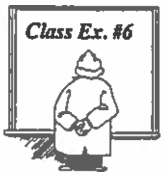
Defining 0!

If we replace r by n in the previous formula, we get the number of permutations of n elements taken n at a time. This we know is $n!$.

$${}_nP_n = n! = \frac{n!}{(n-n)!} = \frac{n!}{0!}$$

For this to be equal to $n!$ the value of $0!$ must be 1.

0! is defined to have a value of 1.



In a region, vehicle license plates consist of 2 different letters followed by 4 different digits. If the letters I, O, Y, and Z are not used, determine how many different license plates are possible by

a) the fundamental counting principle **b)** permutations

In many cases involving simple permutations, the fundamental counting principle can be used in place of the permutation formulas.

Complete Assignment Questions #7 - #14

Assignment

1. Without using a calculator, determine the value of

a) 5!

$5\cdot4\cdot3\cdot2\cdot1$

$= 120$

b) $\frac{10!}{8!}$

$\frac{10\cdot9\cdot\cancel{8!}}{\cancel{8!}} = 90$

c) $\frac{99!}{100!}$

$\frac{\cancel{99!}}{100\cdot\cancel{99!}} = \frac{1}{100}$

2. Express as single factorials.

a) $6\times5\times4\times3\times2\times1$

$6!$

b) $9\times8\times7\times6!$

$9!$

c) $(n+2)(n+1)n(n-1)\ \ldots\times3\times\ 2\times1$

$(n+2)!$

3. Express as a quotient of factorials.

a) $9\times8\times7\times6$

$\frac{9!}{5!}$

b) $20\times19\times18$

$\frac{20!}{17!}$

c) $(n+2)(n+1)n$

$\frac{(n+2)!}{(n-1)!}$

4. Use a calculator to determine the exact value of the following:

a) $10!$ **b)** $\frac{8!}{4!}$ **c)** $\frac{15!}{10!\,5!}$ **d)** $\left(\frac{25!}{21!}\right)\left(\frac{7!}{11!}\right)$

3 628 800 1680 3003 $^{115}/_{3}$

5. Simplify the following expressions. Leave the answer in product form where appropriate.

a) $\frac{n!}{n}$ **b)** $\frac{(n-3)!}{(n-2)!}$ **c)** $\frac{(n+1)!}{(n-1)!}$ **d)** $\frac{(3n)!}{(3n-2)!}$

a) $\frac{\cancel{n}(n-1)!}{\cancel{n}}$

$(n-1)!$

b) $\frac{\cancel{(n-3)!}}{(n-2)\cancel{(n-3)!}}$

$= \frac{1}{n-2}$

c) $\frac{(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}}$

$n(n+1)$

$= n^2+n$

d) $\frac{3n(3n-1)\cancel{(3n-2)!}}{\cancel{(3n-2)!}}$

$3n(3n-1)$

$= 9n^2-3n$

6. Solve the equation.

a) $\frac{(n+1)!}{n!} = 6$

$\frac{(n+1)\cancel{n!}}{\cancel{n!}} = 6$

$n+1 = 6$

$\quad -1 \quad -1$

$\boxed{n = 5}$

b) $(n+1)! = 6(n-1)!$

$(n+1)n\cancel{(n-1)!} = 6\cancel{(n-1)!}$

$n(n+1) = 6$

$n^2+n = 6$

$n^2+n-6 = 0$

$(n+3)(n-2)$

$n = \cancel{-3}, 2$

$\boxed{n=2}$

c) $\frac{(n+2)!}{n!} = 12$

$\frac{(n+2)(n+1)\cancel{n!}}{\cancel{n!}} = 12$

$n^2+3n+2 = 12$

$n^2+3n-10 = 0$

$(n+5)(n-2) = 0$

$n = \cancel{-5}, 2$

$\boxed{n=2}$

d) $\frac{(n+1)!}{(n-2)!} = 20(n-1)$

$\frac{(n+1)(n)\cancel{(n-1)}\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20\cancel{(n-1)}$

