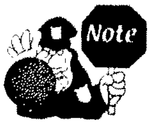

Class Ex. #8 shows that if the sum of the coefficients of a polynomial function is equal to zero, then 1 is a zero of the function.

Complete Assignment Questions #6 - #20

Assignment

1. Use the remainder theorem to find the remainder when each of the following polynomials divided by the binomial.

a) $P(x) = 3x^3 - x^2 + 2x + 1$ is divided by $x + 5$

$P(-5) = 3(-5)^3 - (-5)^2 + 2(-5) + 1$
$= -375 - 25 - 10 + 1$
$= -409$
remaineder = -409.

b) $P(x) = x^4 + x^2 - 8x + 5$ is divided by $x - 4$

$P(4) = (4)^4 + (4)^2 - 8(4) + 5$
$= 256 + 16 - 32 + 5$
$= 245$
$R = 245$.

2. Find the values of p and q if $x^3 + px + q$ yields remainders of -3 and 2 when divided by $x - 2$ and $x + 1$ respectively.

1 $P(2) = -3$
$P(-1) = 2$

2 $-3 = 2^3 + p(2) + q$
$-3 = 8 + 2p + q$
$-11 = 2p + q$

3 $2 = (-1)^3 + p(-1) + q$
$2 = -1 + (-1)p + q$
$3 = -p + q$

4 $2p + q = -11$
subtract. $-p + q = 3$
$3p = -14$
$p = \frac{-14}{3}$

5 $\frac{14}{3} + q = 3$
$q = \frac{-5}{3}$

6 $\boxed{p = \frac{-14}{3}, q = \frac{-5}{3}}$

3. When $P(x) = x^4 + mx^3 - nx^2 + 28x - 24$ is divided by $x - 3$, the remainder is 6. If $P(1) = -4$, find the values of m and n.

1 $P(3) = 6$
$P(1) = -4$

2 $P(3) = 3^4 + m(3)^3 - n(3)^2 + 28(3) - 24 = 6$
$81 + 27 - 9n + 84 - 24 = 6$
$-135 = 27m - 9n$
$13m - n = -15$

3 $P(1) = (1)^4 + m(1)^3 - n(1)^2 + 28(1) - 24$
$-4 = 1 + m - n + 28 - 24$
$-m + n = 9$

4 $3m - n = -15$
$-m + n = 9$
add
$2m = -6$
$m = -3$

5 $-m + n = 9$
$-(-3) + n = 9$
$n = 6$

$\boxed{m = -3, n = 6}$

4. When $x^4 + ax^2 - 16$ is divided by $x + 1$, the remainder is -14.
What is the remainder when $x^4 + ax^2 - 16$ is divided by $x - 2$?

$P(-1) = -14$

$P(-1) = (-1)^4 + a(-1)^2 - 16$

$-14 = 1 + a - 16$

$a = 1$

$P(x) = x^4 + ax^2 - 16$

$P(2) = 2^4 + 2^2 - 16$

$= 4$

$\boxed{R = 4}$

5. $P(x)$ is a polynomial which has a remainder of 2 when it is divided by $x + 3$. Find the remainder when the following polynomials are divided by $x + 3$.

$P(-3) = 2$

(i) $P(x) - 1$

$= 2 - 1$

$R = 1$

(ii) $P(x) + 2x + 6$

$= P(x) + 2(x+3)$

$= 2 + 0$

$R = 2$

(iii) $3P(x)$

$= 3(2)$

$R = 6$

6. If $x - a$ is a factor of the polynomial $P(x)$, what is the remainder obtained when $P(x)$ is divided by $x - a$?

zero

7. Determine which of the following binomials are factors of $P(x) = x^3 - 4x^2 - x + 4$.

a) $x - 1$

$P(1) = 1^3 - 4(1)^2 - 1 + 4$

$= 1 - 4 - 1 + 4$

$P(1) = 0$

$x-1$ is a factor.

b) $x - 2$

$P(2) = 2^3 - 4(2)^2 - 2 + 4$

$= 8 - 16 - 2 + 4$

$= -6$

$P(2) \neq 0$

not a factor.

c) $x + 2$

$P(-2) = (-2)^3 - 4(-2)^2 - (-2) + 4$

$= -8 - 16 + 2 + 4$

$= -18$

$P(-2) \neq 0$, not a factor.

d) $x - 4$

$P(4) = 4^3 - 4(4)^2 - 4 + 4$

$= 64 - 64 - 4 + 4$

$P(4) = 0$

$x-4$ is a factor.

