Note

- When we calculate an expression for a composite function, there may be restrictions on domain and range.
- In most cases the domain and range of a composite function will be different from the domain and range of the original functions.
- The domain of a composite function $f \circ g$ cannot be more than the domain of g. The range of a composite function $f \circ g$ cannot be more than the range of f.

Complete Assignment Questions #7 - #18

Assignment

1. The 305 HP V6 2012 Ford Mustang Sports Car has a highway fuel efficiency of approximately 3.2 gallons per 100 miles. The cost of gasoline is $3.60 per gallon.
   a) The volume, v gallons, of fuel used can be written as a function of the distance, d miles, travelled. Complete the following for v in terms of d.
   $v = f(d) =$ 0.032d

   b) The cost, C dollars, of gasoline used can be written as a function of v. Complete the following for C in terms of v.
   $C = g(v) =$ 3.6v

   c) We can find the cost of gasoline in terms of the distance travelled by combining these two functions. If we substitute the formula for the first function into the formula for th second function, we can write C as a function of d. Complete the following:
   $C = h(d) =$ 0.1152 d

2. The function $p(d) = 0.62d$ converts Canadian dollars into British pounds. The function $e(p) = 1.15p$ converts British pounds into euros.
   a) Determine the function $e(d)$ that converts Canadian dollars into euros.

   e (d) = 0.713d

   b) Use these functions to convert C$2000 into euros.

   = 0.713(2000) = 1426 euros.

   c) Determine the function $d(e)$ that converts euros into Canadian dollars and convert 2000 euros into Canadian dollars (to the nearest cent).

   $d(e) = \frac{e}{0.713}$ = $2805.05

   $= \frac{2000}{0.713}$

3. A composite function $h(x)$ is given. Use the methods of Class Ex. #1 to complete the diagram, and write $h(x)$ as a composition of two functions, f and g, where $h(x) = g(f(x))$.

a) $h(x) = (x - 5)^2$

$x \xrightarrow{f} \_\_\_\_\_ \; \square \xrightarrow{g} \_\_\_\_\_ \; \square$

$f(x) =$ _____ $g(\_\_\_) =$ _____ $g(x) =$ _____

b) $h(x) = x^3 + 6$

$x \xrightarrow{f} \_\_\_\_\_ \; \square \xrightarrow{g} \_\_\_\_\_ \; \square$

$f(x) =$ _____ $g(\_\_\_) =$ _____ $g(x) =$ _____

c) $h(x) = \sqrt{(x - 4)}$

$x \xrightarrow{f} \_\_\_\_\_ \; \square \xrightarrow{g} \_\_\_\_\_ \; \square$

$f(x) =$ _____ $g(\_\_\_) =$ _____ $g(x) =$ _____

4. A composite function $h(x)$ is given. In each case, complete the diagram and write $h(x)$ as a composition of two functions, f and g, where $h(x) = f(g(x))$.

a) $h(x) = \sqrt{x} - 4$

$x \xrightarrow{g} \_\_\_\_\_ \; \square \xrightarrow{f} \_\_\_\_\_ \; \square$

$g(x) =$ _____ $f(\_\_\_) =$ _____ $f(x) =$ _____

b) $h(x) = \dfrac{1}{x + 3}$

$x \xrightarrow{g} \_\_\_\_\_ \; \square \xrightarrow{f} \_\_\_\_\_ \; \square$

$g(x) =$ _____ $f(\_\_\_) =$ _____ $f(x) =$ _____

5. Consider two functions $f(x) = x + 2$ and $g(x) = x^2$.

a) Complete the diagram to determine a formula for the composite function $h(x) = g(f(x)$

$x \xrightarrow{f} \_\_\_\_\_ \; \square \xrightarrow{g} \_\_\_\_\_ \; \square$

$h(x) = g(f(x)) = g($ _____ $) =$ _____

b) Use a similar technique to determine a formula for the composite function $k(x) = f(g($.

