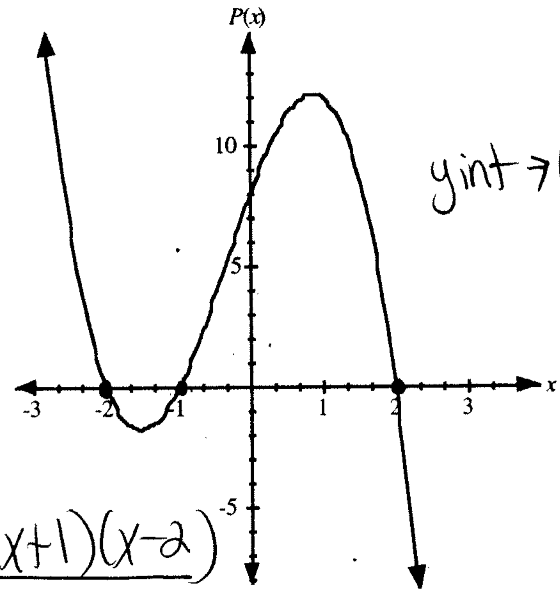
Assignment

1. The graph of the polynomial function shown has integral intercepts.

Determine the equation of the function in factored form.



yint → (0, 8)

$P(x) = c(x+2)(x+1)(x-2)$

$8 = c(0+2)(0+1)(0-2)$

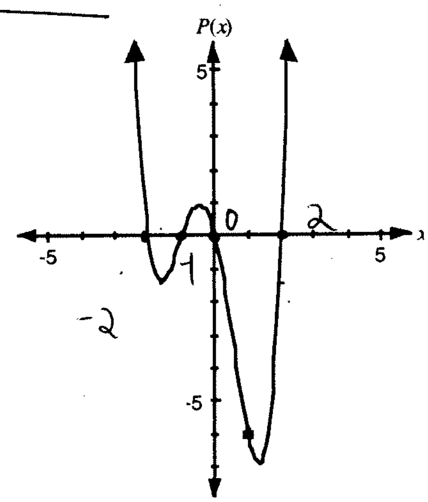
$8 = c(2)(1)(-2)$

$8 = -4c$

$-2 = c \longrightarrow P(x) = -2(x+2)(x+1)(x-2)$

2. The graph passes through the point (1, −6) and has integral *x*-intercepts.

Determine the equation, in factored form, of the polynomial function, *P*(*x*), represented by the graph.



$P(x) = cx(x+2)(x+1)(x-2)$

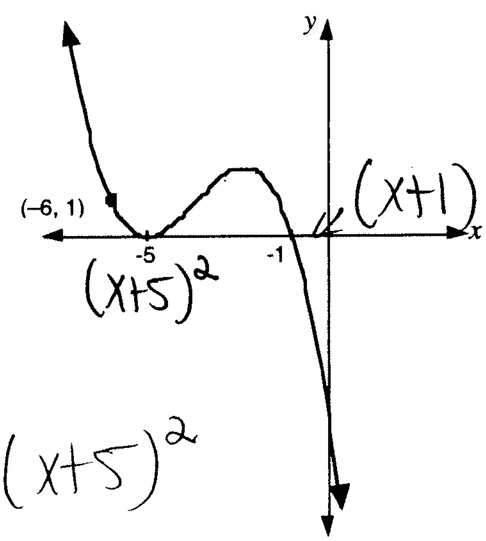
$-6 = c(1)(1+2)(1+1)(1-2)$

$-6 = -6c$

$1 = c$

$P(x) = x(x+2)(x+1)(x-2)$

3. The graph of a third degree polynomial function is shown. The graph passes through the point (−6, 1). If the polynomial function has zeros −5 and −1, determine



a) the equation of the function in factored form;

$P(x) = c(x+1)(x+5)^2$

$1 = c(-6+1)(-6+5)^2$

$1 = -5c$

$-\frac{1}{5} = c \qquad P(x) = -\frac{1}{5}(x+1)(x+5)^2$

b) the *y*-intercept of the graph.

