

Assignment

1. Divide the following polynomials. Express the answer in the form $P = DQ + R$.

a) $\frac{x^2 + 5x + 4}{x + 2}$

$$\begin{array}{r} x+2 \overline{) x^2+5x+4} \\ \underline{x^2+2x} \\ 3x+4 \\ \underline{3x+6} \\ -2 \end{array}$$

$$x^2 + 5x + 4 = (x+2)(x+3) - 2$$

b) $\frac{2x^2 - 5x + 2}{x - 3}$

$$\begin{array}{r} x-3 \overline{) 2x^2-5x+2} \\ \underline{2x^2-6x} \\ x+2 \\ \underline{x-3} \\ 5 \end{array}$$

$$2x^2 - 5x + 2 = (x-3)(2x+1) + 5$$

c) $\frac{6x^2 - 5x - 3}{x - 1}$

$$\begin{array}{r} x-1 \overline{) 6x^2-5x-3} \\ \underline{6x^2-6x} \\ x-3 \\ \underline{x-1} \\ -2 \end{array}$$

$$6x^2 - 5x - 3 = (x-1)(6x+1) - 2$$

2. Divide the following polynomials. Express the answer in the form $\frac{P}{D} = Q + \frac{R}{D}$.

a) $\frac{a^3 - a^2 - 4a + 12}{a - 2}$

$$\begin{array}{r} a-2 \overline{) a^3-a^2-4a+12} \\ \underline{a^3-2a^2} \\ a^2-4a \\ \underline{a^2-2a} \\ -2a+12 \\ \underline{-2a+4} \\ 8 \end{array}$$

$$\frac{a^3 - a^2 - 4a + 12}{a - 2}$$

$$= a^2 + a - 2 + \frac{8}{a-2}$$

b) $\frac{3x^3 - x^2 + 2x + 4}{x + 4}$

$$\begin{array}{r} x+4 \overline{) 3x^3-x^2+2x+4} \\ \underline{3x^3+12x^2} \\ -13x^2+2x \\ \underline{-13x^2-52x} \\ 54x+4 \\ \underline{54x+216} \\ -212 \end{array}$$

$$\frac{3x^3 - x^2 + 2x + 4}{x + 4} = 3x^2 - 13x + 54 - \frac{212}{x+4}$$

3. Determine the quotient and remainder when the following polynomials are divided.

a) $\frac{6x^2 - 5x + 7}{2x - 3}$

$$\begin{array}{r} 3x + 2 \\ 2x - 3 \overline{) 6x^2 - 5x + 7} \\ \underline{6x^2 - 9x} \\ 4x + 7 \\ \underline{2x - 6} \\ 13 \end{array}$$

quotient: $3x + 2$

remainder: 13

b) $\frac{9x^2 - 9}{3x + 1}$

$$\begin{array}{r} 3x - 1 \\ 3x + 1 \overline{) 9x^2 + 0x - 9} \\ \underline{9x^2 + 3x} \\ -3x - 9 \\ \underline{-3x - 1} \\ -8 \end{array}$$

Q: $3x - 1$

R: -8

c) $\frac{12x^3 - 5x^2 + x}{4x - 3}$

$$\begin{array}{r} 3x^2 + x + 1 \\ 4x - 3 \overline{) 12x^3 - 5x^2 + x + 0} \\ \underline{12x^3 - 9x^2} \\ 4x^2 + x \\ \underline{4x^2 - 3x} \\ 4x + 0 \\ \underline{4x - 3} \\ 3 \end{array}$$

Q: $3x^2 + x + 1$

R: 3

4. a) Explain how to determine if $x + 5$ is a factor of $x^3 + 125$.

divide $x^3 + 125$ by $x + 5$, If the remainder is zero then $x + 5$ is a factor of $x^3 + 125$

b) Use the process in a) to determine if $x + 5$ is a factor of $x^3 + 125$.

$$\begin{array}{r} x^2 - 5x + 25 \\ x + 5 \overline{) x^3 + 0x^2 + 0x + 125} \\ \underline{x^3 + 5x^2} \\ -5x^2 + 0x \\ \underline{-5x^2 - 25x} \\ 25x + 125 \\ \underline{25x + 125} \\ 0 \end{array}$$

→ Since remainder is 0 then $x + 5$ is a factor.