6. Consider two functions $f(x) = \sqrt{x}$ and $g(x) = 4x$.

a) Complete the diagram to determine a formula for the composite function $h(x) = g(f(x))$

$$x \xrightarrow{f} \underline{\qquad\qquad} \boxed{} \xrightarrow{g} \underline{\qquad\qquad} \boxed{}$$

$h(x) = g(f(x)) = g(\underline{\qquad\qquad}) = \underline{\qquad\qquad}$

b) Use a similar technique to determine a formula for the composite function $k(x) = f(g(x$

7. For each pair of functions, write a formula for $(f \circ g)(x)$.

a) $f(x) = 2x + 1$, $g(x) = 5x$

$(f \circ g)(x) = 2(5x)+1$

$= 10x+1$

b) $f(x) = 5x - 2$, $g(x) = x^3$

$= 5(x^3)-2$

$= 5x^3-2$

c) $f(x) = 2^x$, $g(x) = x + 4$

$= 2^{x+4}$

8. For each pair of functions, write a formula for $(g \circ f)(x)$.

a) $f(x) = 2 - x$, $g(x) = |x + 2|$

$= |2-x+2|$

$= |4-x|$

b) $f(x) = \boxed{2x + 1}$, $g(x) = x^4$

$= (2x+1)^4$

c) $f(x) = 3^x$, $g(x) = x - 1$

$= 3^x-1$

9. If $f(x) = x + 4$ and $g(x) = x - 1$, determine the value(s) of x for which

a) $f(g(x)) = 50$

$f \circ g$

$= (x-1)+4$

$= x+3$

$50 = x+3$

$47 = x$

b) $(fg)(x) = 50$

$(x+4)(x-1) = 50$

$x^2+3x-4 = 50$

$-50 \quad -50$

$x^2+3x-54 = 00$

$(x+9)(x-6)$

$x = -9, 6$

10. Consider the functions $f(x) = \sqrt{x-3}$ and $g(x) = x^2 + 2$.

a) Find expressions for $(f \circ g)(x)$ and $(g \circ f)(x)$.

$(f \circ g)(x) = \sqrt{x^2+2-3}$
$= \sqrt{x^2-1}$

$(g \circ f)(x) = \sqrt{x-3}^{\,2} + 2$
$= x - 3 + 2$
$= x - 1$

b) Determine the domains of f, g, $f \circ g$, and $g \circ f$.

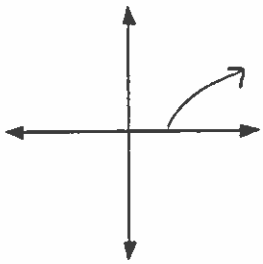
f $x \geq 3$
g $x \in R$
$f \circ g$ $x \mid x \leq -1,\ x \geq 1$
$g \circ f$ $x \mid x \geq 3$

c) Determine the ranges of f, g, $f \circ g$, and $g \circ f$.

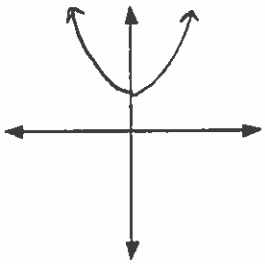
f $y \geq 0$
g $y \geq 2$
$f \circ g$ $y \geq 0$
$g \circ f$ $y \geq 2$

d) In each case, sketch the graph of the function indicated.

