Investigating the Graphs of $y = \log x^n$ and $y = n \log x$

One of the laws of logarithms states that $\log x^n = n \log x$.
However, are the graphs of $y = \log x^n$ and $y = n \log x$ identical?

Investigate by considering odd and even values for n.

Complete Assignment Questions #6 - #17

Assignment

1. a) Complete the following table.

Function	Domain	Range	Asymptote
$f(x) = \log_4 x$	$x \mid x > 0, x \in R$	$y \in R$	$x = 0$
$f(x) = \log_{\frac{1}{4}} x$	$x \mid x > 0, x \in R$	$y \in R$	$x = 0$

b) Describe how the graph of $y = \log_4 x$ is related to the graph of $y = \log_{\frac{1}{4}} x$.

reflection in the x- axis

c) Hence write $y = \log_{\frac{1}{4}} x$ in the form $y = a \log_4 x$.

$y \rightarrow -y \quad y = -\log_4 X$

2. a) Describe the transformation which maps the graph of $\log_9 x$ to the graph of

i) $\log_{\frac{1}{9}} x$ - reflection in x-axis

ii) $\log_{81} x$ vertical stretch by a factor of $\frac{1}{2}$ about x-axis

$9^2 = 81$

iii) $\log_3 x$ - vertical stretch by a factor of 2 about x-axis.

$9^{1/2} = 3$

b) Complete the statements

i) $\log_{\frac{1}{9}} x = \underline{-}\log_9 x$ ii) $\log_{81} x = \underline{1/2}\log_9 x$ iii) $\log_3 x = \underline{2}\log_9 x$

3. a) Describe the transformation which maps the graph of $\log_8 x$ to the graph of

i) $\log_2 x$ - vertical stretch by a factor of 3 about x-axis

$8^{1/3} \to 2$

ii) $\log_{64} x$ - vertical stretch by a factor of $\frac{1}{2}$ about x-axis

$8^2 - 64$

iii) $\log_{\frac{1}{8}} x$ - reflection in x-axis

b) Complete the statements

i) $\log_2 x = \underline{3}\log_8 x$ ii) $\log_{64} x = \underline{1/2}\log_8 x$ iii) $\log_{\frac{1}{8}} x = \underline{-}\log_8 x$

4. a) If $\log_6 x = 6$, state the values of $\log_{\frac{1}{6}} x$, $\log_{36} x$, and $\log_{\sqrt{6}} x$.

-6 $\quad \frac{1}{2}6 = 3$ $\quad 2(6) = 12$

b) Prove the results in a) by converting to exponential form.

$\log_6 X = 6$

$X = 6^6$

let $\log_{1/6} X = a$

$X = \left(\frac{1}{6}\right)^a$

$X = 6^{-a}$

$6^6 = 6^{-a}$

$-a = 6$

$a = -6$

$\log_{1/6} X = -6$

let $\log_{36} X = a$

$X = 36^a$

$X = 6^{2a}$

$6^6 = 6^{2a}$

$3 = a$

$\boxed{\log_{36} X = 3}$

$\log_{\sqrt{6}} X = a$

$X = (\sqrt{6})^a$

$X = 6^{1/2 a}$

$6^6 = 6^{1/2 a}$

$12 = a$

$\log_{\sqrt{6}} X = 12$

Use the following information to answer the next question.

The equations of six logarithmic functions are given below. $y = \log_b X^{-1}$ $-\log b X$

$y = \log_b x$, $y = -\log_b x$, $y = \log_{\frac{1}{b}} x$, $y = -\log_{\frac{1}{b}} x$, $y = \log_b\left(\frac{1}{x}\right)$, $y = \log_{\frac{1}{b}}\left(\frac{1}{x}\right)$

When graphed, the six functions can be arranged into two groups of three identical graphs.

