Assignment

1. For each of the following radical equations:

i) Determine the values of the variable for which the radical equation is defined.

ii) Solve the radical equation algebraically.

a) $(\sqrt{x}+5)^2 = (\sqrt{2x+1})^2$ $\boxed{x \geq 0,}$ $\cancel{x \geq -\frac{1}{2}}$

$(\sqrt{x}+5)(\sqrt{x}+5)$

$x+5\sqrt{x}+5\sqrt{x}+25$

$x+10\sqrt{x}+25 = 2x+1$

$-x \quad -25$

$(10\sqrt{x})^2 = (x-24)^2$

$100x = x^2-48x+576$

$0 = x^2-148x+576$

$(x-144)(x-4)$

$x = 144, 4$

verify

$x=144$

$\sqrt{144}+5 = \sqrt{2(144)+1}$

$17 = 17$ ✓

$x=4$

$\sqrt{4}+5 = \sqrt{2(4)+1}$

$7 = 3$ ✗

$\boxed{x = 144}$

b) $\sqrt{x} + \sqrt{x-4} = 4$ $\cancel{x \geq 0}$ $\boxed{x \geq 4}$

$(\sqrt{x-4})^2 = (4-\sqrt{x})^2$

$(4-\sqrt{x})(4-\sqrt{x})$

$\cancel{x}-4 = 16-8\sqrt{x}+x$

$-\cancel{x} \; -16 \quad -16 \qquad -x$

$(-20)^2 = (-8\sqrt{x})^2$

$\frac{400}{64} = \frac{64x}{64}$

$x = 25/4$

verify

$x=25/4$

$\sqrt{\frac{25}{4}} + \sqrt{\frac{25}{4}-4} = 4$

$2.5 + 1.5$

$4 = 4$ ✓

$\boxed{x = \frac{25}{4}}$

2. Algebraically determine the solution to the following radical equations.

a) $\sqrt{2t+1} - 5 = -\sqrt{t}$

$t \geq \cancel{-\frac{1}{2}}$ $\boxed{t \geq 0}$

$(\sqrt{2t+1})^2 = (-\sqrt{t}+5)^2$

$(-\sqrt{t}+5)(-\sqrt{t}+5)$

$2t+1 = t - 10\sqrt{t} + 25$

$-t \quad -25 \quad -t \quad -25$

$(t-24)^2 = (-10\sqrt{t})^2$

$t^2 - 48t + 576 = 100t$

$t^2 - 148t + 576 = 0$

$(t-144)(t-4)$

$t = 144, 4$

verify

$t = 144$

$\sqrt{2(144)+1} - 5 = -\sqrt{144}$

$17 - 5$

$12 = -12$

false

not 144

$t = 4$

$\sqrt{2(4)+1} - 5 = -\sqrt{4}$

$3 - 5 = -2$

-2 ✓

$\boxed{t = 4}$

b) $\sqrt{2a} = \sqrt{5a+9} - 3$

$\boxed{a \geq 0}$ $\cancel{-\frac{9}{5}}$

$(\sqrt{2a}+3)^2 = (\sqrt{5a+9})^2$

$(\sqrt{2a}+3)(\sqrt{2a}+3) = 5a+9$

$\cancel{2a} + 6\sqrt{2a} + 9 = 5a + 9$

$-2a \quad -9 \quad -2a \quad -9$

$(6\sqrt{2a})^2 = (3a)^2$

$36(2a) = 9a^2$

$72a = 9a^2$

$0 = 9a^2 - 72a$

$9a(a-8)$

$a = 0, 8$

verify

$a = 0$

$\sqrt{2(0)} = \sqrt{5(0)+9} - 3$

$0 = 0$ ✓

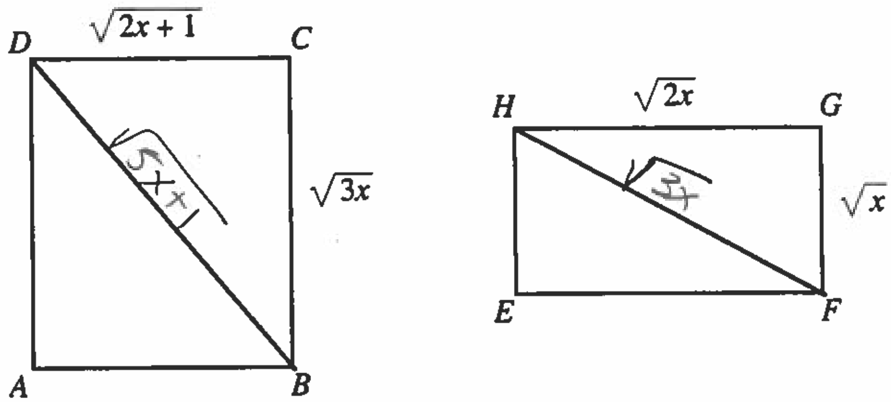
