



The investigative assignment in this lesson will develop the relationships between the multiplicities of the zeros of a polynomial function and its graph.

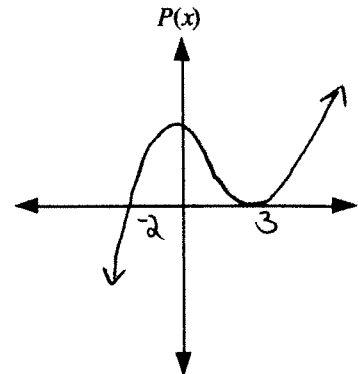
Complete Assignment Questions #1 - #15

Assignment

In this assignment, choose appropriate windows which will enable you to investigate all the characteristics of the functions.

1. a) Graph $P(x) = x^3 - 4x^2 - 3x + 18$ and complete the table.

zero	multiplicity
-2	1
3	2

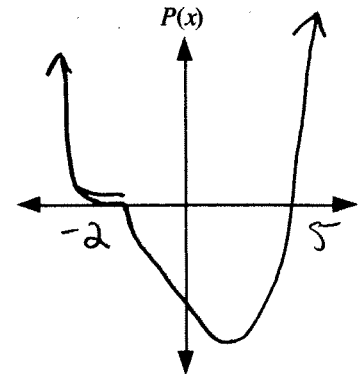


- b) Write the polynomial in the form $P(x) = (x - a)(x - b)^2$, where $a, b \in I$.

$$P(x) = (x + 2)(x - 3)^2$$

2. a) Graph $P(x) = x^4 + x^3 - 18x^2 - 52x - 40$ and complete the table.

zero	multiplicity
-2	3
5	1

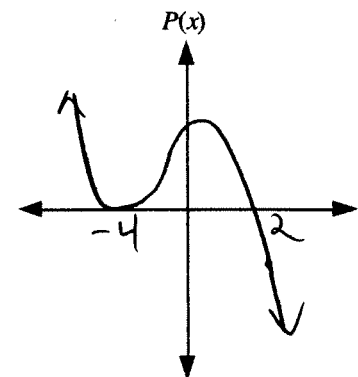


- b) Write the polynomial function in factored form.

$$P(x) = (x - 5)(x + 2)^3$$

3. a) Graph $P(x) = -x^3 - 6x^2 + 32$ and complete the table.

zero	multiplicity
-4	2
2	1



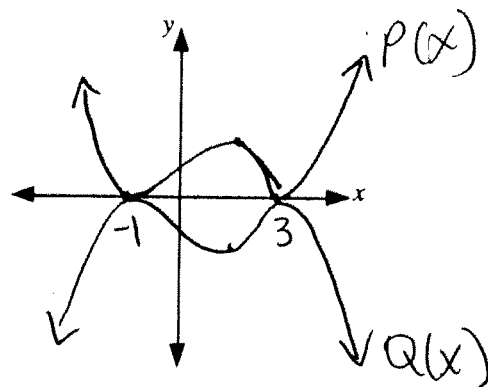
- b) Write the polynomial in the form $P(x) = -(x - a)(x - b)^2$, $a, b \in I$.

$$P(x) = -(x - 2)(x + 4)^2$$

4. a) Sketch the graphs of $P(x) = x^4 - 4x^3 - 2x^2 + 12x + 9$ and $Q(x) = -x^4 + 4x^3 + 2x^2 - 12x - 9$ on the grid.

b) State the zeros, their multiplicities, and the y-intercept of each polynomial function.

	zero	multiplicity	y-int
$P(x)$	-1	2	9
	3	2	
$Q(x)$	-1	2	-9
	3	2	



- c) Write the equations of the polynomials in factored form.

$$P(x) = (x+1)^2(x-3)^2 \quad Q(x) = -(x+1)^2(x-3)^2$$

5. A cubic polynomial function has the equation $P(x) = ax^3 + bx^2 + cx + d$ with a leading coefficient of 1. The zeros of the polynomial are -6, 1, and 3.

