Assignment

1. Emma is determining the zeros of the integral polynomial $P(x) = 4x^5 - 2x + 10$. Which of the following rational numbers are potential zeros ?

$\pm 1, \pm 2, \pm 5, \pm 10$ / $\pm 1, \pm 2, \pm 4$

i) $\frac{5}{4}$ ✓ ii) $\frac{5}{2}$ ✓ iii) $\frac{2}{5}$ no iv) -5 ✓ v) 10 ✓ vi) $-\frac{1}{2}$ ✓ vii) $\frac{1}{4}$ ✓

2. Consider the polynomial $g(x) = 6x^3 + 13x^2 + x - 2$ $\pm 1, \pm 2$ / $\pm 1, \pm 2, \pm 3, \pm 6$

a) List the potential zeros of the polynomial.

$\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}$

b) List the potential binomial factors of the polynomial in the form $ax - b$, where $a \in N$ and $b \in I$.

$(x-1)$ $x-2$ $2x-1$ $3x-1$ $3x-2$ $6x-1$

$(x+1)$ $x+2$ $2x+1$ $3x+1$ $3x+2$ $6x+1$

c) Express the polynomial in factored form.

$-2 \mid 6\ 13\ 1\ -2$

$\quad -12\ -2\ \ 2$

$6\ \ 1\ -1\ \ 0$

$= (x+2)(6x^2 - x - 1)$

$6x^2 + x - 1 = 6x^2 - 2x + 3x - 1$

$= 2x(3x-1) + 1(3x-1)$

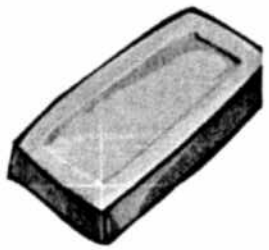
$= (2x+1)(3x-1)$

$g(x) = (x+2)(2x+1)(3x-1)$

d) State **i)** the zeros of $g(x)$ $-2, \frac{1}{3}, -\frac{1}{2}$

ii) the roots of the equation $6x^3 + 13x^2 + x - 2 = 0$ $-2, \frac{1}{3}, -\frac{1}{2}$

3. The volume of a bar of gold is $3x^3 + 23x^2 + 45x + 25$ cm^3. The length, width, and height of the bar can all be expressed in the binomial form $px + q$, where p and q are natural numbers.



a) Determine binomial expressions for the dimensions of the bar.

$-1 \mid 3\ 23\ 45\ 25$

$\quad -3\ -20\ -25$

$3\ \ 20\ \ 25\ \ 0$

$= (x+1)(3x^2 + 20x + 25)$

$3x^2 + 20x + 25$

$= 3x^2 + 15x + 5x + 25$

$= 3x(x+5) + 5(x+5)$

$= (3x+5)(x+5)$

dimensions are : $(x+1)$ cm, $(3x+5)$ cm & $(x+5)$ cm

b) State the dimensions of the bar if $x = 5$.

$(5+1) \cdot (3(5)+5) \quad 5+5$

$= 6 \quad = 20 \quad = 10$

dimensions are $6 \times 10 \times 20$

4. Express $3x^4 - 5x^3 - 17x^2 + 13x + 6$ in factored form.

$$\begin{array}{r|rrrr} 1 & 3 & -5 & -17 & 13 & 6 \\ & & 3 & -2 & -19 & -6 \\ \hline & 3 & -2 & -19 & -6 & 0 \end{array} \quad = (x-1)$$

$$\begin{array}{r|rrrr} -2 & 3 & -2 & -19 & -6 \\ & & -6 & 16 & 6 \\ \hline & 3 & -8 & -3 & 0 \end{array} \quad (x+2)$$

$$= (x-1)(x+2)(3x^2 - 8x - 3)$$

$$3x^2 - 8x - 3$$
$$= 3x^2 - 9x + x - 3$$
$$= 3x(x-3) + 1(x-3)$$
$$= (3x+1)(x-3)$$

$$= (x-1)(x+2)(3x+1)(x-3)$$

5. Consider the function $f(x) = 2x^3 - 9x^2 + 6x - 1$.

a) State the potential zeros of the polynomial.