$a = 8$

$\sqrt{2(8)} = \sqrt{5(8)+9} - 3$

$4 = 4$ ✓

$\boxed{a = 0, 8}$

3. Consider the two rectangles shown.



a) Determine the exact length of diagonal *BD*.

$(\sqrt{2x+1})^2 + (\sqrt{3x})^2 = DB^2$

$2x+1+3x = DB^2$

$\sqrt{5x+1} = DB$

b) Determine the exact length of diagonal *FH*.

$(\sqrt{2x})^2 + \sqrt{x}^2 = FH^2$

$2x + x = FH^2$

$\sqrt{3x} = FH$

c) If *BD* is 1 unit longer than *FH*, determine the length and width of each rectangle.

$(\sqrt{3x}+1)^2 = (\sqrt{5x+1})^2$ $\quad$ ~~$x \geq -\frac{1}{5}$~~ $\boxed{x \geq 0}$ $\quad$ verify

$(\sqrt{3x}+1)(\sqrt{3x}+1$

$\sqrt{3x}$

$3x + 2\sqrt{3x} + 1 = 5x + 1$

$-3x \quad -1 \quad -3x \quad -1$

$x \neq 0$ because can't have $\ominus$ length so $\boxed{x=3}$

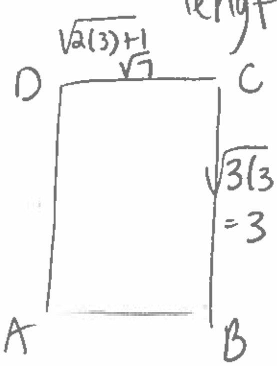
$(2\sqrt{3x})^2 = (2x)^2$

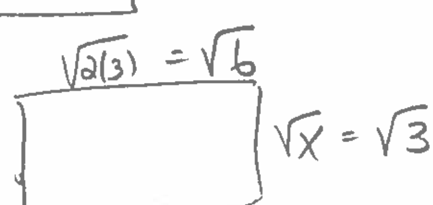
$4(3x) = 4x^2$

$12x = 4x^2$

$0 = 4x^2 - 12x$

$= 4x(x-3)$





width $= \sqrt{6}$
length $= \sqrt{3}$

$x = \cancel{0}, 3$ $\quad$ width $= \sqrt{7}$, length $= 3$

4. When the square root of one more than a number is subtracted from the square root of six less than double the number, the result is two.

a) Write a radical equation to represent this information.

$\sqrt{2x-6} - \sqrt{x+1} = 2$

b) Solve the equation to determine the number and state the extraneous root.

$(\sqrt{2x-6})^2 = (2+\sqrt{x+1})^2$

$(2+\sqrt{x+1})(2+\sqrt{x+1})$

$4 + 2\sqrt{x+1} + 2\sqrt{x+1} + x + 1$

$2x - 6 = 5 + 4\sqrt{x+1} + x$

$-x \quad -5$

$(x-11)^2 = (4\sqrt{x+1})^2$

$x^2 - 22x + 121 = 16(x+1)$

$x^2 - 22x + 121 = 16x + 16$

-16

$x^2 - 38x + 105 = 0$

$(x-35)(x-3)$

$x = 35, 3$

verify

$x = 35$

$\sqrt{2(35)-6} - \sqrt{35+1} = 2$

$8 - 6 = 2$ ✓

$x = 3$

$\sqrt{2(3)-6} - \sqrt{3+1} = 2$

$-2 = 2$ X

$\boxed{x = 35}$

3 is extraneous

5. A number, n, is non-negative. The difference between the square root of nine more than five times the number and the square root of twice the number is three.

a) Write a radical equation to represent this information.