$P(0) = -\frac{1}{5}(0+1)(0+5)^2$

$= -5$

$\boxed{\text{y-int} = -5}$

4. Find the equation of a quartic function whose graph has a point of inflection at the origin and passes through (4, 0) and (−1, 10).

multiplicity of 3 x^3

$(x-4)$

$P(x) = cx^3(x-4)$

$10 = c(-1)^3(-1-4)$

$10 = 5c$

$2 = c$

$P(x) = 2x^3(x-4)$

5. The design of a route for a cross country ski course was drawn on a Cartesian plane. The route is tangent to the x-axis at (1, 0) and (−3, 0). It crosses the x-axis at (−5, 0) and also passes through the point (−2, 9). Determine the equation of the fifth degree polynomial function that will meet these conditions.

$(x-1)^2 \quad (x+3)^2 \quad (x+5)$

$P(x) = c(x+5)(x-1)^2(x+3)^2$

$9 = c(-2+5)(-2-1)^2(-2+3)^2$

$9 = c(3)(9)(1)$

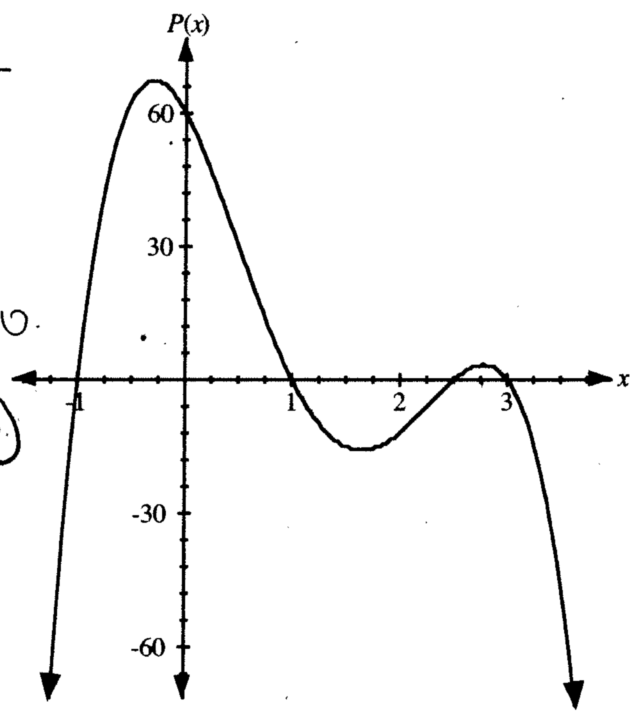
$9 = 27c$

$\frac{1}{3} = c$

$P(x) = \frac{1}{3}(x+5)(x-1)^2(x+3)^2$

6. The graph shown has x-intercepts of −1, 1, 2.5, and 3 and a y-intercept of 60.

Determine the equation of the graph in factored form using integral factors.



$(x+1)(x-1)(2x-5)(x-3)$

$2.5 = \frac{5}{2} = x$

$2x = 5$

$2x - 5 = 0$

$P(x) = c(x+1)(x-1)(2x-5)(x-3)$

$60 = c(0+1)(0-1)(2(0)-5)(0-3)$

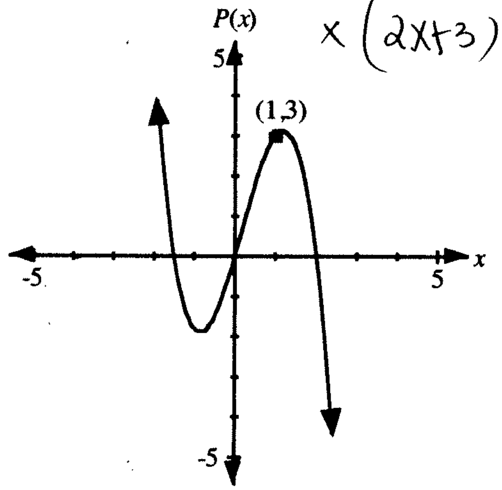
$60 = c(1)(-1)(-5)(-3)$

$60 = -15c$

$-4 = c$

7. The graph below has x-intercepts $-\frac{3}{2}$, 0, and 2 and passes through the point (1, 3). Determine the equation of the graph using integral factors.

$x(2x+3)(x-2)$



$P(x) = cx(2x+3)(x-2)$

$3 = c\cancel{(1)}(2(1)+3)(1-2)$

$3 = c(5)(1)(-1)$

$3 = -5c$

$\frac{-3}{5} = c$

$P(x) = -\frac{3}{5}x(2x+3)(x-2)$

8. Determine the equation of a fifth degree polynomial function whose graph has a point of inflection at (3, 0), is tangent to the x-axis at $\left(\frac{1}{2}, 0\right)$, and passes through (2, 1).