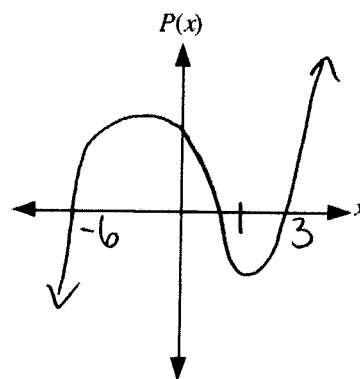
- a) Sketch the graph of $P(x)$ and write the equation of the polynomial in factored form.

$$P(x) = (x+6)(x-1)(x-3)$$

- b) Determine the values of $a, b, c,$ and d in $P(x)$.

$$\begin{aligned} P(x) &= (x+6)(x^2-4x+3) \\ &= x^3 - 4x^2 + 3x + 6x^2 - 24x + 18 \\ &= x^3 + 2x^2 - 21x + 18 \end{aligned}$$

$$a=1 \quad b=2 \quad c=-21 \quad d=18$$



6. A cubic polynomial function has the equation $P(x) = ax^3 + bx^2 + cx + d$ with a leading coefficient of 1. The function has two real distinct zeros. The zero 2 has multiplicity one, and the zero -3 has multiplicity two.

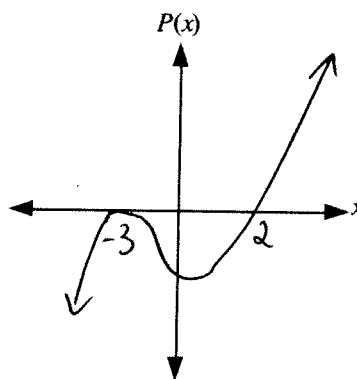
- a) Sketch the graph of the function and write the equation of the polynomial in factored form.

$$P(x) = (x-2)(x+3)^2$$

- b) Determine the values of $a, b, c,$ and d in $P(x)$.

$$\begin{aligned} P(x) &= (x-2)(x+3)(x+3) \\ &= (x-2)(x^2+6x+9) \\ &= x^3 + 6x^2 + 9x - 2x^2 - 12x - 18 \\ &= x^3 + 4x^2 - 3x - 18 \end{aligned}$$

$a=1 \quad b=4 \quad c=-3 \quad d=-18$



- c) A new function is formed by changing the signs of each of the values of $a, b, c,$ and d . Describe how the graph of the new function compares to the graph of $P(x)$.

The graph of the new function is a reflection in the x -axis of the graph of $P(x)$.

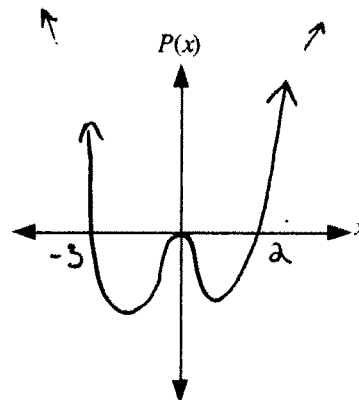
7. A polynomial function has the equation

$$P(x) = x^2(x - 2)(x + 3).$$

a) Make a rough sketch without using a graphing calculator. Verify using a graphing calculator.

b) State the zeros, their multiplicities, and the y-intercept of $P(x)$.

zero	multiplicity	y-intercept
-3	1	0
0	2	
2	1	



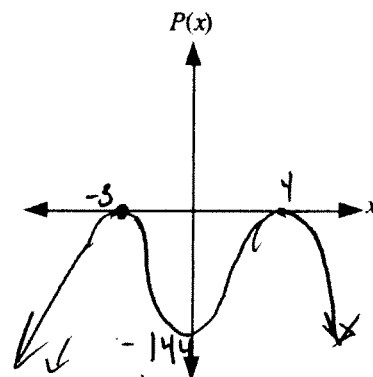
8. A polynomial function has the equation

$$P(x) = -(x - 4)^2(x + 3)^2.$$

a) Make a rough sketch without using a graphing calculator. Verify using a graphing calculator.

b) State the zeros, their multiplicities, and the y-intercept of $P(x)$.

z	m	y-int
-3	2	$-(-4)^2(3)^2$
4	2	$= -144$



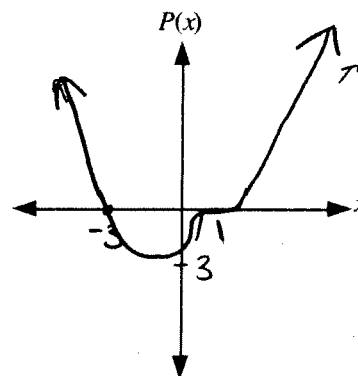
9. A polynomial function has the equation

$$P(x) = (x - 1)^3(x + 3).$$

a) Make a rough sketch without using a graphing calculator. Verify using a graphing calculator.