$\sqrt{5x+9} - \sqrt{2x} = 3$

b) Solve the equation to determine all possible values of the number.

$\boxed{x \geq 0}, -9/5$

$(\sqrt{5x+9})^2 = (3+\sqrt{2x})^2 \rightarrow (3+\sqrt{2x})(3+\sqrt{2x})$

$9 + 6\sqrt{2x} + 2x$

$5x + 9 = 9 + 6\sqrt{2x} + 2x$

$-2x \quad -9 \quad -9 \quad -2x$

$(3x)^2 = (6\sqrt{2x})^2$

$9x^2 = 36(2x)$

$9x^2 - 72x = 0$

$9x(x-8) = 0$

$x = 0, 8$

verify

$x = 0$

$\sqrt{5(0)+9} - \sqrt{2(0)} = 3$

$\sqrt{9} - 0 = 3$ ✓

$x = 8$

$\sqrt{5(8)+9} - \sqrt{2(8)} = 3$

$7 - 4 = 3$ ✓

$\boxed{x = 0, 8}$

6. Algebracially solve and verify the following radical equations.

[redacted] $\sqrt{x+11} - \sqrt{x-9} = 2$

$\cancel{x \geq -11}$, $\boxed{x \geq 9}$

$(\sqrt{x+11})^2 = (2 + \sqrt{x-9})^2$

$x+11 = 4 + 4\sqrt{x-9} + x - 9$

$\frac{16}{4} = \frac{4\sqrt{x-9}}{4}$

$4 = \sqrt{x-9}$

$16 = x - 9$

$25 = x$

verify

$x = 25$

$\sqrt{25+11} - \sqrt{25-9} = 2$

$\sqrt{36} - \sqrt{16} = 2$

$6 - 4 = 2$ ✓

$\boxed{x = 25}$

b) $(\sqrt{x+3} + 2)^2 = (\sqrt{x+11})^2$ $\boxed{x \geq -3}$, $\cancel{x \geq -11}$

$x + 3 + 4\sqrt{x+3} + 4 = x + 11$

$-x \quad -3 \quad\quad -4 \quad\quad -x \quad -3 \quad -4$

$\frac{4\sqrt{x+3}}{4} = \frac{4}{4}$

$\sqrt{x+3}^2 = 1^2$

$x + 3 = 1$

$-3 \quad\quad -3$

$x = -2$

verify $x = -2$

$\sqrt{-2+3} + 2 = \sqrt{-2+11}$

$\sqrt{1} + 2 = \sqrt{9}$

$3 = 3$ ✓

$\boxed{x = -2}$

c) $(\sqrt{4p+5})^2 = (2+\sqrt{2p-1})^2$ $\boxed{p \geq \frac{1}{2}} \; p \geq -5/4$ $\rightarrow p = 5, 1$

