Assignment

1. Consider the polynomial $P(x) = x^3 + x^2 - 4x - 4$

a) List the potential zeros of the polynomial.

$\pm 1, \pm 2, \pm 4$

b) List the potential factors of the polynomial.

$(x+1)(x-1)(x+2)(x-2)(x+4)(x-4)$

c) Express the polynomial in factored form.

① try $x+1$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -4 & -4 \\ & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$(x+1)$ is a factor

② $P(x) = (x+1)(x^2-4)$

③ $P(x) = (x+1)(x-2)(x+2)$

2. Consider the polynomial $P(x) = x^3 - 2x^2 - 5x + 6$

a) List the potential zeros of the polynomial.

$\pm 1, \pm 2, \pm 3, \pm 6$

b) List the potential factors of the polynomial.

$x+1, x-1, x+2, x-2, x+3, x-3, x+6, x-6$.

c) Express the polynomial in factored form.

① try $x+1$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -5 & 6 \\ & & -1 & 3 & 2 \\ \hline & 1 & -3 & -2 & 8 \end{array}$$

② try $x-1$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

$(x-1)$ is a factor

③ $P(x) = (x-1)(x^2-x-6)$

④ $P(x) = (x-1)(x+2)(x-3)$

d) State the zeros of $P(x)$.

$1, -2, 3$

e) State the roots of the equation $x^3 - 2x^2 - 5x + 6 = 0$.

$1, -2, 3$

3. Determine the smallest positive value of k if the possible integral zeros of $P(x) = x^3 - 7x^2 + k$ are

a) $\pm 1, \pm 2, \pm 7$

$k = 14$

what # are these all factors of

b) $\pm 1, \pm 2, \pm 5$, and ± 10

$k = 10$

4. Factor the following polynomials algebraically.

a) $P(x) = x^3 + x^2 - 5x + 3$ $\pm 1, \pm 3$

$P(1) = 1^3 + 1^2 - 5(1) + 3$
$= 1 + 1 - 5 + 3$
$= 0$

1	1	1	-5	3
		1	2	-3
	1	2	-3	0

$P(x) = (x-1)(x^2+2x-3)$
$= (x-1)(x-1)(x+3)$

b) $P(x) = x^3 + x^2 + 2x + 2$ $\pm 1, \pm 2$

$P(1) = 1^3 + 1^2 + 2(1) + 2 = 4$
$P(-1) = (-1)^3 + (-1)^2 + 2(-1) + 2 = 0$

-1	1	1	2	2
		-1	0	-2
	1	0	2	0

$P(x) = (x+1)(x^2+2)$

5. Algebraically determine the zeros of the function $f(x) = x^4 + x^3 - 7x^2 - x + 6$. $\pm 1, \pm 2, \pm 3, \pm 6$

① $f(1) = 1^4 + 1^3 - 7(1)^2 - 1 + 6 = 0$

1	1	1	-7	-1	6
		1	2	-5	-6
	1	2	-5	-6	0

② $f(x) = (x-1)(x^3+2x^2-5x-6)$

③ factor x^3+2x^2-5x-6

-1	1	2	-5	-6
		-1	-1	6
	1	1	-6	0

$f(x) = (x-1)(x+1)(x^2+x-6)$
$f(x) = (x-1)(x+1)(x-2)(x+3)$

6. Algebraically determine the x-intercepts of the graph of $y = x^4 - 2x^3 - 9x^2 + 2x + 8$. $\pm 1, \pm 2, \pm 4, \pm 8$.

