Assignment

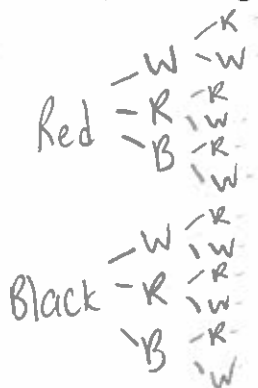1. A football team has the following kit :   jersey: red or black
                                             pants: white, red, or black
                                             socks: red or white

   The team plays in a different uniform each week until it has to repeat a previous uniform.

   Determine how many weeks the team can play before repeating a previous uniform by using

   a) a tree diagram   b) the fundamental counting principle

   $$\frac{2}{J} \times \frac{3}{P} \times \frac{2}{S}$$

   $$= 12.$$

2. How many ways are there of arranging 6 different books side by side on a shelf?

   $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 120$

3. With the new renovations completed at Prestwick High School, there will be seven entrances. In how many different ways can a student coming for Math tutorials

   a) enter and exit through any entrance? $\frac{7}{enter} \; \frac{7}{exit} = 49$

   b) enter the school and exit through a different entrance? $\frac{7}{enter} \; \frac{6}{exit} = 42$

   c) enter and exit through the same entrance? $\frac{7}{entr} \; \frac{1}{exit} = 7$

4. The score at the end of the second period of a hockey game is: Flames 6   Oilers 3.

   Jarome was attempting to determine how many different possibilities there are for the score at the end of the first period. He used the fundamental counting principle and multiplied 6 by 3 to get an answer of 18. Explain the error in his reasoning.

   $\frac{7}{Flams} \times \frac{4}{Oilers} = 28$

   Flames could have
   Opts 1pt 2pts 3pts 4pts 5pts 6plz
                                    7 options

5. If each of the students in a class of (30) students is capable of winning any of the class prizes, how many ways are there of awarding

   a) a first prize, a second prize, and a third prize in Mathematics? $30 \times 29 \times 28 = 24360$

   can't win 1st & 2nd

   b) a Mathematics prize, a Chemistry prize, and a Physics prize?   can win each subject

   $30 \times 30 \times 30 = 27000$

6. Three digit numbers are formed using only the digits 2, 3, 5, 6, 7, and 9.   6 #s

　　a) If repetitions are (not) permitted, how many 3-digit numbers can be formed?

$$\underline{6 \times 5 \times 4} \qquad = 120$$

　　b) How many of these are

i) less than 400?	**ii)** even?	**iii)** odd?	**iv)** multiples of 5?
—must start with 2 or 3	end in 2|6	end in 3,5,7,9	end in 5 ← must be 5
$\underline{2 \times 5 \times 4}$	$\underline{5\ \ 4\ \ 2}$	$\underline{5\ \ 4\ \ 4}$	$\underline{5\ \ 4\ \ 1}$
$= 40$	$= 40$	$= 80$	$= 20$

7. A vehicle license plate consists of 3 letters followed by 3 digits. How many different license plates are possible if:

　　a) there are <u>no restrictions</u> on the letters or digits used?

$$\underset{L\ \ \ L\ \ \ L}{\underline{26\ \ 26\ \ 26}}\ \ \underset{\#\ \ \ \#\ \ \ \#}{\underline{10\ \ 10\ \ 10}} \quad = 17\ 576\ 000$$

　　b) no letters may be repeated?

$$\underline{26\ \ 25\ \ 24}\ \ \underline{10\ \ 10\ \ 10} \quad = 15\ 600\ 000$$

　　c) the first digit cannot be zero and no (digits) can be repeated?

