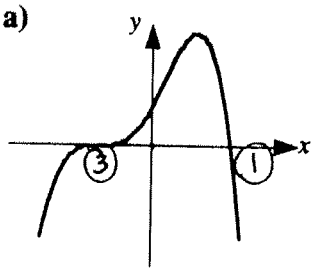
b) Which graph(s) could represent a seventh-degree polynomial function? ii or iii

c) Which graph(s) could **not** represent a polynomial function of degree 10? i, iii, iv.

d) In which of the graphs is the leading coefficient negative? iii

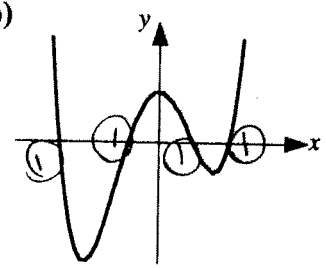
3. The following graphs represent functions of lowest possible degree. State the degree in each case.

a)



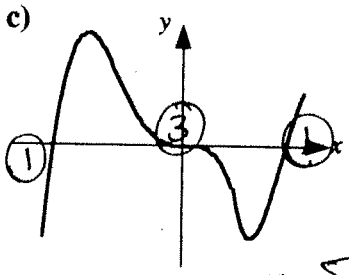
= 4

b)



= 4

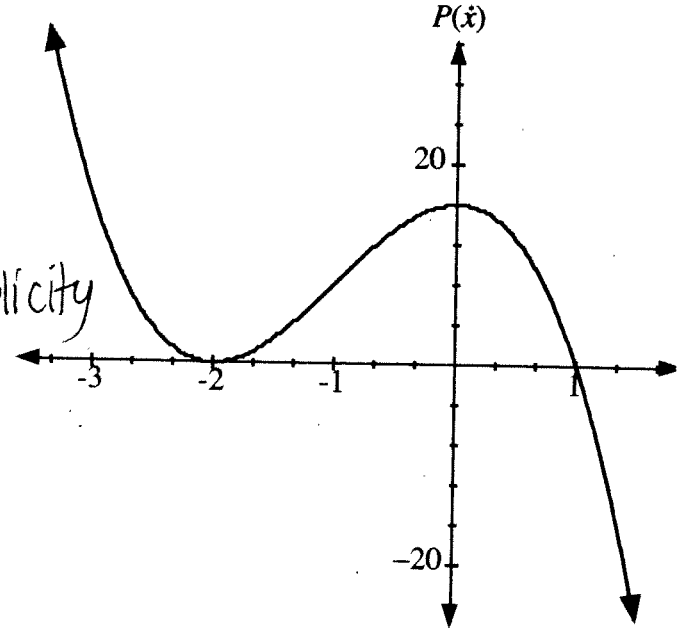
c)



= 5

4. The graph represents a polynomial function $P(x)$ of degree 5.

Write the equation of $P(x)$ in factored form if the leading coefficient is –1.



1 → multiplicity of 1
therefore, -2 → multiplicity of 4

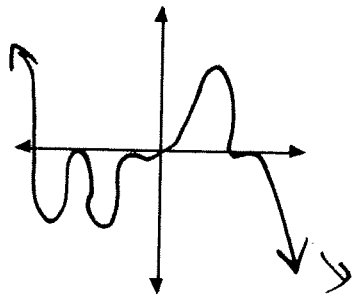
$P(x) = (x-1)(x+2)^4$

5. a) On the grid, sketch a graph of a polynomial function satisfying the given conditions.

many answers
even + odd

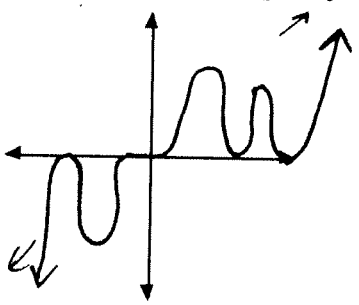
i)
- negative leading coefficient
- one real zero of multiplicity 1
- one real zero of multiplicity 2
- two real zeros of multiplicity 3

min degree 9



ii)
- positive leading coefficient
- two real zeros of multiplicity 2 $2 \cdot 2 = 4$
- one real zero of multiplicity 3 - 3
- one real zero of multiplicity 6 - 6

$4+3+6 = 13$

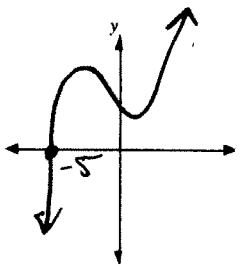


b) State the degree of each polynomial function.

$1+2+3+3 = 9$

$2+3+6+2 = 13$

6. The cubic function $f(x) = x^3 - 12x + 65$ has one real zero of multiplicity 1 and two non-real zeros.



a) Use a graphing calculator to graph the function and make a sketch of the graph on the grid.

b) Use synthetic division to determine the real zero of the function.

$$\begin{array}{r|rrrr} -5 & 1 & 0 & -12 & 65 \\ & \downarrow & -5 & 25 & -65 \\ \hline & 1 & -5 & 13 & \textcircled{0} \end{array}$$

$f(x) = (x+5)(x^2-5x+13)$

c) Determine the two non-real zeros of the function.

solve $x^2-5x+13$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{5 \pm \sqrt{(-5)^2-4(1)(13)}}{2(1)} = \frac{5 \pm \sqrt{-27}}{2} = \frac{5 \pm 3\sqrt{-3}}{2}$$

