Assignment

1. In the partially completed synthetic division below, a polynomial $P(x)$ is divided by $x - 2$.

$$\begin{array}{c|cccc} & 1 & -2 & 6 & 3 \\ \hline & 1 & 0 & 6 & 15 \end{array}$$

a) State the polynomial $P(x)$.   b) State the quotient.   c) State the remainder.

$$x^3 - 2x^2 + 6x + 3 \qquad\qquad x^2 + 6 \qquad\qquad 15$$

d) Write the above synthetic division in the form of the division algorithm.

$$x^3 - 2x^2 + 6x + 3 = (x-2)(x^2+6) + 15$$

2. Use synthetic division to divide the polynomial by the binomial and express each in the form $P(x) = D(x) \cdot Q(x) + R(x)$.

a) $x^3 + 2x^2 + 3x + 6; \quad x - 2$

$$\begin{array}{c|cccc} 2 & 1 & 2 & 3 & 6 \\ & \downarrow & 2 & 8 & 22 \\ \hline & 1 & 4 & 11 & \boxed{28} \end{array}$$

$$x^3 + 2x^2 + 3x + 6 = (x+4x+11)$$
$$(x-2)(x^2+4x+11) + 28$$

b) $2x^3 - 4x^2 - 5x + 9; \quad x + 2$

$$\begin{array}{c|cccc} -2 & 2 & -4 & -5 & 9 \\ & \downarrow & -4 & 16 & -22 \\ \hline & 2 & -8 & 11 & \boxed{-13} \end{array}$$

$$2x^3 - 4x^2 - 5x + 9 = x^2 - 8x$$
$$(x+2)(x^2 - 8x + 11) - 13$$

c) $x^4 - x^2 + 7; \quad x + 1$

$$\begin{array}{c|ccccc} -1 & 1 & 0 & -1 & 0 & 7 \\ & \downarrow & -1 & 1 & 0 & 0 \\ \hline & 1 & -1 & 0 & 0 & 7 \end{array}$$

$$x^4 - x^2 + 7 = (x+1)(x^3 - x^2) + 7$$

d) $2y^4 - y^5 - y^3 + 4y; \quad y - 3 \qquad -y^5 + 2y^4 - y^3 + 4y$

$$\begin{array}{c|cccccc} 3 & -1 & 2 & -1 & 0 & 4 & 0 \\ & \downarrow & -3 & -3 & -12 & -36 & -96 \\ \hline & -1 & -1 & -4 & -12 & -32 & \boxed{-96} \end{array}$$

$$2y^4 - y^5 - y^3 + 4y = (y-3)(-y^4 - y^3 - 4y^2 - 12y - 32) - 96$$

3. Determine p, q, and r in the partially completed synthetic division below in which the divisor is $x - 1$.

$$p = 2 + 3 = 5$$
$$q + 5 = 7$$
$$q = 2$$
$$r = 8$$

$$\begin{array}{c|cccc} 1 & 2 & 3 & q & 1 \\ & \downarrow & 2 & 5 & 7 \\ \hline & 2 & p & 7 & r \end{array}$$

$$p = 5, \quad q = 2, \quad r = 8$$

4. Determine m and n in the partially completed synthetic division below in which the divisor is $x + 2$.

$$m + -4 = -4 \quad m = 0$$

$$n + (-10) = 0$$
$$n = 10$$

$$\boxed{m = 0, \; n = 10}$$

$$-2 \,|\, \begin{array}{cccc} 2 & m & -3 & n \\ & -4 & 8 & -10 \end{array}$$
$$\begin{array}{cccc} 2 & -4 & 5 & m \; 0 \end{array}$$

5. Find the remainder on dividing $x^3 - 3x^2 + x + 8$ by $x - 2$. Compare this with $f(2)$ where $f(x) = x^3 - 3x^2 + x + 8$.

$$2 \,|\, \begin{array}{cccc} 1 & -3 & 1 & 8 \\ & 2 & -2 & -2 \end{array}$$
$$\begin{array}{cccc} 1 & -1 & -1 & \boxed{6} \end{array}$$

remainder = $\underline{6}$

$$f(2) = 2^3 - 3(2)^2 + 2 + 8$$
$$= 8 - 12 + 2 + 8$$
$$= 6$$

they match

6. Find the remainder on dividing $12 - 5x + 3x^2 + 2x^3$ by $x + 3$. Compare this with $P(-3)$ where $P(x) = 12 - 5x + 3x^2 + 2x^3$.

