Assignment

1. Without using a calculator, evaluate each of the following.

a) $\log_4 8 + \log_4 0.5$

$= \log_4(8 \cdot 0.5)$

$= \log_4 4$

$= 1$

b) $\log_5 100 - \log_5 4$

$= \log_5\left(\frac{100}{4}\right)$

$= \log_5 25 = \log_5 5^2$

$= 2\log_5 5 = 2(1) = 2$

c) $\log_6 9 + \log_6 8 - \log_6 2$

$\log_6(9 \cdot 8) - \log_6 2$

$= \log_6 72 - \log_6 2$

$= \log_6\left(\frac{72}{2}\right) = \log_6 36$

$= \log_6 6^2 = 2\log_6 6 = 2(1) = 2$

d) $\log 2 + \log 10 - \log \frac{1}{5}$

$= \log(2 \cdot 10) - \log \frac{1}{5}$

$= \log 20 - \log \frac{1}{5}$

$= \log\left(\frac{20}{1/5}\right) = \log 100$

$= \log 10^2 = 2\log_{10} 10$

$= 2(1) = 2$

e) $\log 8 - \log \frac{2}{5} + \log 5$

$= \log\left(\frac{8}{2/5}\right) + \log 5$

$= \log 20 + \log 5 = \log(20 \cdot 5)$

$= \log 100 = \log 10^2$

$= 2\log_{10} 10 = 2(1) = 2$

f) $\log 3 + \log 4 + \log \frac{1}{2} + \log \frac{1}{6}$

$= \log\left(3 \cdot 4 \cdot \frac{1}{2} \cdot \frac{1}{6}\right)$

$= \log 1$

$= 0$

2. In each case, use laws of logarithms to write each expression as a single logarithm and evaluate for the given value of the variable.

a) $\log_x\left(\frac{4}{3}\right) + \log_x 768$, for $x = 2$

$= \log_x\left(\frac{4}{3} \cdot 768\right)$

$= \log_x(1024)$

$\log_2 1024 = \frac{\log 1024}{\log 2} = 10$

b) $\log_a\left(\frac{7}{2}\right) - \log_a 56$, for $a = 4$

$\log_4\left(\frac{7/2}{56}\right) = \log_4 \frac{1}{16}$

$= \log_4 1/16 = \frac{\log 1/16}{\log 4} = -2$

c) $\log_b 9 - \log_b\left(\frac{1}{3}\right)$, for $b = 3$

$= \log_b\left(9 / \frac{1}{3}\right)$

$= \log_b 27$

$= \log_3 27$

$= \frac{\log 27}{\log 3} = 3$

d) $\log_n 3 + \log_n 2 - \log_n 27 - \log_n 6$, for $n = 3$

$= \log_n 3 + \log_n 2 - (\log_n 27 + \log_n 6)$

$= \log_n(3 \cdot 2) - \log_n(27 \cdot 6)$

$= \log_n 6 - \log_n 162$

$= \log_n\left(\frac{6}{162}\right) = \log_n\left(\frac{1}{27}\right)$

$\log_3\left(\frac{1}{27}\right) = \frac{\log 1/27}{\log 3} = -3$

3. Use the laws of logarithms to identify which of the following statements are true for logarithms to ***every base***. Do not use a calculator.

a) $\log_b 2 + \log_b 3 = \log_b 5$

is $\log_b(2 \cdot 3) = \log_b 5$

$\log_b 6 \neq \log_b 5$

false

b) $\log_b 3 + \log_b 4 = \log_b 12$

$\log_b(3 \cdot 4) = \log_b 12$

$\log_b 12 = \log_b 12$

true

c) $\log_b 8 = \log_b 4 + \log_b 2$

$\log_b 8 = \log_b(4 \cdot 2)$

true

d) $\log_b 10 + \log_b 10 = \log_b 100$

$\log_b(10 \cdot 10) = \log_b 100$

true

e) $\log_b 2 \times \log_b 3 = \log_b 6$

$\log_b 2 + \log_b 3 = \log_b 6$

false

f) $\dfrac{\log_b 8}{\log_b 2} = \log_b 4$

$\log_b 8 - \log_b 2 = \log_b 4$

false

g) $\log_b 3^2 + \log_b 3^{-2} = 0$

$\log_b(3^2 \cdot 3^{-2}) = 0$

$\log_b 1 = 0$

true

h) $\log_b \dfrac{5}{3} = \dfrac{\log_b 5}{\log_b 3}$

$\log_b 5 - \log_b 3 \neq \dfrac{\log_b 5}{\log_b 3}$

false

i) $\log_b \dfrac{1}{8} = -\log_b 8$

$\log_b 1 - \log_b 8 = -\log_b 8$

$0 - \log_b 8 = -\log_b 8$

true

4. a) Determine the value of $\log_2 p - \log_2 q$ if $\dfrac{p}{q} = 64$.