5. Given that the degree of $D(x) = 4$, state the possible degrees of $R(x)$ in $P(x) = D(x) \cdot Q(x) + R(x)$.

0, 1, 2, 3

$$A = l w$$

6. A rectangular carpet has an area of $x^3 - 5x + 12$ square metres.

a) If the width of the carpet is $x + 3$ metres, determine an expression for the length of the carpet.

$$l = \frac{A}{w} = \frac{x^3 - 5x + 12}{x + 3}$$

$$l = \underline{x^2 - 3x + 4}$$

$$x+3 \overline{) \begin{array}{r} x^2 - 3x + 12 \\ x^3 + 0x^2 + 5x + 12 \\ \hline \end{array}}$$

$$-3x^2 + 5x$$

$$-3x^2 - 9x$$

$$14x + 12$$

$$14x + 12$$

0

b) If the width of the carpet is 8 metres, determine the length of the carpet.

$$w = x + 3 = 8$$

$$x = 5$$

$$l = 5^2 - 3(5) + 4 = \underline{\underline{14 \text{ metres}}}$$

7. The division shows a polynomial expression in x , written as $P(x)$, being divided by a binomial.

$$3x - 2 \overline{) \begin{array}{r} x^2 - 3x + 2 \\ \hline \end{array}} \begin{array}{l} Q \\ P(x) \end{array}$$

a) Write $P(x)$ in the form $D(x) \cdot Q(x) + R(x)$.

$$P(x) = (3x - 2)(x^2 - 3x + 2) + 17$$

17 R

b) Write $P(x)$ in the form $ax^3 + bx^2 + cx + d$.

$$P(x) = 3x^3 - 9x^2 + 6x - 2x^2 + 6x - 4 + 17$$

$$P(x) = \underline{\underline{3x^3 - 11x^2 + 12x + 13}}$$

8. When a third degree polynomial is divided by $x + 5$, the quotient is $x^2 - 2x - 1$ and the remainder is 7. Express the polynomial in the form $ax^3 + bx^2 + cx + d$.

$$P(x) = (x + 5)(x^2 - 2x - 1) + 7$$

$$= x^3 - 2x^2 - x + 5x^2 - 10x - 5 + 7$$

$$= \underline{\underline{x^3 + 3x^2 - 11x + 2}}$$

Multiple choice

9. When $(3z^4 + 6z^3 - 18z)$ is divided by $(z + 3)$, the remainder is

- A. -45
- B. 45
- C. -135
- D. 135**

$$\begin{array}{r}
 3z^3 - 3z^2 + 9z - 45 \\
 \hline
 z+3 \overline{) 3z^4 + 6z^3 + 0z^2 - 18z + 0} \\
 \underline{3z^4 + 9z^3} \\
 -3z^3 + 0z^2 \\
 \underline{-3z^3 + 9z^2} \\
 9z^2 - 18z \\
 \underline{9z^2 + 27z} \\
 -45z + 0 \\
 \underline{-45z - 135} \\
 135
 \end{array}$$

Numerical Response

10. A rectangle has an area of $8x^2 - 14x - 15$ cm² and a length of $4x + 3$ cm. The perimeter of the rectangle can be written in the form $ax + b$ cm. The value of $a + b$ is _____.

(Record your answer in the numerical response box from left to right.)

8			
---	--	--	--

① find width ② calculate perimeter.

$$w = \frac{A}{l} = \frac{8x^2 - 14x - 15}{4x + 3}$$

$$\begin{array}{r}
 2x - 5 \\
 \hline
 4x + 3 \overline{) 8x^2 - 14x - 15} \\
 \underline{8x^2 + 6x} \\
 -20x - 15 \\
 \underline{-20x - 15} \\
 0
 \end{array}$$

$$w = 2x - 5$$

$$\begin{aligned}
 P &= 2l + 2w \\
 P &= 2(4x + 3) + 2(2x - 5) \\
 &= 8x + 6 + 4x - 5 \\
 &= 12x - 4
 \end{aligned}$$

a: 12
b: -4

$$12 + (-4) = 8$$

11. When the polynomial $ax^3 + bx^2 + cx + d$ is divided by $3x - 2$, the quotient is $2x^2 + 2x + 3$ and the remainder is 7.

Record the value of a in the first box.
Record the value of b in the second box.
Record the value of c in the third box.
Record the value of d in the fourth box.

6	2	5	1
---	---	---	---

$$\begin{aligned}
 &(3x - 2)(2x^2 + 2x + 3) + 7 \\
 &= 6x^3 + 6x^2 + 9x - 4x^2 - 4x - 6 + 7 \\
 &= 6x^3 + 2x^2 + 5x + 1
 \end{aligned}$$