$4p+5 = 4 + 4\sqrt{2p-1} + 2p - 1$

$(2p+2)(2p+2)$

$(2p+2)^2 = (4\sqrt{2p-1})^2$

$4p^2+8p+4 = 16(2p-1)$

$4p^2+8p+4 = 32p-16$

$4p^2-24p+20 = 0$

$4(p^2-6p+5) = 0$

$4(p-5)(p-1) = 0$

verify $p=5$

$\sqrt{4(5)+5} = 2+\sqrt{2(5)-1}$

$\sqrt{25} = 2+\sqrt{9}$

$5 = 5$ ✓

$p=1$

$\sqrt{4(1)+5} = 2+\sqrt{2(1)-1}$

$\sqrt{9}$

$3 = 2+\sqrt{1}$

3 ✓

$\boxed{p = 5, 1}$

d) $(\sqrt{3-a} - 3)^2 = (-\sqrt{2a+3})^2$ $a \geq -\frac{3}{2}$ $a \leq 3$

$(\sqrt{3-a} - 3)(\sqrt{3-a} - 3)$

$3 - a - 6\sqrt{3-a} + 9$

$-a - 6\sqrt{3-a} + 12 = 2a+3$

$(-6\sqrt{3-a})^2 = (3a-9)^2$

$36(3-a) = 9a^2-54a+81$

$108-36a = 9a^2-54a+81$

$0 = 9a^2-18a-27$

$= 9(a^2-2a-3)$

$= 9(a-3)(a+1)$

$a = 3, -1$

verify

$a=3$

$\sqrt{3-3} - 3 = -\sqrt{2(3)+3}$

$-3 = -3$ ✓

$a=-1$

$\sqrt{3-(-1)} - 3 = -\sqrt{2(-1)+3}$

$-1 = -1$ ✓

$\boxed{a = 3, -1}$

omit until after Quad Functions.

Numerical Response

7. Jasmine uses an algebraic procedure to determine the solution to the equation $2\sqrt{x} - \sqrt{x+4} = 3$. The solution, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right.)

1	2	.	4

$(-\sqrt{x+4})^2 = (3 - 2\sqrt{x})^2$

$x+4 = 9 - 12\sqrt{x} + 4x$

$(-3x-5)^2 = (-12\sqrt{x})^2$

$9x^2 + 30x + 25 = 144x$

$9x^2 - 114x + 25 = 0$ $\quad -114 \mid \frac{x}{225}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{114 \pm \sqrt{(-114)^2 - 4(9)(25)}}{2(9)}$

$= \frac{114 \pm \sqrt{12096}}{18}$

$\cancel{x = 0.223}, \ x = 12.443$

Group Work

Algebraically determine the roots of the equation $(\sqrt{x+3} + \sqrt{x+8})^2 = (\sqrt{5x+20})^2$

$x+3 + 2\sqrt{x+3}\sqrt{x+8} + x + 8 = 5x + 20$

$(2\sqrt{x+3}\sqrt{x+8}) = (3x+9)^2$

$4(x+3)(x+8) = 9x^2 + 54x + 81$

$4(x^2 + 11x + 24) = 9x^2 + 54x + 81$

$4x^2 + 44x + 96 = 9x^2 + 54x + 81$

$= 5x^2 + 10x - 15$

$= 5(x^2 + 2x - 3)$

$= 5(x+3)(x-1)$

$x = -3, 1$

verify $x = -3$

$\sqrt{-3+3} + \sqrt{-3+8} = \sqrt{5(-3)+20}$

$\sqrt{5} = \sqrt{5}$ ✓

$x = 1$

$\sqrt{1+3} + \sqrt{1+8} = \sqrt{5(1)+20}$

$\sqrt{4} + \sqrt{9} = \sqrt{25}$

$2 + 3 = 5$

$5 = 5$ ✓

$\boxed{x = -3, 1}$

Answer Key

1. a) i) $\{x \mid x \geq 0, x \in R\}$ ii) 144 **b)** i) $\{x \mid x \geq 4, x \in R\}$ ii) $\frac{25}{4}$

2. a) 4 **b)** 0, 8

3. a) $\sqrt{5x+1}$ **b)** $\sqrt{3x}$ **c)** rectangle *ABCD* has dimensions $\sqrt{7}$ by 3
rectangle *EFGH* has dimensions $\sqrt{6}$ by $\sqrt{3}$

4. a) $\sqrt{2n-6} - \sqrt{n+1} = 2$ **b)** The number is 35 and the extraneous root is 3.

5. a) $\sqrt{5n+9} - \sqrt{2n} = 3$ **b)** 0 or 8

6. a) $x = 25$ **b)** $x = -2$ **c)** $p = 1, 5$ **d)** $a = -1, 3$

7.

1	2	.	4

Group Work $-3, 1$