$= \log_2\left(\frac{p}{q}\right) = \log_2 64 = \dfrac{\log 64}{\log 2} = 6$

b) Determine the value of $4 \log_3 a + 4 \log_3 b$ if $ab = 81$.

$4(\log_3 a + \log_3 b) = 4 \log_3(ab) = 4 \log_3 81 = 4 \dfrac{\log 81}{\log 3} = 4(4) = 16$

c) Determine the value of $5 \log_5 Q - 5 \log_5 R$ if $Q = 5R$.

$= 5\left(\log_5\left(\frac{Q}{R}\right)\right) = 5 \log_5\left(\frac{5R}{R}\right) = 5 \log_5 5 = 5(1) = 5$

5. Without using a calculator, evaluate each of the following.

a) $\log_2 8^{15}$

$= 15 \log_2 8$

$= 15(3 \log_2 2)$

$= 45 \log_2 2$

$= 45(1)$

$= 45$

b) $\log_7 49^{20}$

$= 20 \log_7 49$

$= 20 \log_7 7^2$

$= 20 \cdot 2 \log_7 7$

$= 20 \cdot 2 \cdot 1$

$= 40$

c) $\log_{49} 7^{20}$

$= 20 \log_{49} 7$

$= 20 \log_{49} 49^{1/2}$

$= 20 \cdot 1/2 \log_{49} 49$

$= 20 \cdot \frac{1}{2} \cdot 1$

$= 10$

d) $\log 10^{15}$

$15 \log_{10} 10$

$= 15 \cdot 1$

$= 15$

6. Use the laws of logarithms to identify which of the following statements are true for logarithms to ***every base***. Do not use a calculator.

a) $\log 5^{-2} = -2 \log 5$

$-2\log 5 = -2\log 5$

true

b) $\log 4 = \frac{2}{3}\log 8$

$\log 4 = \log 8^{2/3}$ $\quad (\sqrt[3]{8})^2 = 4$

$\log 4 = \log 4$ true

c) $\log 125 = \frac{3}{2}\log 25$

$\log 125 = \log 25^{3/2}$ — $\sqrt{25}^3 = 125$

$\log 125 = \log 125$

true

d) $\frac{1}{3}\log 11 = \log \frac{11}{3}$

$\log 11^{1/3} \neq \log \frac{11}{3}$

false

e) $\log 5 = \frac{1}{2}\log 10$

$\log 5 \neq \log 10^{1/2}$

false

f) $\log 2 - \log \sqrt{2} = \log \sqrt{2}$

$\log \frac{2}{\sqrt{2}} = \log \sqrt{2}$

true

g) $\log \frac{1}{5} - \log 5 = -\log 25$

$\log\left(\frac{1}{5}\right) = -\log 25$

$\log \frac{1}{25} = -\log 25$

$\log 1 - \log 25 = -\log 25$

$0 - \log 25 = -\log 25$ → true

h) $\frac{\log \sqrt{2}}{\log \sqrt{8}} = \frac{1}{3}$

$\frac{\log \sqrt{2}}{\log (\sqrt{2})^3} = \frac{1}{3}$ → $\frac{\log \sqrt{2}}{3\log \sqrt{2}} = \frac{1}{3}$

true

7. a) Explain why $\log 81 = 4 \log 3$.

$\log 81 = \log 3^4$

$= 4\log 3$

b) Hence simplify: **(i)** $\log 81 - \log 27$ **(ii)** $\frac{\log 81}{\log 27}$

$\log 81 = 4\log 3$

$\log 27 = \log 3^3 = 3\log 3$

(i) $4\log 3 - 3\log 3$

$= \log 3$

(ii) $= \frac{4\log 3}{3\log 3}$

$= \frac{4}{3}$

8. Determine the greatest of $\frac{1}{3}\log x$, $\frac{2}{3}\log x$, $\frac{4}{3}\log x$ if

a) $x = 2$

$\frac{1}{3}\log 2 = 2^{1/3}$

$\frac{2}{3}\log 2 = 2^{2/3}$

$\frac{4}{3}\log 2 = \boxed{2^{4/3}}$

$\frac{4}{3}\log x$ → largest

b) $x = 1$?