8. Find the value of a so that $x + 1$ is a factor of $x^4 + 4x^3 + ax^2 + 4x + 1$.

$P(-1) = (-1)^4 + 4(-1)^3 + a(-1)^2 + 4(-1) + 1$

$0 = 1 - 4 + a - 4 + 1$

$0 = a - 6$

$\boxed{6 = a}$

9. When $P(x) = 2x^3 + ax^2 + bx + 6$ is divided by $x + 2$, the remainder is -12. If $x - 1$ is a factor of the polynomial, find the values of a and b.

$P(-2) = -12$

$-12 = 2(-2)^3 + a(-2)^2 + b(-2) + 6$

$-12 = -16 + 4a - 2b + 6$

$-2 = 4a - 2b$

$2a - b = -1$

$P(1) = 0$

$0 = 2(1)^3 + a(1)^2 + b(1) + 6$

$0 = 2 + a + b + 6$

$-8 = a + b$

$2a - b = -1$

add $a + b = -8$

$3a = -9$

$a = -3$

$a + b = -8$

$-3 + b = -8$

$b = -5$

$\boxed{a = -3,\ b = -5}$

10. If $P(x) = x^3 + kx^2 - x - 2$ and $P(-2) = 0$, determine the complete factoring of $P(x)$.

$P(-2) = (-2)^3 + k(-2)^2 - (-2) - 2$

$0 = -8 + 4k + 2 - 2$

$8 = 4k$

$2 = k$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -1 & -2 \\ & & -2 & 0 & 2 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$P(x) = (x+2)(x^2 - 1)$ $x^2 - 1$

$\boxed{P(x) = (x+2)(x-1)(x+1)}$

11. Show that -4 is a zero of $P(x) = 6x^3 + 25x^2 + 2x - 8$ and find the other zeros.

① $P(-4) = 0$

$0 = 6(-4)^3 + 25(-4)^2 + 2(-4) - 8$

$0 = -384 + 400 - 8 - 8$

$= 0$

-4 is a zero

②
$$\begin{array}{r|rrrr} -4 & 6 & 25 & 2 & -8 \\ & & -24 & -4 & 8 \\ \hline & 6 & 1 & -2 & 0 \end{array}$$

$= 6x^2 + x - 2$

③ $6x^2 + x - 2$ ← factor to get zeros.

$6x^2 - 3x + 4x - 2$

$3x(2x - 1)\ 2(2x - 1)$

$= (3x + 2)(2x - 1)$ → other zeros are $-\frac{2}{3}, \frac{1}{2}$

12. Given that $x^2 + 2x - 3$ is a factor of $f(x) = x^4 + 2x^3 - 7x^2 + ax + b$, find a and b and hence factor $f(x)$ completely.

① $x^2+2x-3 = (x+3)(x-1)$

$x = -3, 1$

$f(1)=0$

② $f(-3)=0$

$0 = (1)^4 + 2(1)^3 - 7(1)^2 + a(1) + b$

③ $0 = 1 + 2 - 7 + a + b$

$4 = a + b$

$0 = (-3)^4 + 2(-3)^3 - 7(-3)^2 + a(-3) + b$

④ $0 = 81 - 54 - 63 - 3a + b$

$3a - b = -36$

⑤ $3a - b = -36$

add $a + b = 4$

$4a = -32$

$a = -8$

⑥ $a + b = 4$

$-8 + b = 4$

$b = 12$

$f(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$

⑦
$$\begin{array}{r|rrrrr} -3 & 1 & 2 & -7 & -8 & 12 \\ & & -3 & & 12 & -12 \\ \hline & 1 & -1 & -4 & 4 & 0 \end{array}$$