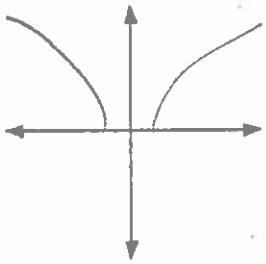
i) $y = f(x)$



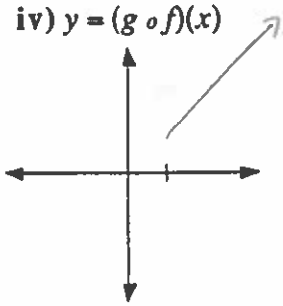
ii) $y = g(x)$



iii) $y = (f \circ g)(x)$



iv) $y = (g \circ f)(x)$



11. If $f(x) = 2x + 3$ and $g(x) = 5 - 2x$, determine the value of:

a) $f(g(5))$ $f \circ g$

$g(5) = 5 - 2(5)$
$= -5$
$f(-5)$
$= 2(-5) + 3$
$= -7$

b) $g(f(-3))$

$f(-3) = 2(-3) + 3$
$= -3$
$g(3)$
$= 5 - 2(3)$
$= -11$

c) $(f \circ g)(0)$

$g(0) = 5 - 2(0)$
$= 5$
$f(5)$
$= 2(5) + 3$
$= 13$

d) $-2(g \circ f)(0)$

$f(0) = 2(0) + 3$
$= 3$
$g(3) = 5 - 2(3)$
$= -1$
$-2(-1) = 2$

12. If $f(x) = 2\sqrt{x}$ and $g(x) = 2 + 2x$, determine the value of:

a) $f(g(7))$

$g(7) = 2 + 2(7)$
$= 16$
$f(16)$
$= 2\sqrt{16}$
$= 8$

b) $g\left(f\left(\frac{1}{4}\right)\right)$

$f(1/4)$
$= 2\sqrt{1/4}$
$= 1$
$g(1) =$
$2 + 2(1)$
$= 4$

c) $(f \circ g)(5)$

$g(5)$
$= 2 + 2(5)$
$= 12$
$f(12)$
$2\sqrt{12}$
$4 \cdot 3$
$= 2 \cdot 2\sqrt{3}$
$= 4\sqrt{3}$

d) $3(g \circ f)(5)$

$f(5) = 2\sqrt{5}$
$g(2\sqrt{5})$
$2 + 2(2\sqrt{5})$
$= 2 + 4\sqrt{5}$
$3(2 + 4\sqrt{5})$
$= 6 + 12\sqrt{5}$

13. Find $(f \circ g)(x)$, $(g \circ f)(x)$, and $(f \circ f)(x)$ for the following. State any domain restrictions.

a) $f(x) = -2x$, $g(x) = x^2 - 3$

$(f \circ g)(x) = -2(x^2-3)$
$= -2x^2+6$

$(g \circ f)(x) = (-2x)^2 - 3$
$= 4x^2-3$

$(f \circ f)(x) = -2(-2x)$
$= 4x$

b) $f(x) = \frac{1}{3-x}$, $g(x) = x^2$

$(f \circ g)(x) = \frac{1}{3-x^2}$

$x \neq \pm 3$

$(g \circ f)(x) = \left(\frac{1}{3-x}\right)^2 = \frac{1}{(3-x)^2}$

$x \neq 3$

$(f \circ f)(x) = \frac{1}{3 - \frac{1}{3-x}}$

$= 1 \div 3\frac{(3-x)}{1} - \frac{1}{3-x}$

$= 1 \div \frac{9-3x-1}{3-x} = \frac{1 \cdot 3-x}{8-3x} = \frac{3-x}{8-3x}$ $x \neq 8/3, 3$

c) $f(x) = 3x$, $g(x) = \sqrt{x-2}$

$(f \circ g)(x) = 3\sqrt{x-2}$, $x \geq 2$

$(g \circ f)(x) = \sqrt{3x-2}$, $x \geq \frac{2}{3}$

$(f \circ f)(x) = 3(3x)$
$= 9x$

Multiple Choice

14. Given $f(x) = 4 - x$ and $g(x) = 3\sqrt{5x}$, then $(f \circ g)(5)$ is equal to

A. -71
B. -11
C. -1
D. 35

$g(5) = 3\sqrt{5(5)}$
$= 15$
$f(15) = 4-15$
$= -11$

15. Given $f(x) = \frac{1}{x+5}$ and $g(x) = 6x - 1$, then $(g \circ f)(-2)$ is equal to

A. 1
B. $\frac{16}{3}$
C. $-\frac{1}{8}$
D. -3

$f(-2) = \frac{1}{-2+5} = \frac{1}{3}$

$g(1/3) = 6(1/3) - 1$
$= 2 - 1 = 1$

16. Given that $p(x) = 2x + 1$ and $q(x) = x^2 - 1$, then $p(q(x))$ equals

A. $2x^2 + 1$
B. $2x^2 - 1$
C. $4x^2$
D. $4x^2 + 4x$

$2(x^2-1)+1$
$= 2x^2-2+1$
$= 2x^2-1$

Numerical Response

17. The functions f, g, and h are given by $f(x) = x^2 - 1$, $g(x) = 3x + 2$, and $h(x) = |x + 2|$. The value of $(f \circ g \circ h)(-8)$, to the nearest whole number, is _____ .

(Record your answer in the numerical response box from left to right.)

3	9	9	

$h(-8) = |-8+2| = 6$

$g(6) = 3(6)+2 = 20$

$f(20) = 20^2 - 1$

$= 399$

18. The functions f and g are given by $f(x) = \frac{1}{x}$, and $g(x) = \frac{1}{x+1}$.

If $(f \circ f)(x) = (g \circ g)\left(\frac{1}{2}\right)$, then the value of x, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right.)