b) State the zeros, their multiplicities, and the y-intercept of $P(x)$.

z	m	y-int
-3	1	$(-1)^3(3)$
1	3	$= -3$



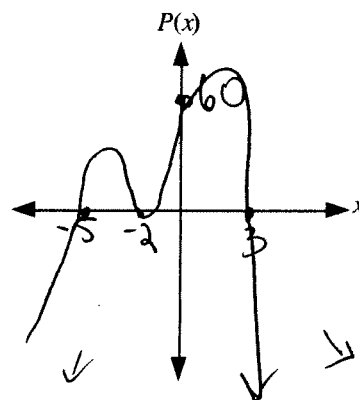
10. A polynomial function has the equation

$$P(x) = (x + 2)^2(x + 5)(3 - x).$$

a) Make a rough sketch without using a graphing calculator. Verify using a graphing calculator.

b) State the zeros, their multiplicities, and the y-intercept of $P(x)$.

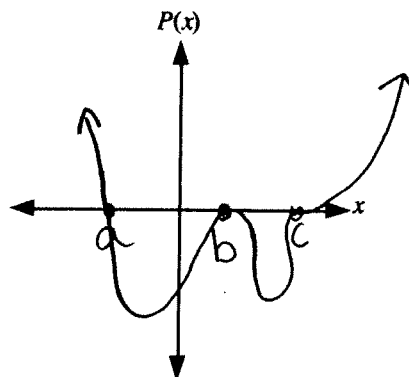
z	m	y-int
-5	1	$(2^2)(5)(3)$
-2	2	$= 60$
3	1	



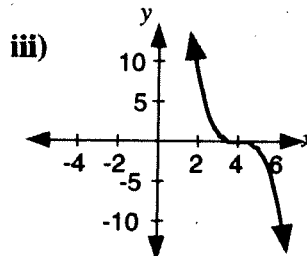
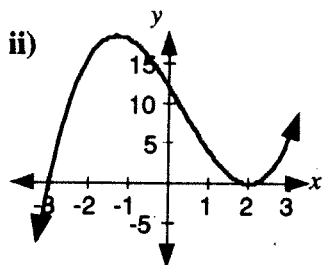
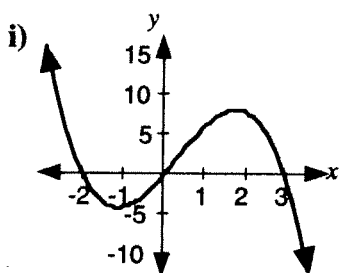
11. Complete the following based on your observations from questions #1 - #10.

- a) If a polynomial function has a zero of multiplicity 1 at $x = a$, then the graph of the function at $x = a$ passes through the x-axis.
- b) If a polynomial function has a zero of multiplicity 2 at $x = b$, then the graph of the function at $x = b$ is tangent to the x-axis.
- c) If a polynomial function has a zero of multiplicity 3 at $x = c$, then the graph of the function at $x = c$ has a point of inflection.
- d) A polynomial function with a leading coefficient of 1 has three distinct zeros.
 - a zero of multiplicity 1 at $x = a$
 - a zero of multiplicity 2 at $x = b$
 - a zero of multiplicity 3 at $x = c$

If $a < b < c$, make a rough sketch of a polynomial which satisfies these conditions.



12. The graphs shown each represent a cubic polynomial function with equation $P(x) = ax^3 + bx^2 + cx + d$, where a is 1 or -1 . The x -intercepts on the graphs are integers



In each case, write $P(x)$ in factored form and determine the values of a, b, c , and d .