$(x-3)^3$ $\quad(2x-1)^2$ $\quad x = \frac{1}{2}$ $\quad 2x = 1$ $\quad 2x-1 = 0$

$P(x) = c(2x-1)^2(x-3)^3$

$1 = c(2(2)-1)^2(2-3)^3$

$1 = c(-1)(9)$

$1 = -9c$

$-1/9 = c$

$P(x) = -\frac{1}{9}(x-3)^3(2x-1)^2$

Multiple Choice

9. If the zeros of a polynomial are -1, $\frac{1}{2}$ and $\frac{2}{3}$, then the polynomial could be

A. $12x^3 - 2x^2 + 10x - 4$
B. $6x^3 + x^2 - 5x + 2$
C. $30x^3 - 5x^2 - 25x + 10$
D. $18x^3 + 3x^2 - 15x - 6$

$p(x) = c(x+1)(2x-1)(3x-2)$

$= c(x+1)(6x^2 - 7x + 2)$

$= c(6x^3 - 7x^2 + 2x + 6x^2 - 7x + 2)$

$= c(6x^3 - x^2 - 5x + 2)$

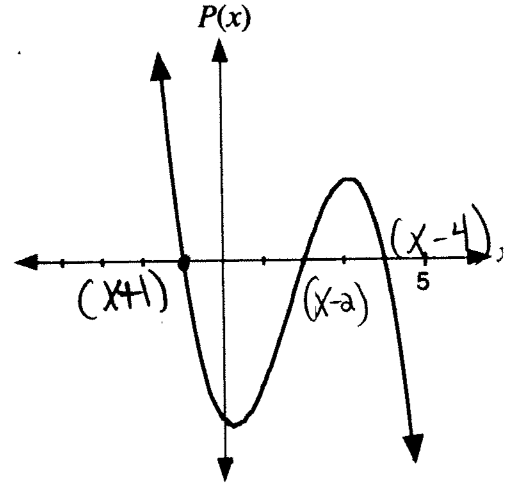
- the only multiple of $6x^3 - x^2 - 5x + 2$ is D

$3(6x^3 - x^2 - 5x + 2 = 18x^3 + 3x^2 - 15x - 6$

10. $P(x) = -3x^3 + bx^2 + cx + d$ is an integral polynomial function with zeros $2, -1$, and 4. A sketch of $y = P(x)$ is shown.

At which of the following points does the graph of $P(x)$ cross the y-axis?

A. $(0, -8)$ **B.** $(0, -15)$
C. $(0, -16)$ **D.** $(0, -24)$



$P(x) = c(x+1)(x-2)(x-4)$
$= c(x+1)(x^2-6x+8)$
$= c(x^3-6x^2+8x+x^2-6x+8)$
$= c(x^3-5x^2+2x+8)$ $c=-3$
y-int $= P(0)$ $-3(8) = -24$

11. The graph of a fourth degree polynomial function has x-intercepts $-2, -1, 0$, and 1. If the graph passes through the point $(-3, -48)$, then the coefficient of the third degree term of $P(x)$ is

A. -4
B. -2
C. -1
D. 2

$(x+2)(x+1)x(x-1)$
$P(x) = cx(x+2)(x+1)(x-1)$
$-48 = c(-3)(-3+2)(-3+1)(-3-1)$
$-48 = c(-3)(-1)(-2)(-4)$
$-48 = 24c$
$-2 = c$
$P(x) = -2x(x+2)(x+1)(x-1)$
✓ Factor
$P(x) = -2x(x+2)(x^2-1)$
$= -2x(x^3+2x^2-x-2$
$= -2x^4 - 4x^3 + 2x^2 + 4x$

12. A third degree polynomial, $f(x)$, has three distinct zeros, $-3, -1$, and 2. If a new polynomial, $g(x)$, is found by multiplying $f(x)$ by $(x + 1)$, then which of the following statements is true?