$\pm 1, \pm \frac{1}{2}$ → $\neq \pm 1$ must be $\pm \frac{1}{2}$.

b) Given that there are no integral zeros, determine a rational zero of the polynomial.

$$f\left(\tfrac{1}{2}\right) = 2\left(\tfrac{1}{2}\right)^3 - 9\left(\tfrac{1}{2}\right)^2 + 6\left(\tfrac{1}{2}\right) - 1$$
$$= \tfrac{1}{4} - \tfrac{9}{4} + 3 - 1 = 0$$

$\frac{1}{2}$ is a zero

c) The zero in b) is the only rational zero. Use the quadratic formula to determine the two irrational zeros.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -9 & 6 & -1 \\ & & 1 & -4 & 1 \\ \hline & 2 & -8 & 2 & 0 \end{array}$$

$$f(x) = \left(x - \tfrac{1}{2}\right)(2x^2 - 8x + 2)$$
$$= \left(x - \tfrac{1}{2}\right)(2)(x^2 - 4x + 1)$$
$$= (2x-1)(x^2 - 4x + 1)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

irrational zeros are: $2 + \sqrt{3}$, $2 - \sqrt{3}$

d) State the roots as exact values of the equation $2x^3 - 9x^2 + 6x - 1 = 0$.

$x = \frac{1}{2}, 2 + \sqrt{3}, 2 - \sqrt{3}$

6. State the zeros of the polynomial function $g(x) = (2x - 5)(x - 7)(3x + 1)$.

$\frac{5}{2}, 7, -\frac{1}{3}$

7. The function $P(x) = 12x^3 - 16x^2 - 7x + 6$ has no integral zeros.

a) State all possible rational zeros of the function.

$\pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}, \pm\frac{2}{3}, \pm\frac{3}{2}, \pm\frac{3}{4}$

$\pm 1, \pm 2, \pm 3, \pm 6$ / $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

b) Express the function in factored form.

$\frac{1}{2}$ | 12 -16 -7 6
6 -5 -6
12 -10 -12 0

$P(x) = (x-\frac{1}{2})(12x^2-10x-12)$ ×2

$= (2x-1)(6x^2-5x-6)$

$6x^2-5x-6 = 6x^2-9x+4x-6$
$= 3x(2x-3)+2(2x-3)$
$= (3x+2)(2x-3)$

$P(x) = (2x-1)(3x+2)(2x-3)$

8. Solve the equation $8x^4 - 12x^3 + 6x^2 - x = 0$.

$x(8x^3-12x^2+6x-1) = 0$

$\frac{1}{2}$ | 8 -12 6 -1
4 -4 1
8 -8 2 0

$= x(x-\frac{1}{2})(8x^2-8x+2)$
$= x(x-\frac{1}{2})(2)(4x^2-4x+1)$
$= x(2x-1)(4x^2-4x+1)$

$4x^2-4x+1 = 4x^2-2x-2x+1$
$= 2x(2x-1)-1(2x-1)$
$= (2x-1)(2x-1)$

$= x(2x-1)(2x-1)(2x-1) = 0$

$x = 0, \frac{1}{2}$

Multiple Choice

9. Jenny is attempting to algebraically find the factors of $P(x) = 6x^3 - 7x^2 - x + 2$. Which of the following factors should she **NOT** consider as a possible factor?