$$\underline{26\ \ 26\ \ 26}\ \ \underline{9\ \ 9\ \ 8} \quad = 11\ 389\ 248$$

8. How many ways are there of getting from A to C in each diagram, passing through each point at most once?

Diagram 1

Diagram 2

Diagram 3

Answer to Diagram 1

$$5 \times 3 = \boxed{15}$$

Answer to Diagram 2

through B₁　$\underline{3} \times \underline{3} = 9$

through B₂　$\underline{3} \times \underline{2} = + 6$

$9 + 6 = \boxed{15}$
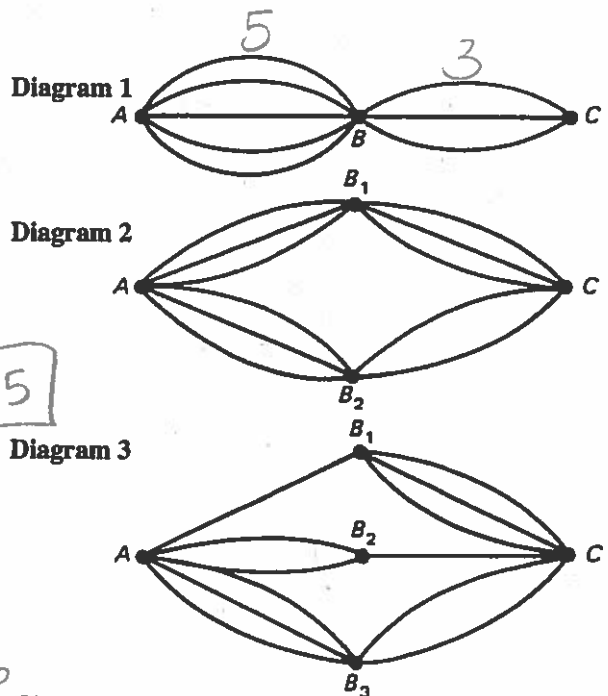
Answer to Diagram 3

through B₁ 　$\underline{1}\quad\underline{3} = 3$

B₂ 　$\underline{2}\quad\underline{1} = 2$

B₃ 　$\underline{3}\quad\underline{2} = 6$

$3 + 2 + 6$
$= \boxed{11}$

9. Determine the number of four letter "words" that can be formed from the letters of the word **PRODUCE** if 7 letters

   a) each letter can only be used once $\underline{7}$ $\underline{6}$ $\underline{5}$ $\underline{4}$ = 840

   b) each letter can only be <u>used once</u> and the "<u>word</u>" <u>must</u>

   i) contain <u>only consonants</u> 4C
   $\underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 24$

   ii) begin and end with a consonant
   $\underline{\underset{C}{4}}$ $\underline{\underset{C or V}{5}}$ $\underline{\underset{C or V}{4}}$ $\underline{\underset{C}{3}}$ = 240

   iii) begin with a vowel
   $\underline{3}$ $\underline{6}$ $\underline{5}$ $\underline{4}$ = 360

   iv) contain the letter **P**
   $\underline{P}$ $\underline{6}$ $\underline{5}$ $\underline{4}$ = 120
   $\underline{6}$ $\underline{P}$ $\underline{5}$ $\underline{4}$ = 120
   $\underline{6}$ $\underline{5}$ $\underline{P}$ $\underline{4}$ = 120   = 480
   $\underline{6}$ $\underline{5}$ $\underline{4}$ $\underline{P}$ = 120

   v) begin with **D** and end with a vowel
   $\underline{D}$ $\underline{5}$ $\underline{4}$ $\underline{3}$ = 60

10. a) How many different three-digit numerals can be formed from the digits 1, 5, and 8 if the digits cannot be <u>repeated</u>?
   $\underline{3}$ $\underline{2}$ $\underline{1}$ = 6

   b) How many different three-digit numerals can be formed using the digits 1, 3, 5, 7, and 9 if the digits <u>may be repeated</u>?
   $\underline{5}$ $\underline{5}$ $\underline{5}$ = 125

   c) How many four-digit numerals can be formed from the digits 0, 2, and 3 if the digits may be repeated?   (Note: 0223 is classified as the 3-digit numeral 223.)
   $\underline{\underset{not 3}{2}}$ $\underline{3}$ $\underline{3}$ $\underline{3}$ = 54

   d) How many different non-zero numerals are possible using some or all of the numerals 0, 1, 2, and 3 if the digits <u>cannot be repeated</u>?
   4 digit or 3 digit or 2 digit or 1 digit
   $\underline{\underset{not 0}{3}}$ $\underline{3}$ $\underline{2}$ $\underline{1}$ + $\underline{3}$ $\underline{\underset{not 0}{3}}$ $\underline{2}$ + $\underline{3}$ $\underline{\underset{not 0}{3}}$ + $\underline{\underset{not 0}{3}}$ = 18 + 18 + 9 + 3 = $\boxed{48}$

11. Mr. and Mrs. McDonald want a family picture taken with their children, <u>Hamish, Flora and James</u>. In how many different ways can all <u>five line up in a straight line</u> for the picture if

   a) there are no restrictions? $\underline{5}$ $\underline{4}$ $\underline{3}$ $\underline{2}$ $\underline{1}$ = 120

   b) the parents must be at either end of the line?
   $\underline{\underset{Parent}{2}}$ $\underline{\underset{C}{3}}$ $\underline{\underset{C}{2}}$ $\underline{\underset{C}{1}}$ $\underline{\underset{parent}{1}}$ = 12

   c) baby James must be in the middle?
   $\underline{4}$ $\underline{3}$ $\underline{J}$ $\underline{2}$ $\underline{1}$ = 24

   d) the children alternate with the adults?
   must be
   $\underline{\underset{C}{3}}$ $\underline{\underset{A}{2}}$ $\underline{\underset{C}{2}}$ $\underline{\underset{A}{1}}$ $\underline{\underset{C}{1}}$ = 12

Use the following information to answer the next question.