0	.	6	

$g(1/2) = \frac{1}{1/2+1} = 2/3$

$g(2/3) = \frac{1}{2/3+1} = 3/5$

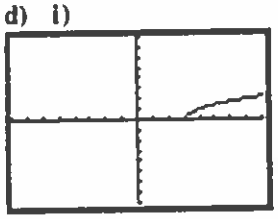
$(f \circ f)(3/5) = 1/5$

$f(3/5) = 5/3$

$f(5/3) = 3/5 = 0.6$

Answer Key

1. a) $v = 0.032d$ b) $C = 3.6v$ c) $C = 0.1152d$
2. a) $e(d) = 0.713d$ b) 1426 euros c) $d(e) = \frac{e}{0.713}$, \$2805.05
3. a) $f(x) = x - 5$, $g(x) = x^2$ b) $f(x) = x^3$, $g(x) = x + 6$ c) $f(x) = x - 4$, $g(x) = \sqrt{x}$
4. a) $f(x) = x - 4$, $g(x) = \sqrt{x}$ b) $f(x) = \frac{1}{x}$, $g(x) = x + 3$
5. a) $h(x) = (x + 2)^2$ b) $k(x) = x^2 + 2$
6. a) $h(x) = 4\sqrt{x}$ b) $k(x) = 2\sqrt{x}$
7. a) $(f \circ g)(x) = 10x + 1$ b) $(f \circ g)(x) = 5x^3 - 2$ c) $(f \circ g)(x) = 2^{x+4}$
8. a) $(g \circ f)(x) = |4 - x|$ b) $(g \circ f)(x) = (2x + 1)^4$ c) $(g \circ f)(x) = 3^x - 1$
9. a) $x = 47$ b) $x = -9, 6$
10. a) $(f \circ g)(x) = \sqrt{x^2 - 1}$, $x \le -1$ or $x \ge 1$, $(g \circ f)(x) = x - 1$, $x \ge 3$
    b) Domain: f: $\{x \mid x \ge 3, x \in R\}$ g: $\{x \mid x \in R\}$
    $f \circ g$: $\{x \mid x \le -1 \text{ or } x \ge 1, x \in R\}$ $g \circ f$: $\{x \mid x \ge 3, x \in R\}$
    c) Range: f: $\{y \mid y \ge 0, y \in R\}$ g: $\{y \mid y \ge 2, y \in R\}$
    $f \circ g$: $\{y \mid y \ge 0, y \in R\}$ $g \circ f$: $\{y \mid y \ge 2, y \in R\}$
    d) i)



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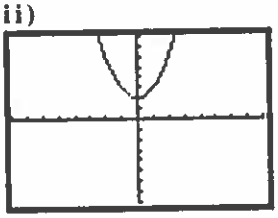

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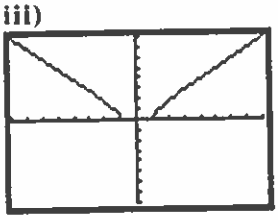

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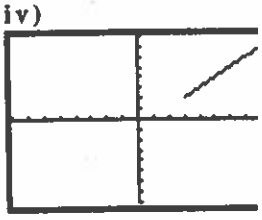

11. a) -7 b) 11 c) 13 d) 2
12. a) 8 b) 4 c) $4\sqrt{3}$ d) $6 + 12\sqrt{5}$
13. a) $(f \circ g)(x) = -2x^2 + 6$ $(g \circ f)(x) = 4x^2 - 3$ $(f \circ f)(x) = 4x$
    b) $(f \circ g)(x) = \frac{1}{3 - x^2}$, $x \ne \pm\sqrt{3}$ $(g \circ f)(x) = \frac{1}{(3 - x)^2}$, $x \ne 3$ $(f \circ f)(x) = \frac{3 - x}{8 - 3x}$, $x \ne \frac{8}{3}$
    c) $(f \circ g)(x) = 3\sqrt{x - 2}$, $x \ge 2$ $(g \circ f)(x) = \sqrt{3x - 2}$, $x \ge \frac{2}{3}$ $(f \circ f)(x) = 9x$

14. B 15. A 16. B 17. 399 18. 0.6