1	1	-2	-9	2	8
		1	-1	-10	-8
	1	-1	-10	-8	0

$y = (x-1)(x^3 - x^2 - 10x - 8)$

factor $x^3 - x^2 - 10x - 8$

-1	1	-1	-10	-8
		-1	2	8
	1	-2	-8	0

$y = (x-1)(x+1)(x^2-2x-8)$
$y = (x-1)(x+1)(x+2)(x-4)$

x-int = $\pm 1, -2, 4$

7. Factor the polynomial $x^5 + 2x^4 - 12x^3 - 14x^2 + 11x + 12$.

1	1	2	-12	-14	11	12
		1	3	-9	-23	-12
	1	3	-9	-23	-12	0

$= (x-1)(x^4 + 3x^3 - 9x^2 - 23x - 12)$

-1	1	3	-9	-23	-12
		-1	-2	11	12
	1	2	-11	-12	0

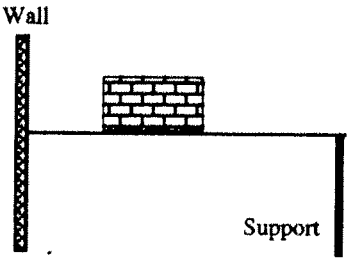
$= (x-1)(x+1)(x^3 + 2x^2 - 11x - 12)$

-1	1	2	-11	-12
		-1	-1	12
	1	1	-12	0

$= (x-1)(x+1)(x+1)(x^2 - x - 12)$
$= (x-1)(x+1)^2(x-3)(x+4)$

Use the following information to answer the next question.

A horizontal beam 8 metres long has its left end built into a wall and its right end resting on a support. A pile of bricks is placed on the beam and the beam sags downward according to the formula $y = f(x) = x^4 - 26x^3 + 224x^2 - 640x$, where x is the distance in metres from the wall and y is the sag in hundredths of a millimetre.



8. a) State a suitable domain for x.

$x \mid 0 \le x \le 8,\ x \in R$.

b) Determine the sag at the midpoint of the beam.

$x=4 \quad f(4) = 4^4 - 26(4)^3 + 224(4)^2 - 640(4) = -384 = 3.84\text{mm}$

c) The function has a zero when $x = 10$. Determine the other zeros of the function and explain what the zeros represent in the context of the question.

$f(x) = x(x^3 - 26x^2 + 224x - 640)$

$= x(x-10)(x^2 - 16x + 64)$

$= x(x-10)(x-8)(x-8)$

10	1	-26	224	-640
		10	-160	640
	1	-16	64	0

Zeros are 0, 8 + 10

The zeros 0 + 8 indicate there is no sag where beam is attached to the wall + where it is supported

The zero of 10 - is outside domain + is meaningless.

9. Consider the polynomial function $P(x) = x^3 + 10x^2 + 8x - 16$.

a) Find the zeros of $P(x)$, to the nearest hundredth, using a graphing calculator.

$-8.90, -2.00, 0.90$

b) Use the integral zero in a) and synthetic division to determine the exact value of the zeros.

$x^3 + 10x^2 + 8x - 16$

$= (x+2)(x^2 + 8x - 8)$

-2	1	10	8	-16
		-2	-16	16
←	1	8	-8	0

Solve $x^2 + 8x - 8$ with quadratic formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-8 \pm \sqrt{(8)^2 - 4(1)(-8)}}{2(1)}$

$= \dfrac{-8 \pm \sqrt{96}}{2} = \dfrac{-8 \pm 4\sqrt{6}}{2} = -4 \pm 2\sqrt{6}$

The zeros are: $-2, -4 - 2\sqrt{6}, -4 + 2\sqrt{6}$

Graph it to help you get 1 zero, use this to divide.

10. Determine the exact roots of the following equations.

a) $x^3 + 4x^2 + 3x - 2 = 0$

$\pm 1, \pm 3, \pm 5, \pm 15$

$-2 \mid 1 \;\; 4 \;\; 3 \;\; -2$
$\quad\quad -2 \;\; -4 \;\; 2$
$\quad 1 \;\; 2 \;\; -1 \;\; (0)$

$= (x+2)(x^2+2x-1)$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-2 \pm \sqrt{2^2-4(1)(-1)}}{2(1)}$

$x = \dfrac{-2 \pm \sqrt{8}}{2} = \dfrac{-2 \pm 2\sqrt{2}}{2}$

$= -1 \pm \sqrt{2}$

roots are: $-2, -1+\sqrt{2}, -1-\sqrt{2}$

b) $x^4 - 12x^2 + 4x + 15 = 0.$

$-1 \mid 1 \;\; 0 \;\; -12 \;\; 4 \;\; 15$
$\quad\quad -1 \;\; 1 \;\; 11 \;\; -15$
$\quad 1 \;\; -1 \;\; -11 \;\; 15 \;\; (0)$