The word **PRODUCT** has been spelled using letter tiles. An illustration is shown.

$$P \quad R \quad O \quad D \quad U \quad C \quad T$$

5C
⑦ 2V

12. Using only these tiles, determine the number of four letter arrangements if the arrangement

a) has any letter in any position

$$\underline{7} \; \underline{6} \; \underline{5} \; \underline{4} \quad = 840$$

b) begins with **PR**

$$\underline{P} \; \underline{R} \; | \; \underline{5} \; \underline{4} = 20.$$

c) has two vowels in the middle

$$\underline{5} \; \frac{\underline{2}}{V} \; \frac{\underline{1}}{V} \; \underline{4} = 40$$

d) has two consonants in the middle

$$\underline{5} \; \frac{\underline{5}}{C} \; \frac{\underline{4}}{C} \; \underline{4} = 400$$

13. Ocean-going ships use coloured flags hung vertically for signalling. By changing the order of the coloured flags, the ships can send out different signals.

If ships carry six different coloured flags, one flag of each colour, how many different signals are possible if

a) all six flags are used?

$$\underline{6} \; \underline{5} \; \underline{4} \; \underline{3} \; \underline{2} \; \underline{1} = 720$$

b) four flags are used?

$$\underline{6} \; \underline{5} \; \underline{4} \; \underline{3} = 360$$

c) at least two flags are used?

$$\underline{6} \; \underline{5} \quad \text{or} \quad \underline{6} \; \underline{5} \; \underline{4} \quad \text{or} \quad \underline{6} \; \underline{5} \; \underline{4} \; \underline{3} \quad \text{or} \quad \underline{6} \; \underline{5} \; \underline{4} \; \underline{3} \; \underline{2} \quad \text{or} \quad \underline{6} \; \underline{5} \; \underline{4} \; \underline{3} \; \underline{2} \; \underline{1}$$

$$30 \; + \; 120 \; + \; 360 \; + \; 720 + 720 = 1950$$

14. a) How many odd six digit numbers have no repeating digits?

$$\underline{8} \; \underline{8} \; \underline{7} \; \underline{6} \; \underline{5} \; \underset{odd}{\underline{5}} = 67\,200$$

b) Consider the question "How many even six digit numerals have no repeating digits?" Explain why we need to consider two separate cases to determine the answer.

→ account for when it ends in zero or when it ends in 2/4/6/8

c) How many even six digit numerals have no repeating digits?

$$\underline{9} \; \underline{8} \; \underline{7} \; \underline{6} \; \underline{5} \; \underset{end \; in \; 0}{\underline{1}} + \underline{8} \; \underline{8} \; \underline{7} \; \underline{6} \; \underline{5} \; \underset{2/4/6/8}{\underline{4}}$$

$$15120 + \underset{not \; 0}{} = 68880$$
$$53760$$

15. In the final of a 100-metre race there are 8 competitors. The number of possible ways in which the gold, silver, and bronze medals can be awarded is

A. 21   B. 24 $\underline{8} \times \underline{7} \times \underline{6}$

Ⓒ 336   D. 512

16. How many even 5-digit whole numbers are there? Note that 31248 is acceptable, but 01248 is not.

A. 13 776

B. 15 120

C. 45 000

D. 50 000

9 10 10 10 5
not 0   0/2/4/6/8

= 45 000

Numerical Response **17.** Sandra is taking an examination which consists of two parts, A and B, with the following instructions.

- Part A consists of three questions and the student must do two.
- Part B consists of four questions and the student must do two.
- Part A must be completed before starting Part B.
- At the end of the exam the student has to list the order in which she attempted the questions.

The number of different possible orders is _____ .

(Record your answer in the numerical response box from left to right.)

3 2 × 4 3
Part A and Part B
then

7 2

6 × 12 = 72

Answer Key

1. 12   2. 720   3. a) 49 b) 42   c) 7

4. He should have multiplied 7 by 4 to get 28.   5. a) 24360 b) 27000

6. a) 120   b) i) 40   ii) 40   iii) 80   iv) 20

7. a) 17 576 000   b) 15 600 000   c) 11 389 248

8. Diagram 1 → 15   Diagram 2 → 15   Diagram 3 → 11

9. a)   840   b)   i)24   ii) 240   iii) 360   iv) 480   v) 60

10.a)   6   b) 125   c)   54   d) 48

11.a) 120   b) 12   c) 24   d) 12   12.a) 840   b) 20   c) 40   d) 400

13.a) 720   b) 360   c) 1950

14.a) 67 200
   b) If the last digit is zero, there are nine possible choices for the first digit.
      If the last digit is 2, 4, 6, or 8, there are only eight choices for the first digit since zero cannot be used.
   c) 68 880

15. C   16. C   17. | 7 | 2 | | |