5. Which functions are in each group?

① $y = \log_b X$, $y = -\log_{\frac{1}{b}} X$, $y = \log_{\frac{1}{b}}\left(\frac{1}{X}\right)$

② $y = -\log_b X$, $y = \log_{\frac{1}{b}} X$, $y = \log_b \frac{1}{X}$

6. Describe how the graphs of the following functions relate to the graph of $y = \log x$:

a) $y = 5\log x - 2$

$y + 2 = 5\log x$

$y \to \frac{1}{5}y,\ y \to y + 2.$

- vertical stretch by a factor of 5, vertical trans 2 units down.

b) $\frac{1}{7}y = \log 2(x - 3)$

$y \to \frac{1}{7}y$

$x \to x - 3$

$x \to 2x$

- vertical stretch by a factor of 7, horizontal stretch by a factor of $\frac{1}{2}$, hor. trans 3 units right

c) $y = \log \frac{1}{3}x + 1$

$x \to \frac{1}{3}x$

$y \to y - 1$

horizontal stretch by a factor of 3 about y-axis, vertical translation 1 unit up.

d) $y = \log(3x + 1)$

$y = \log 3(x + \frac{1}{3})$

$x \to 3x$

$x \to x + \frac{1}{3}$

- horiz. stretch by a factor of $\frac{1}{3}$ + translation $\frac{1}{3}$ unit left.

7. Complete the following table .

Function	Domain	Range	Asymptote
$f(x) = \log x$	$x \mid x > 0, x \in R$	$y \in R$	$x = 0$
$f(x) = 5\log x + 2$	$x \mid x > 0, x \in R$.	$y \in R$	$x = 0$
$f(x) = 5\log(x + 2)$ 2uL	$x \mid x > -2, x \in R$	$y \in R$	$x = -2$
$f(x) = -\log x$	$x \mid x > 0, x \in R$	$y \in R$	$x = 0$
$f(x) = \log(-x)$ $x \to -x$ (reflecting)	$x \mid x < 0, x \in R$	$y \in R$	$x = 0$
$f(x) = 2\log 3(x - 6) - 1$ 6uR	$x \mid x > 6, x \in R$	$y \in R$	$x = 6$
$f(x) = \log(3x - 6)$ $\log 3(x - 2)$	$x \mid x > 2, x \in R$	$y \in R$	$x = 2$.

8. Consider the graph of the function $f(x) = a\log_c b(x - h) + k$, with $a, b > 0$
Which of the parameters a, b, c, h, k, affect the

a) domain

h

(horiz. translation)

b) range

none

c) asymptote

h.

9. Consider the graph of the function $f(x) = a \log_c b(x - h) + k$, with $a, b > 0$.

a) If a is changed to a negative value, does this affect the domain, range, or asymptote?

vert. stretch no

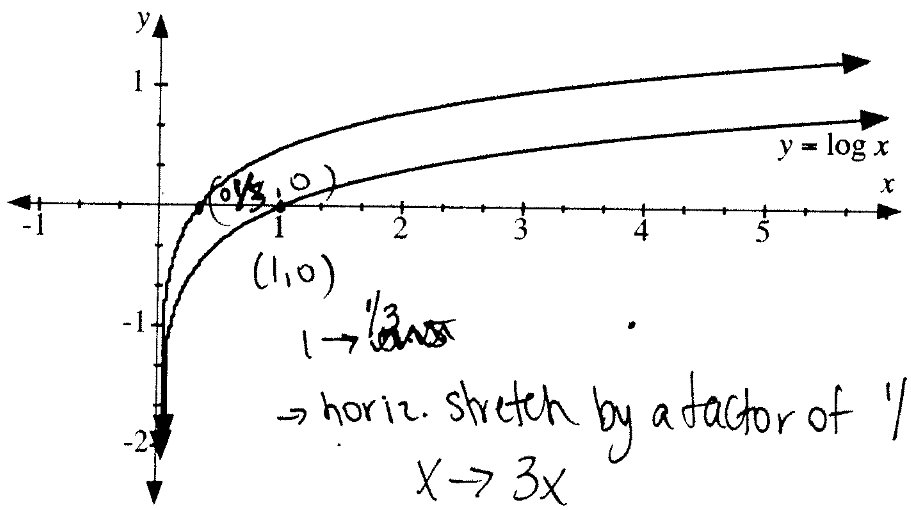
b) If b is changed to a negative value, does this affect the domain, range, or asymptote?