i) leading coefficient is negative

zero	m
-2	1
0	1
3	1

$$P(x) = -x(x+2)(x-3)$$

$$P(x) = -x(x^2 - x - 6)$$

$$= -x^3 + x^2 + 6x$$

$$a = -1 \quad b = 1 \quad c = 6 \quad d = 0$$

ii) leading coefficient is positive

zero	m
-3	1
2	2

$$P(x) = (x+3)(x-2)^2$$

$$= (x+3)(x^2 - 4x + 4)$$

$$= x^3 - 4x^2 + 4x + 3x^2 - 12x + 12$$

$$= x^3 - x^2 - 8x + 12$$

$$a = 1, \quad b = -1 \quad c = -8 \quad d = 12$$

iii) leading coefficient is negative

zero	multiplicity
4	3

$$P(x) = -(x-4)^3$$

$$= -(x-4)(x^2 - 8x + 16)$$

$$= -(x^3 - 8x^2 + 16x - 4x^2 + 32x - 64)$$

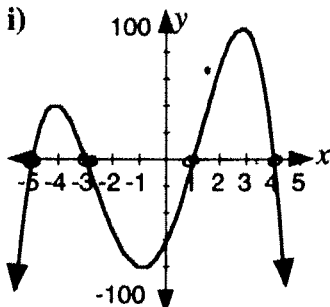
$$= -x^3 + 12x^2 - 48x + 64$$

$$a = -1 \quad b = 12$$

$$c = -48 \quad d = 64$$

13. The graphs shown below each represent a quartic polynomial function with equation $P(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a is 1 or -1 . The zeros of the functions are integers.

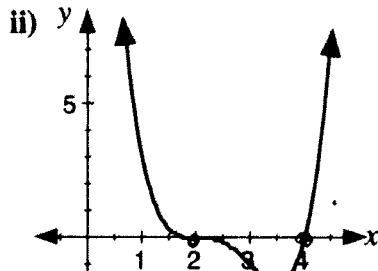
In each case, write the equation of the polynomial function in factored form and determine the value of e .



$$P(x) = -(x+5)(x+3)(x-1)(x-4)$$

$$e = -(5)(3)(-1)(-4)$$

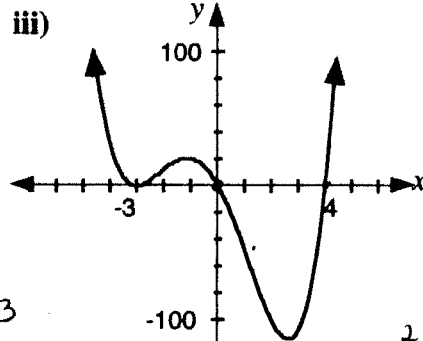
$$= -60$$



$$P(x) = (x-4)(x-2)^3$$

$$e = (-4)(-2)^3$$

$$e = 32$$



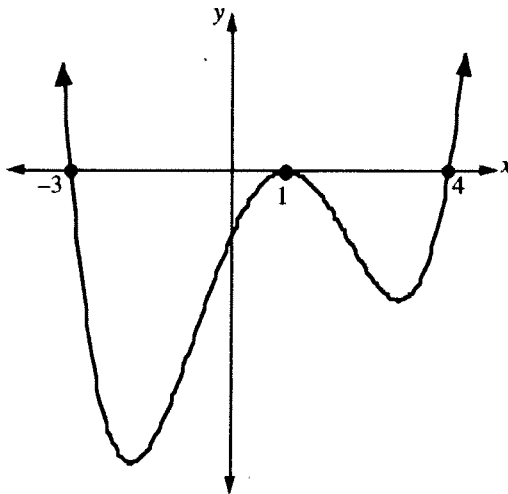
$$P(x) = x(x+3)^2(x-4)$$

$$= 0(3)^2(-4)$$

$$= -\cancel{36} \quad 0$$

Use the following information to answer questions #14 and #15.

The partial graph of a fourth degree polynomial function $P(x)$ is shown. The leading coefficient is 1 and the x -intercepts of the graph are integers.



$$P(x) = (x+3)(x-1)^2(x-4)$$

$$P(x) = (x-1)^2(x-4)(x+3)$$

$$c=1 \quad a=1 \quad b=4 \quad d=3$$

hint =

Numerical Response

14. If the polynomial function is written in the form $P(x) = c(x-a)^2(x-b)(x+d)$, where a, b, c , and d are all positive integers, then the respective numerical values of a, b, c, d from left to right are _____.

(Record your answer in the numerical response box from left to right.)

1	4	1	3
---	---	---	---

15. The graph crosses the y -axis at $(0, -m)$. The value of m is _____.

(Record your answer in the numerical response box from left to right.)

1	2		
---	---	--	--

$$y\text{-int} = (-1)^2(-4)(3)$$

$$= -12$$