A. The x-intercepts of the graph of $y = g(x)$ will be $-4, -2$, and 1.
B. The x-intercepts of the graph of $y = g(x)$ will be $-2, 0$, and 3.
C. The y-intercept of the graph of $y = g(x)$ will be the negative of the y-intercept of the graph of $y = f(x)$.
D. The x and y-intercepts of the graph of $y = g(x)$ are the same as the x and y intercep of the graph of $y = f(x)$.

$f(x) = c(x+3)(x+1)(x-2)$ x-int $= -3, -1, 2$ y-int $= c(3)(1)(-2) = -6$
$g(x) = c(x+3)(x+1)^2(x-2)$ x-int $= -3, -1, 2$ y-int $= c(3)(1)^2(-2) = -6$

Numerical Response

13. Consider a fourth degree polynomial function whose graph is tangent to the x-axis at b
(3, 0) and (−4, 0) and passes through (2, 9). The y-intercept of the graph is _____ .
$(x-3)^2$ $(x+4)^2$
(Record your answer in the numerical response box from left to right.)

3	6		

$P(x) = c(x-3)^2(x+4)^2$

$9 = c(2-3)^2(2+4)^2$

$9 = c(1)(36)$

$9 = 36c$

$c = \frac{1}{4}$

$\rightarrow P(x) = \frac{1}{4}(x-3)^2(x+4)^2$

$P(0) = \frac{1}{4}(0-3)^2(0+4)^2$ (y-int)

$= \frac{1}{4}(-3)^2(4)^2$

$= 36$

14. $P(x) = ax^3 + bx^2 + cx + d, a > 0$, is an integral polynomial function with 2 and 5 as its zeros. The graph of $y = P(x)$ is shown.

degree 3

$P(x) = a(x-2)^2(x-5)$

y-int

$P(0) = a(0-2)^2(0-5)$

$= a(4)(-5)$

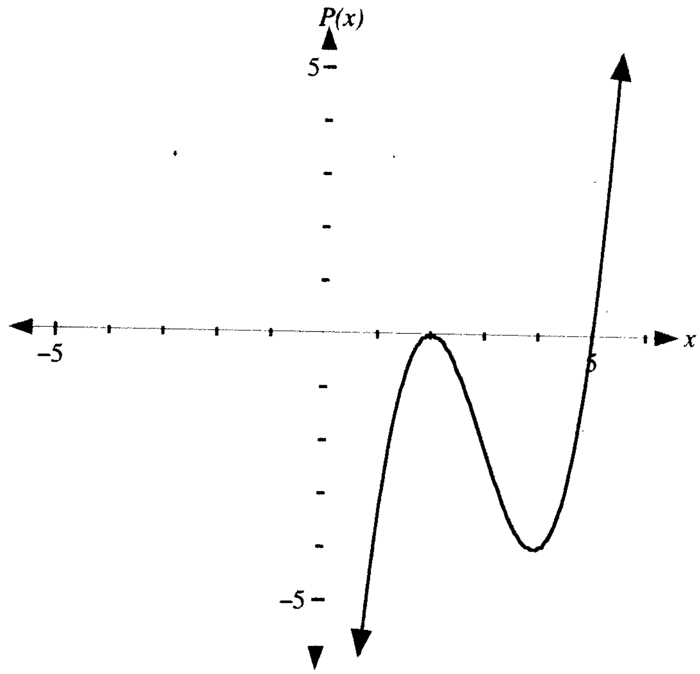
$= -20a$

$a \rightarrow$ positive integer.

max y-int $= -20(1) = -20$

$-m = -20$

$m = 20$



If the maximum y-intercept is at the point $(0, -m)$, then m is _____ .
(Record your answer in the numerical response box from left to right.)

2	0		

Answer Key

1. $P(x) = -2(x + 2)(x + 1)(x - 2)$
2. $P(x) = x(x + 2)(x + 1)(x - 2)$
3. a) $P(x) = -\frac{1}{5}(x + 1)(x + 5)^2$ b) −5
4. $P(x) = 2x^3(x - 4)$
5. $P(x) = \frac{1}{3}(x + 5)(x - 1)^2(x + 3)^2$
6. $P(x) = -4(x + 1)(x - 1)(2x - 5)(x - 3)$
7. $P(x) = -\frac{3}{5}x(2x + 3)(x - 2)$
8. $P(x) = -\frac{1}{9}(2x - 1)^2(x - 3)^3$
9. C
10. D
11. A
12. D

13.

3	6		

14.

2	0		