A. $x-1$
B. $x-6$ too big
C. $x+1$
D. $x+2$

$\pm 1, \pm 2$ / $\pm 1, \pm 2, \pm 3, \pm 6$

10. The zeros of the function $f(x) = 2x^3 - x^2 - 18x + 9$ are

A. $\frac{1}{2}, -3, 3$

B. $\frac{1}{2}, 3, 3$

C. $-\frac{1}{2}, 3, 3$

D. $-\frac{1}{2}, -3, 3$

$3 \mid 2 \quad -1 \quad -18 \quad 9$
$\quad\quad 6 \quad 15 \quad -9$
$2 \quad 5 \quad -3 \quad 0$

$= (x-3)(2x^2+5x-3)$

$2x^2 - x + 6x - 3$
$x(2x-1) + 3(2x-1)$
$(x+3)(2x-1)$

$f(x) = (x-3)(x+3)(2x-1)$
$= \pm 3, \frac{1}{2}$

11. If $P(x) = 6x^3 - 11x^2 - x + 6$ and $P\left(\frac{3}{2}\right) = 0$, then the factorization of $P(x)$ is

A. $(2x - 3)(3x + 2)(x - 1)$

B. $(3x - 2)(2x + 3)(x - 1)$

C. $((2x + 3)(3x - 2)(x + 1)$

D. $(3x + 2)(2x - 3)(x + 1)$

$\frac{3}{2} \mid 6 \quad -11 \quad -1 \quad 6$
$\quad\quad 9 \quad -3 \quad -6$
$6 \quad -2 \quad -4 \quad 0$

$P(x) = (x - \frac{3}{2})(6x^2 - 2x - 4)$
$= (2x-3)(3x^2 - x - 2)$

$3x^2 - 3x + 2x - 2$
$3x(x-1) + 2(x-1)$
$(x-1)(3x+2)$

$= (2x-3)(x-1)(3x+2)$

Numerical Response

12. The trinomial $x^2 - 3x - 4$ is a factor of the polynomial $2x^3 - 12x^2 + cx + d$, where c and d are integers. The value of $c + d$ is _____.

(Record your answer in the numerical response box from left to right.)

3	4		

$x^2 - 3x - 4 = (x-4)(x+1)$

$4 \mid 2 \quad -12 \quad c \quad d$
$\quad\quad 8 \quad -16 \quad 4c-64$
$2 \quad -4 \quad c-16 \quad 4c+d-64$

$-1 \mid 2 \quad -12 \quad c \quad d$
$\quad\quad -2 \quad 14 \quad -c-14$
$2 \quad -14 \quad c+14 \quad -c+d-14$

$4c + d - 64 = 0$
$4c + d = 64$

$-c + d - 14 = 0$
$-c + d = 14$

$4c + d = 64$
$-c + d = 14$
subtract $5c = 50$
$c = 10$

$-c + d = 14$
$-10 + d = 14$
$d = 24$

$c + d = 10 + 24 = 34$

Answer Key

1. i), ii), iv), v), vi), vii)

2. **a)** $\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$

b) $x - 1, x + 1, x - 2, x + 2, 2x - 1, 2x + 1, 3x - 1, 3x + 1, 3x - 2, 3x + 2, 6x - 1, 6x + 1$

c) $g(x) = (x + 2)(2x + 1)(3x - 1)$ **d)** $-2, -\frac{1}{2}, \frac{1}{3}$ **e)** $-2, -\frac{1}{2}, \frac{1}{3}$

3. **a)** $3x + 5$ cm, $x + 1$ cm, $x + 5$ cm **b)** 20 cm, 10 cm, 6 cm **4.** $(x - 1)(x + 2)(x - 3)(3x + 1$

5. **a)** $\pm 1, \pm\frac{1}{2}$ **b)** $\frac{1}{2}$ **c)** $2 \pm \sqrt{3}$ **d)** $\frac{1}{2}, 2 \pm \sqrt{3}$ **6.** $-\frac{1}{3}, \frac{5}{2}, 7$

7. **a)** $\pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}, \pm\frac{2}{3}, \pm\frac{3}{2}, \pm\frac{3}{4}$ **b)** $P(x) = (2x - 1)(2x - 3)(3x + 2)$

8. $0, \frac{1}{2}$ **9.** B **10.** A **11.** A **12.**

3	4		