Class Ex. #5

Factor the expression $36(x + 5)^2 - 49(x - 8)^2$.

Complete Assignment Questions #5 - #11

Assignment

1. Factor:

a) $16x^2 - 49y^2$

$(4x-7y)(4x+7y)$

b) $25a^2 - 121y^2$

$(5a-11y)(5a+11y)$

c) $p^2q^2 - r^2s^2$

$(pq-rs)(pq+rs)$

d) $16x^2 - 4y^2$

$4(4x^2-y^2)$

$4(2x-y)(2x+y)$

e) $9a^2b^2 - 36c^2$

$9(a^2b^2-4c^2)$

$9(ab-2c)(ab+2c)$

f) $12a^2 - 75p^2q^2$

$3(4a^2-25p^2q^2)$

$3(2a-5pq)(2a+5pq)$

g) $4xy^3 - 169x^3y$

$xy(4y^2-169x^2)$

$xy(2y-13x)(2y+13x)$

h) $60a^2b^2 - 15a^4b^4$

$15a^2b^2(4-a^2b^2)$

$15a^2b^2(2-ab)(2+ab)$

i) $4b^2g^2 - 49t^2z^2$

$(2bg-7tz)(2bg+7tz)$

j) $25x^2 + 100y^2$

$25(x^2+4y^2)$

k) $225a^2c^2 - 16b^2d^2$

$(15ac-4bd)(15ac+4bd)$

l) $xw^2y^2 - x^3z^2$

$x(w^2y^2-x^2z^2)$

$x(wy-xz)(wy+xz)$

m) $1 - \cos^2 x$

$(1-\cos x)(1+\cos x)$

n) $\sin^2 x - \cos^2 x$

$(\sin x-\cos x)(\sin x+\cos x)$

o) $\frac{x^2}{64} - \frac{y^2}{49}$

$\left(\frac{x}{8}-\frac{y}{7}\right)\left(\frac{x}{8}+\frac{y}{7}\right)$

The floor of a classroom is rectangular with an area of $81m^2 - 4n^2$ square metres.

a) The length and width of the floor can both be written in the form $(am + bn)$ where a and b are integers. Determine appropriate values for a and b, and hence write expressions in m and n for the length and width of the floor. $(9m-2n)(9m+2n)$

b) If the perimeter of the floor is 72 metres, form an equation in m and n and solve for m.
$72 = 2(9m-2n) + 2(9m+2n)$
$72 = 18m - 4n + 18m + 4n$
$72 = 36m$
$2 = m$

c) Determine the length and width of the floor if the length is 25% greater than the width.
length $= 1.25$ width
$9m + 2n = 1.25(9m-2n)$
$9(2) + 2n = 1.25(9(2)-2n)$
$18 + 2n = 1.125(18-2n)$
$18 + 2n = 22.5 - 2.5n$
$m = 2$
$4.5n = 4.5$
$n = 1$
length $= 9m+2n$
$9(2)+2(1) = 20m$
width $= 9m-2n = 16m$
$9(2)-2(1)$

3. Factor.

a) $x^4 - y^4$
$(x^2-y^2)(x^2+y^2)$
$(x-y)(x+y)(x^2+y^2)$

b) $a^4 - 256b^4$
$(a^2-16b^2)(a^2+16b^2)$
$(a-4b)(a+4b)(a^2+16b^2)$

c) $2z^4 - 162$
$2(z^4-81)$
$2(z^2-9)(z^2+9)$
$2(z-3)(z+3)(z^2+9)$

d) $48x^4 - 3y^4$
$3(16x^4-y^4)$
$3(4x^2-y^2)(4x^2+y^2)$
$3(2x-y)(2x+y)(4x^2+y^2)$

$9a^4b^4 - 144c^4d^4$
$9(a^4b^4-16c^4d^4)$
$9(a^2b^2-4c^2d^2)(a^2b^2+4c^2d^2)$

$z^8 - 256$
$(z^4-16)(z^4+16)$
$(z^2-4)(z^2+4)(z^4+16)$
$(z-2)(z+2)(z^2+4)(z^4+16)$

4. Factor each expression.

a) $81a^4 - 16b^4$

$(9a^2 - 4b^2)(9a^2 + 4b^2)$

$(3a - 2b)(3a + 2b)(9a^2 + 4b^2)$

● $16p^4 - \frac{1}{81}q^4$ $\left(4p^2 - \frac{1}{9}q^2\right)\left(4p^2 + \frac{1}{9}q^2\right)$