7. In each case, sketch the graph of the polynomial function using a graphing calculator window of x:[–8, 8, 1] and y:[–50, 50, 10] and write a statement which describes the number, type, and multiplicity of the zeros.

a) $P(x) = x^3 - 6x^2 + 6x - 5$

one real zero multiplicity 1
2 non-real zeros - multiplicity 1

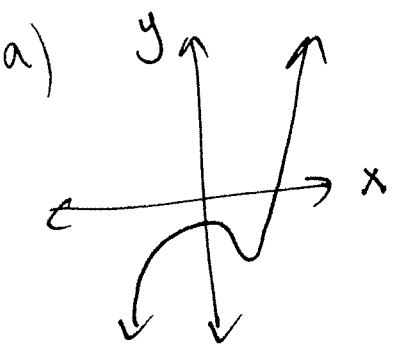
b) $Q(x) = x^4 - 11x^3 + 36x^2 - 35x + 25$

One real zero multiplicity 2
Two non-real zeros, each of multiplicity 1

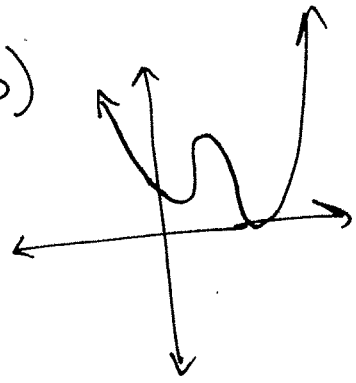
c) $R(x) = -x^5 + 13x^4 - 61x^3 + 124x^2 - 112x + 64$

One real zero of multiplicity 3
two non-real zeros, each of multiplicity 1

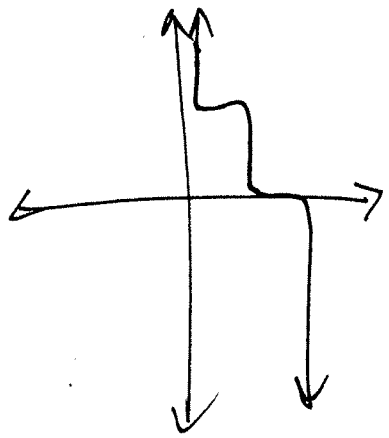
a)



b)

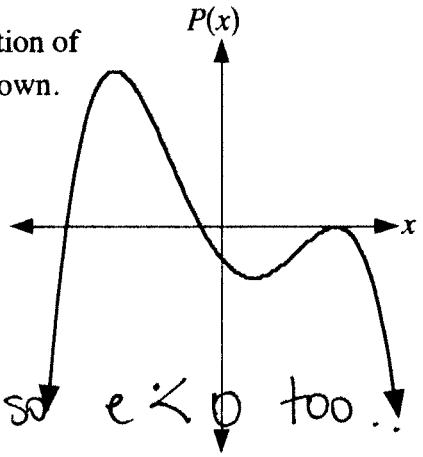


c)



Use the following information to answer the next two questions.

The graph of a fourth degree polynomial function of the form $P(x) = ax^4 + bx^3 + cx^2 + dx + e$ is shown.



- even degree
- leading coefficient is negative so -
a < 0, y-int is ⊖ so e < 0 too ∴

Multiple Choice

8. The values a and e must satisfy

A. $a>0, e<0$ **B.** $a<0, e>0$ **C.** $a>0, e>0$ **D.** $a<0, e<0$

9. If $P(x) = 0$ has exactly three different solutions, then which one of the following statements about the roots of $P(x) = 0$ is true?

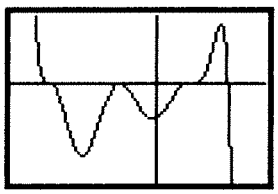
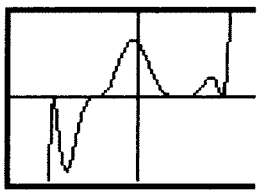
A. Two roots are real, equal and negative, and two roots are real, not equal and positive.
B. Two roots are real, equal and positive and two roots are real, not equal and negative.
C. Two roots are real and negative, and two roots are not real.
D. Two roots are real and positive, and two roots are not real.

Answer Key

1. - number of zeros refers to how many distinct zeros the function has
- multiplicity refers to the number of times a zero repeats

2. **a)** **i)** two zeros, the zero –1 has a multiplicity of 3 or 5 or 7 ..., the zero 2 has a multiplicity of 1.
ii) two zeros, the zero 1 has a multiplicity of 2 or 4 or 6 ..., the zero 3 has a multiplicity of 1.
iii) one zero, the zero 0 has a multiplicity of 3 or 5 or 7 ... .
iv) two zeros, both the zeros –1 and 2 have a multiplicity of 2 or 4 or 6 ...,
b) ii), iii) **c)** ii), iii), iv) **d)** iii)

3. **a)** 4 **b)** 4 **c)** 5 **4.** $P(x) = -(x+2)^4(x-1)$

5. **a)** Answers may vary → → → → → → **i)**  **ii)** 
b) **i)** 9 **ii)** 13

6. **b)** –5 **c)** $\frac{5-3\sqrt{-3}}{2}$ and $\frac{5+3\sqrt{-3}}{2}$

7. **a)** One real zero of multiplicity 1, and two non-real zeros, each of multiplicity 1.
b) One real zero of multiplicity 2, and two non-real zeros, each of multiplicity 1.
c) One real zero of multiplicity 3, and two non-real zeros, each of multiplicity 1.

8. D **9.** B