$= (x+1)(x^3 - x^2 - 11x + 15)$

$3 \mid 1 \;\; -1 \;\; -11 \;\; 15$
$\quad\quad 3 \;\; +6 \;\; -15$
$\quad 1 \;\; +2 \;\; -5 \;\; (0)$

$= (x+1)(x-3)(x^2+2x-5)$

$x = \dfrac{-2 \pm \sqrt{2^2-4(1)(-5)}}{2(1)} = \dfrac{-2 \pm \sqrt{24}}{2}$

$= \dfrac{-2 \pm 2\sqrt{6}}{2} = -1 \pm \sqrt{6}$

roots are: $-1, 3, -1+\sqrt{6}, -1-\sqrt{6}$

Multiple Choice **11.** Which of the following is not a potential zero of $P(x) = x^5 - 3x^3 - 6x - 27$.

A. 1 **B.** –3
C. –6 **D.** –27

$\pm 1, \pm 3, \pm 9, \pm 27$

12. Which of the following is not a root of the equation $x^4 - 8x^3 + 13x^2 + 12x - 18 = 0$?

A. 1
B. 3
C. $2 + 2\sqrt{5}$
D. $2 - \sqrt{10}$

$1 \mid 1 \;\; -8 \;\; 13 \;\; 12 \;\; -18$
$\quad\quad 1 \;\; -7 \;\; 6 \;\; 18$
$\quad 1 \;\; -7 \;\; 6 \;\; 18 \;\; (0)$

$3 \mid 1 \;\; -7 \;\; 6 \;\; 18$
$\quad\quad 3 \;\; -12 \;\; -18$
$\quad 1 \;\; -4 \;\; -6 \;\; (0)$

$= (x-1)(x^3 - 7x^2 + 6x + 18) = (x-1)(x-3)(x^2 - 4x - 6)$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2(1)} = \dfrac{4 \pm \sqrt{40}}{2} = \dfrac{4 \pm 2\sqrt{10}}{2}$

$= 2 \pm \sqrt{10}$ roots are: $1, 3, 2-\sqrt{10}, 2+\sqrt{10}$

Numerical Response **13.** The polynomial function $P(x) = x^3 - 7x^2 + 16x - 12$ can be written in the form $P(x) = (x - a)(x - b)^2$ where a and b are integers. The value of b is _____ .

(Record your answer in the numerical response box from left to right.)

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$$2 \mid 1 \quad -7 \quad 16 \quad -12$$
$$\quad\quad\;\; 2 \quad -10 \quad 12$$
$$\quad 1 \quad -5 \quad 6 \quad 0$$

$= (x-2)(x^2-5x+6)$

$= (x-2)(x-2)(x-3)$

$= (x-2)^2(x-3)$

$a = 3 \quad b = 2$

Answer Key

1. a) $\pm1, \pm2, \pm4$ **b)** $x-1, x+1, x-2, x+2, x-4, x+4$ **c)** $P(x) = (x-2)(x+2)(x+1$

2. a) $\pm1, \pm2, \pm3, \pm6$ **b)** $x-1, x+1, x-2, x+2, x-3, x+3, x-6, x+6$
c) $P(x) = (x-1)(x+2)(x-3)$ **d)** $-2, 1, -3$ **e)** $-2, 1, -3$

3. a) 14 **b)** 10

4. a) $P(x) = (x-1)^2(x+3)$ **b)** $(x+1)(x^2+2)$

5. $-3, -1, 1, 2$

6. $-2, -1, 1, 4$

7. $(x-1)(x+1)^2(x-3)(x+4)$

8. a) $\{x \mid 0 \le x \le 8, x \in R\}$ **b)** 3.84 mm
c) The zeros are 0, 8 (twice), and 10. The zeros at 0 and 8 indicate that there is no sag where the beam is attached to the wall and where it is supported. The zero at 10 is outside the domain and has no meaning in the context of this question.

9. a) $-8.90, -2.00, 0.90$ **b)** $-2, -4 - 2\sqrt{6}, -4 + 2\sqrt{6}$

10. a) $-2, -1 - \sqrt{2}, -1 + \sqrt{2}$ **b)** $-1, 3, -1 - \sqrt{6}, -1 + \sqrt{6}$

11. C **12.** C **13.**

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