⑧
$$\begin{array}{r|rrrr} 1 & 1 & -1 & -4 & 4 \\ & & 1 & 0 & -4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

⑨ $f(x) = (x+3)(x-1)(x^2-4)$

$\boxed{= (x+3)(x-1)(x-2)(x+2)}$

13. Find the remainder when

a) $2x^4 + x^3 - 3x^2 + 3x - 4$ is divided by $2x - 1$

$P(\tfrac{1}{2}) = 2(\tfrac{1}{2})^4 + (\tfrac{1}{2})^3 - 3(\tfrac{1}{2})^2 + 3(\tfrac{1}{2}) - 4$

$= \tfrac{1}{8} + \tfrac{1}{8} - \tfrac{3}{4} + \tfrac{3}{2} - 4$

$\boxed{R = -3}$

b) $3t^3 - 2t + 2$ is divided by $3t + 1$

$P(-\tfrac{1}{3}) = 3(-\tfrac{1}{3})^3 - 2(-\tfrac{1}{3}) + 2$

$= -\tfrac{1}{9} + \tfrac{2}{3} + 2$

$\boxed{R = \tfrac{23}{9}}$

14. For $P(x) = x^2 - x + 1$, find a if $P(a) = 3$.

$3 = a^2 - a + 1$

$0 = a^2 - a - 2$

$= (a-2)(a+1)$

$a = 2$ or -1

$\boxed{a = -1, 2}$

Multiple Choice **15.** When a polynomial $P(x)$ is divided by $x - 2$, the remainder is 3. If the polynomial $A(x) = 2P(x)$ is divided by $x - 2$, the remainder will be

A. 1.5
B. 2
C. 3
D. 6

$2(3) = 6$

when $x = 0$

16. If a polynomial $P(x)$ has $P(0) = 8$, then which of the following statements **must** be true

A. The constant term in $P(x)$ is 8. B. The constant term in $P(x)$ is -8.
C. A factor of $P(x)$ is $x + 8$. D. A factor of $P(x)$ is $x - 8$.

17. If a polynomial $P(x)$ has $P(8) = 0$, then which of the following statements **must** be true

$(x-8)$

A. The constant term in $P(x)$ is 8. B. The constant term in $P(x)$ is -8.
C. A factor of $P(x)$ is $x + 8$. D. A factor of $P(x)$ is $x - 8$.

18. When a polynomial $P(x)$ is divided by $x + 5$, the remainder is -2. Which of the following statements is true?

$P(-5) = -2$

A. $P(-2) = -5$ B. $P(-2) = 5$ C. $P(5) = -2$ D. $P(-5) = -2$

Numerical Response

19. The polynomial $P(x) = 2x^3 - ax^2 - 11x + 2a$ has a remainder of 126 when divided by $x - 5$. The value of a, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right.)

3	.	0	

$p(5) = 2(5^3) - a(5)^2 - 11(5) + 2a$

$126 = 250 - 25a - 55 + 2a$

$23a = 69$

$a = 3$

20. When $x^3 - 4x^2 + 3$ and $x^3 - 3x^2 - 8x + 19$ are each divided by $x - a$, the remainders are equal. To the nearest tenth, the value of a is _____ .

$P(a) = 0$

(Record your answer in the numerical response box from left to right.)

4	.	0	

$a^3 - 4a^2 + 3 = a^3 - 3a^2 - 8a + 19$

$0 = a^2 - 8a + 16$

$0 = (a-4)(a-4)$

$a = 4$

Answer Key

1. a) -409 b) 245
2. $p = -\frac{14}{3}, q = -\frac{5}{3}$
3. $m = -3, n = 6$
4. 4
5. (i) 1 (ii) 2 (iii) 6
6. 0
7. a and d
8. $a = 6$
9. $a = -3, b = -5$
10. $(x + 2)(x + 1)(x - 1)$
11. $-\frac{2}{3}, \frac{1}{2}$
12. $a = -8, b = 12$ $(x + 3)(x - 1)(x + 2)(x - 2)$
13. a) -3 b) $\frac{23}{9}$
14. $a = -1$ or $a = 2$
15. D
16. A
17. D
18. D
19. 3 . 0
20. 4 . 0