$$-3 \,|\, \begin{array}{cccc} 2 & 3 & -5 & 12 \\ & -6 & 9 & -12 \end{array}$$
$$\begin{array}{cccc} 2 & -3 & 4 & \boxed{0} \end{array}$$

remainder = 0

$$P(-3) = 12 - 5(-3) + 3(-3)^2 + 2(-3)^3$$
$$= 12 + 15 + 27 - 54$$
$$= 0$$

→ remainder on dividing $P(x)$ by $x + 3 = P(-3)$

7. When $2x^3 + ax^2 - 3x + 4$ is divided by $x + 1$, the remainder is 8. Determine the value of a.

$$-1 \,|\, \begin{array}{cccc} 2 & a & -3 & 4 \\ & -2 & -a+2 & a+1 \end{array}$$
$$\begin{array}{cccc} 2 & a-2 & -a-1 & \boxed{a+5} \end{array}$$

$$a + 5 = 8$$
$$\underline{a = 3}$$

8. Divide $9x^3 + 18x^2 - 13x + 5$ by $3x - 1$ using synthetic division and write the division in the form $P = DQ + R$.

$$3x - 1)$$
$$x = \frac{1}{3}$$

$$\tfrac{1}{3} \,|\, \begin{array}{cccc} 9 & 18 & -13 & 5 \\ & 3 & 7 & -2 \end{array}$$
$$\begin{array}{cccc} 9 & 21 & -6 & \boxed{3} \end{array}$$

$$P = \left(x - \tfrac{1}{3}\right)\left(9x^2 + 21x - 6\right) + 3$$
$$P = \left(x - \tfrac{1}{3}\right)(3)\left(3x^2 + 7x - 2\right) + 3$$
$$P = (3x - 1)\left(3x^2 + 7x - 2\right) + 3$$

$$9x^3 + 18x^2 - 13x + 5 = (3x - 1)(3x^2 + 7x - 2) + 3.$$

9. Divide $4x^3 + 11x^2 - 14x - 9$ by $4x + 3$ using synthetic division and write the division in the form $P = DQ + R$.

$$\cdot 4x + 3$$

$$x = -\frac{3}{4}$$

$$-\frac{3}{4} \,\Big|\; \begin{array}{cccc} 4 & 11 & -14 & -9 \\ & -3 & -6 & 15 \\ \hline 4 & 8 & -20 & \boxed{6} \end{array}$$

$$P = \left(x + \frac{3}{4}\right)\left(4x^2 + 8x - 20\right) + 6$$

$$P = \left(x + \frac{3}{4}\right)(4)(x^2 + 2x - 5) + 6$$

$$P = (4x + 3)(x^2 + 2x - 5) + 6$$

$$4x^3 + 11x^2 - 14x - 9 = (4x + 3)(x^2 + 2x - 5) + 6$$

Multiple Choice

10. When the polynomial $2a^3 - 7a + 6$ is divided by $a - 4$, the remainder is

- **A.** -94
- **B.** 10
- **C.** 66
- **D.** 106

$$4 \,\Big|\; \begin{array}{cccc} 2 & 0 & -7 & 6 \\ & 8 & 32 & 100 \\ \hline 2 & 8 & 25 & \boxed{106} \end{array}$$

Numerical Response

11. When the polynomial $3y^3 - 4y^2 + by + 6$ is divided by $y + 2$, the remainder is -40. The value of b is _____.

(Record your answer in the numerical response box from left to right.)

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$$-2 \,\Big|\; \begin{array}{cccc} 3 & -4 & b & 6 \\ & -6 & 20 & -2b-40 \\ \hline 3 & -10 & b+20 & \boxed{-2b-34} \end{array}$$

$$-2b - 34 = -40$$
$$-2b = -6$$
$$b = 3$$

Answer Key

1. a) $x^3 - 2x^2 + 6x + 3$ **b)** $x^2 + 6$ **c)** 15 **d)** $x^3 - 2x^2 + 6x + 3 = (x - 2)(x^2 + 6) + 15$

2. a) $x^3 + 2x^2 + 3x + 6 = (x - 2)(x^2 + 4x + 11) + 28$
b) $2x^3 - 4x^2 - 5x + 9 = (x + 2)(2x^2 - 8x + 11) - 13$
c) $x^4 - x^2 + 7 = (x + 1)(x^3 - x^2) + 7$
d) $-y^5 + 2y^4 - y^3 + 4y = (y - 3)(-y^4 - y^3 - 4y^2 - 12y - 32) - 96$

3. $p = 5, q = 2, r = 8$ **4.** $m = 0, n = 10$ **5.** $6, 6$ **6.** $0, 0$ **7.** 3

8. $9x^3 + 18x^2 - 13x + 5 = (3x - 1)(3x^2 + 7x - 2) + 3$

9. $4x^3 + 11x^2 - 14x - 9 = (4x + 3)(x^2 + 2x - 5) + 6$ **10.** D **11.**

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