-horiz. stretch → affects domain if $h \neq 0$.

Use the following information to answer the next question.

Two students, Andy and Holly, were given the following question on an exam.

" A transformation is applied to the graph of $y = \log x$ and a sketch of both graphs is shown. Write the equation of the graph which represents the transformed image."



$1 \to 1/3$

→ horiz. stretch by a factor of 1/3.

$X \to 3X$

10. a) Andy ~~correctly~~ answered the question with the equation of the transformed image as $y = \log 3x$. Explain from the given sketch how Andy arrived at his solution.

yes → $y = \log 3x$ is a hor. stretch by a factor of 1/3.

b) Holly also correctly answered the question, but with an equation in the form $y = \log x + k$. Find the value of k.

$\log 3x = \log x + K$

$\log 3 + \log x = \log x + K$ ⤻ $K = \log 3$

11. Explain why $2\log_b x = \log_b x^2$, but the graphs of $y = 2\log_b x$ and $y = \log_b x^2$ are not identical.

$2\log_b X = \log_b X^2$ b/c of product law of logrithms.

$y = 2\log_b X$ - has a doman $X > 0, X \in R$ + $y = \log_b X^2$ has a doman $X \neq 0, X \in R$.

→ domains are different, the graphs are not identical.

Multiple Choice

12. The x-intercept of the graph of the function $f(x) = \log_a(x - d)$ is

let $y = 0$, solve for x
$0 = \log_a(x-d)$
$x - d = a^0$ $x = a^0 + d$ $x = 1 + d$.

A. d **B.** $-d$

C. $1 + d$ **D.** $1 - d$

13. The graph of $y = \log x$ is transformed to the graph of $y = \log(2x + 5)$ by a horizontal stretch about the y-axis by a factor of p followed by a horizontal translation of q units le The value of $p + q$ is

$x \to 2x$ → h.s. factor of $\frac{1}{2}$ $(0.5) = p$
$x \to x + 2.5$ trans 2 units left $= 2 = q$
$p + q = 0.5 + 2.5 = 3$

A. 3

B. 4.5

C. 5.5

D. 7

14. If $\log_6 x = k \log_{216} x$, then the value of k is

$6^3 = 216$
$6 = 216^{1/3}$ $- 3\log_{216} x = \log_6 x$
$\log_{216} x = \log_{6^3} x = \frac{1}{3}\log_6 x$ $k = 3$

A. 3 **B.** -3

C. $\frac{1}{3}$ **D.** 36

15. An equation of the asymptote of $y = 3\log_4(x + 2) - 6$ is

$x \to x + 2$ → 2 units left → moves from 0 to -2 $x = -2$.

A. $x = -6$ **B.** $y = -6$

C. $x = -2$ **D.** $y = -2$

Numerical Response

16. If the graph of $y = \log_7 x$ is reflected in the x-axis, the equation of the image can be writ in the form $y = \log_c x$. The value of c, to the nearest hundredth, is ______ .

(Record your answer in the numerical response box from left to right.)

0	.	1	4

$c = \frac{1}{7} = 0.142857$

17. The graph of $y = \log_5 x$ is translated 2 units down. A student writes the equation of the transformed image in the form $y = \log_5 kx$. The value of k, to the nearest hundredth, is ______ .

(Record your answer in the numerical response box from left to right.)

0	.	0	4

$y \to y + 2$
$y + 2 = \log_5 x$
$y = \log_5 x - 2$

$\log_5 kx = \log_5 x - 2$
$\log_5 k + \cancel{\log_5 x} = \cancel{\log_5 x} - 2$
$\log_5 k = -2$
$k = 5^{-2} = 0.04$