$n(n+1) = 20$

$n^2+n = 20$

$n^2+n-20 = 0$

$(n+5)(n-4) = 0$

$n = \cancel{-5}, 4$

$\boxed{n=4}$

7. Determine the number of arrangements that can be made using all of the letters in the word

a) DOG b) DUCK c) SANDWICH d) CANMORE

$3! = 6$ $4! = 24$ $8! = 40\,320$ $7! = 5040$

8. Consider the number of five-digit numbers that can be made from the digits 2, 3, 4, 7, and 9 if no digit can be repeated. Express your answer using

a) factorial notation b) $_nP_r$ notation c) the fundamental counting principle

$5! = 120$ $_5P_5 = 120$ $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$

9. a) Use the formula for $_nP_r$ to show that $_7P_0 = 1$.

$_nP_r = \frac{n!}{(n-r)!}$

$\frac{7!}{(7-0)!} = \frac{7!}{7!} = 1$

b) Explain why n must be greater than or equal to r in the notation $_nP_r$.

You can't pick more than what you have.

10. In each case determine the number of arrangements of the given letters by

i) using the fundamental counting principle ii) writing in $_nP_r$ form and evaluating

a) two letters from the word **GOLDEN**

$_6P_2 = 30$

or

$\underline{6}\ \underline{5} = 30$

b) three letters from the word **CHAPTERS**

$_8P_3 = 336$

or

$\underline{8} \cdot \underline{7} \cdot \underline{6} = 336$

c) four letters from the word **WEALTH**

$_6P_4 = 360$

or

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3} = 360$

d) one letter from the word **VALUE**

$_5P_1 = 5$

or $\underline{5} = 5$.

11. How many numbers (up to a maximum of four digit numbers) can be made from the digits 2, 3, 4, and 5 if no digit can be repeated?

1 digit or 2 digits or 3 digits or 4 digits.

$_4P_1 + {_4P_2} + {_4P_3} + {_4P_4}$

$= 4 + 12 + 24 + 24 = 64$

Multiple Choice

12. In a ten-team basketball league, each team plays every other team twice, once at home and once away. The number of games that are scheduled is

A. 45

B. 90

C. 100

D. 180

→ 2 teams per game

$_{10}P_2 = 90$

13. The value of $_nP_2$ is

A. $\frac{n}{n-2}$

B. $\frac{n!}{2!}$

C. $\frac{n}{2}$

D. $n(n-1)$

$_nP_r = \frac{n!}{(n-r)!}$

$\frac{n!}{(n-2)!} = \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}$

Numerical Response

14. In a competition on the back of a cereal packet, seven desirable qualities for a kitchen (eg. spaciousness, versatility, etc.) must be put in order of importance. The number of different entries that must be completed in order to ensure a winning order is _____ .

(Record your answer in the numerical response box from left to right.)

5	0	4	0

7 | 6 _ _ _ _ _ _

Answer Key

1. a) 120 b) 90 c) $\frac{1}{100}$
2. a) 6! b) 9! c) $(n+2)!$
3. a) $\frac{9!}{5!}$ b) $\frac{20!}{17!}$ c) $\frac{(n+2)!}{(n-1)!}$
4. a) 3 628 800 b) 1680 c) 3003 d) $\frac{115}{3}$
5. a) $(n-1)!$ b) $\frac{1}{n-2}$ c) $n(n+1)$ d) $3n(3n-1)$
6. a) $n=5$ b) $n=2$ c) $n=2$ d) $n=4$
7. a) 6 b) 24 c) 40 320 d) 5040
8. a) 5! b) $_5P_5$ c) $5 \times 4 \times 3 \times 2 \times 1 = 120$
9. a) $_7P_0 = \frac{7!}{(7-0)!} = \frac{7!}{7!} = 1$

   b) You cannot arrange more elements than the number of elements there are to begin with.
10. a) $_6P_2 = 30$ b) $_8P_3 = 336$ c) $_6P_4 = 360$ d) $_5P_1 = 5$
11. 64 12. B 13. D 14.

5	0	4	0