$\left(2p - \frac{1}{3}q\right)\left(2p + \frac{1}{3}q\right)\left(4p^2 + \frac{1}{9}q^2\right)$

c) $16a^4 - 121b^2c^2$

$(4a^2 - 11bc)(4a^2 + 11bc)$

d) $z^6 - 9$

$(z^3 - 3)(z^3 + 3)$

● $1 - a^{16}$

$(1 - a^8)(1 + a^8)$

$(1 - a^4)(1 + a^4)(1 + a^8)$

$(1 - a^2)(1 + a^2)(1 + a^4)(1 + a^8)$

$(1 - a)(1 + a)(1 + a^2)(1 + a^4)(1 + a^8)$

f) $x^4 - 0.0256y^4$

$(x^2 - 0.16y^2)(x^2 + 0.16y^2)$

$(x - 0.4y)(x + 0.4y)(x^2 + 0.16y^2)$

5. Factor each expression.

a) $(a - b)^2 - c^2$

$[(a - b) - c][(a - b) + c]$

$(a - b - c)(a - b + c)$

b) $a^2 - (b + c)^2$

$(a - (b + c))(a + (b + c))$

$(a - b - c)(a + b + c)$

c) $(x + y)^2 - x^2$

$((x + y) - x)((x + y) + x)$

$y(2x + y)$

● $x^2 - (x - y)^2$ $(x^2 - (x - y))(x + (x - y))$

$y(2x - y)$

e) $\sqrt{4(p + q)^2} - \sqrt{25}$

$[2(p + q) - 5][2(p + q) + 5]$

$(2p + 2q - 5)(2p + 2q + 5)$

f) $\sqrt{36(a + b)^2} - \sqrt{(p + q)^2}$

$[6(a + b) - (p + q)][6(a + b) + (p + q)]$

$(6a + 6b - p - q)(6a + 6b + p + q)$

g) $(x + 5)^2 - (x - 5)^2$

$[(x + 5) - (x - 5)][(x + 5) + (x - 5)]$

$(10)(2x) = 20x$

● $9(a + b + c)^2 - 4(a - b + c)^2$ $(3A - 2B)(3A + 2B)$

$[3(a + b + c) - 2(a - b + c)][3(a + b + c) + 2(a - b + c)]$

$(3a + 3b + 3c - 2a + 2b - 2c)(3a + 3b + 3c + 2a - 2b + 2c)$

$(a + 5b + c)(5a + b + 5c)$

● $256(a - 4)^2 - 100(a - 6)^2$

$4\left(64(a - 4)^2 - 25(a - 6)^2\right)$

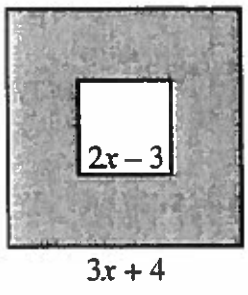
$4[8(a - 4) - 5(a - 6)][8(a - 4) + 5(a - 6)]$

$4(8a - 32 - 5a + 30)(8a - 32 + 5a - 30)$

$4(3a - 2)(13a - 62)$

Use the following information to answer the next question.

The diagram shows a square of side $(2x - 3)$ metres inside a square of side $(3x + 4)$ metres. Students were asked to determine the area of the shaded region in factored form.



6. **a)** Rhonda chose to solve the problem by finding the area of each square, subtracting, and factoring the result. Use Rhonda's method to determine the area.

$\text{Area}_{big} - \text{Area}_{small}$

$(9x^2+24x+16) - (4x^2-12x+9)$

$5x^2 + 36x + 7$

$5x^2+35x+1x+7$

$5x(x+7)+1(x+7)$

$(5x+1)(x+7)$

$(3x+4)(3x+4)$

$= 9x^2+12x+12x+16$

$= 9x^2+24x+16$

$(2x-3)(2x-3)$

$4x^2-12x+9$

b) Soyee used the factoring technique of difference of squares to determine the answer. Use Soyee's method to determine the area.

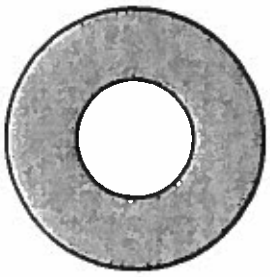
$(3x+4)^2 - (2x-3)^2 = (x+7)(5x+1)$

$[(3x+4)-(2x-3)][(3x+4)+(2x-3)]$

c) Which method do you prefer?

b) because it is FUNNER

7. The diagram shows a washer with outer radius $(r + 3)$ mm, and an inner radius of $(r - 1)$ mm.



a) Write, but do not simplify, an expression for the area of the surface of the washer

$A = \pi r^2$ $\quad \text{Area}_{big} - \text{Area}_{small}$

$A = \pi(r+3)^2 - \pi(r-1)^2$

b) Factor this expression completely.

$\pi[(r+3)^2 - (r-1)^2]$

$= \pi[(r+3)-(r-1)][(r+3)+(r-1)]$

$= \pi(4)(2r+2) = \pi(4)(2(r+1)] = 8\pi(r+1)$

c) Use the factored expression to determine the area of the washer if $r = 5$.

$A = 8\pi(r+1)$ $\quad 8(\pi)(6)$

$= 8\pi(5+1) =$ $\quad = 48\pi \text{ mm}^2$

Multiple Choice

8. One factor of $y^4 - 81$ is

$(y^2-9)(y^2+9)$

$(y-3)(y+3)(y^2+9)$

A. $y + 9$

B. $y + 3$

C. $y^2 - 3$

D. $y^2 + 3$

9. The polynomials $4x^2 + 8xy - 5y^2$ and $24x^2 - 6y^2$ have in common a factor of

A. $4x + y$

B. $4x - y$

C. $2x + y$

D. $2x - y$

$4x^2+8xy-5y^2$ -20 $\overset{+1}{\underset{10,-2}{8}}$ x

$4x^2-2xy+10xy-5y^2$

$2x(2x-y)+5y(2x-y)$

$(2x+5y)(2x-y)$

$6(4x^2-y^2)$

$6(2x-y)(2x+y)$

Numerical Response

When fully factored, the expression $a^4 - (9a + 18)^2$ can be written as the product of two binomial factors and one trinomial factor. If the binomial factors are $a + p$ and $a + q$, the value of pq is _____.

(Record your answer in the numerical response box from left to right.)

1	8		

B

$(a^2-b)(a^2+b)$

$(a^2-(9a+18))(a^2+9a+18)$

$(a^2-9a-18)(a^2+9a+18)$

$(a^2-9a-18)(a+6)(a+3)$

p q

11. The expression $81(x-3)^2 - 144(x-2)^2$ can be factored into the form $-a(x+b)(cx-d)$, where $a, b, c, d \in N$. The value of $a+b+c+d$ is _____.

(Record your answer in the numerical response box from left to right.)

3	4		

$81(x-3)^2 - 144(x-2)^2$

$[9(x-3) - 12(x-2)][9(x-3) + 12(x-2)]$

$(9x - 27 - 12x + 24)(9x - 27 + 12x - 24)$

$(-3x - 3)(21x - 51)$

$-3(x+1)3(7x - 17)$

$-9(x+1)(7x-17)$

$a \quad b \quad c \quad d$

$9 + 1 + 7 + 17$

Answer Key

1. a) $(4x - 7y)(4x + 7)$ b) $(5a - 11y)(5a + 11y)$ c) $(pq + rs)(pq - rs)$
d) $4(2x - y)(2x + y)$ e) $9(ab - 2c)(ab + 2c)$ f) $3(2a - 5pq)(2a + 5pq)$
g) $xy(2y - 13x)(2y + 13x)$ h) $15a^2b^2(2 - ab)(2 + ab)$ i) $(2bg - 7tz)(2bg + 7tz)$
j) $25(x^2 + 4y^2)$ k) $(15ac - 4bd)(15ac + 4bd)$ l) $x(wy - xz)(wy + xz)$
m) $(1 - \cos x)(1 + \cos x)$ n) $(\sin x - \cos x)(\sin x + \cos x)$ o) $\left(\frac{x}{8} + \frac{y}{7}\right)\left(\frac{x}{8} - \frac{y}{7}\right)$

2. a) $(9m + 2n)$ metres, $(9m - 2n)$ metres b) $2(9m + 2n) + 2(9m - 2n) = 72$, $m = 2$
c) Length = 20 metres, Width = 16 metres.

3. a) $(x - y)(x + y)(x^2 + y^2)$ b) $(a - 4b)(a + 4b)(a^2 + 16b^2)$
c) $2(z - 3)(z + 3)(z^2 + 9)$ d) $3(2x - y)(2x + y)(4x^2 + y^2)$
e) $9(ab - 2cd)(ab + 2cd)(a^2b^2 + 4c^2d^2)$ f) $(z - 2)(z + 2)(z^2 + 4)(z^4 + 16)$

4. a) $(3a - 2b)(3a + 2b)(9a^2 + 4b^2)$ b) $\left(2p - \frac{1}{3}q\right)\left(2p + \frac{1}{3}q\right)\left(4p^2 + \frac{1}{9}q^2\right)$
c) $(4a^2 - 11bc)(4a^2 + 11bc)$ d) $(z^3 - 3)(z^3 + 3)$
e) $(1 - a)(1 + a)(1 + a^2)(1 + a^4)(1 + a^8)$ f) $(x - 0.4y)(x + 0.4y)(x^2 + 0.16y^2)$

5. a) $(a - b - c)(a - b + c)$ b) $(a - b - c)(a + b + c)$
c) $y(2x + y)$ d) $y(2x - y)$
e) $(2p + 2q - 5)(2p + 2q + 5)$ f) $(6a + 6b - p - q)(6a + 6b + p + q)$
g) $20x$ h) $(a + 5b + c)(5a + b + 5c)$
i) $4(3a - 2)(13a - 62)$

6. a) $(x + 7)(5x + 1)$ b) $(x + 7)(5x + 1)$

7. a) $\pi(r + 3)^2 - \pi(r - 1)^2$ b) $8\pi(r + 1)$ c) 48π mm^2

8. B **9.** D **10.**

1	8		

11.

3	4		