$\frac{1}{3}\log 1 = 0$

$\frac{2}{3}\log 1 = 0$

$\frac{4}{3}\log 1 = 0$

all 0

c) $x = \frac{1}{2}$

$\frac{1}{3}\log \frac{1}{2} = \frac{1}{2}^{1/3}$ ⟸ largest

$\frac{2}{3}\log \frac{1}{2} = \frac{1}{2}^{2/3}$

$\frac{4}{3}\log \frac{1}{2} = \frac{1}{2}^{4/3}$

9. State the value of the following without the use of a calculator.

a) $\log_5 5^7$ (7) **b)** $10^{\log 6}$ (6) **c)** $\ln e^4$ (4) **d)** $\log_c c^t$ (t) **e)** $e^{\ln 7}$ (7)

7 6 4 t 7

Use the following information to answer the next question.

Three students were asked to find an alternative expression for $\log\left(\frac{1}{x}\right)$, $x > 0$.

- Alex gave the answer as $-\log x$.
- Bahman gave the answer as $\log(-x)$.
- Connor gave the answer as $\log(x^{-1})$.

- not defined, can't be negative

Multiple Choice

10. The correct alternative was given by

$\frac{1}{x} = x^{-1}$

$\log \frac{1}{x} = \log x^{-1} = -\log x$

A. Connor only

B. Alex and Connor only

C. Bahman and Connor only

D. some other combination of the students

11. $\log x + \log(x + 4)$ is equal to

$\log(x \cdot (x+4))$
$= \log(x^2 + 4x)$

A. $\log(2x + 4)$ **B.** $\log(x^2 + 4x)$

C. $\log(x^2 + 4)$ **D.** $\log(x) \log(x + 4)$

12. $\log(x^2 - 4) - \log(x - 2)$ is equal to

$\log\left(\frac{x^2-4}{x-2}\right) = \log\left[\frac{(x-2)(x+2)}{(x-2)}\right]$

$= \log(x+2)$

A. $\log(x + 2)$

B. $\log(x^2 - x - 2)$

C. $\log(x - 2)$

D. $\frac{\log(x^2 - 4)}{\log(x - 2)}$ ← tricksy

13. $(\log 2x)^2$ is equivalent to

$(\log 2 + \log x)^2$
$= (\log 2 + \log x)(\log 2 + \log x)$
$= (\log 2)^2 + \log 2 \log x + \log 2 \log x + (\log x)^2$

A. $2 \log 2x$

B. $\log 4x^2$

C. $2 \log 4x$

D. $(\log 2)^2 + 2 \log 2 \log x + (\log x)^2$

Numerical Response

14. The value of $\dfrac{3^{\log_2 4\sqrt{5}}}{3^{\log_2 \sqrt{5}}}$ to the nearest tenth is _____ .

(Record your answer in the numerical response box from left to right.)

9	.	0	

$$3^{\log_2 4\sqrt{5} - \log_2 \sqrt{5}}$$

$$= 3^{\log_2\left(\frac{4\sqrt{5}}{\sqrt{5}}\right)}$$

$$= 3^{\log_2 4}$$

$$= 3^{\log_2 2^2}$$

$$= 3^{2\log_2 2}$$

$$= 3^{2(1)}$$

$$= 3^2 = 9$$

Answer Key

1. a) 1 **b)** 2 **c)** 2 **d)** 2 **e)** 2 **f)** 0

2. a) $\log_x 1024, 10$ **b)** $\log_a\left(\frac{1}{16}\right), -2$ **c)** $\log_b 27, 3$ **d)** $\log_n\left(\frac{1}{27}\right), -3$

3. a) F **b)** T **c)** T **d)** T **e)** F **f)** F **g)** T **h)** F **i)** T

4. a) 6 **b)** 16 **c)** 5

5. a) 45 **b)** 40 **c)** 10 **d)** 15

6. a) T **b)** T **c)** T **d)** F **e)** F **f)** T **g)** T **h)** T

7. a) $\log 81 = \log 3^4 = 4 \log 3$ **b) (i)** $\log 3$ **(ii)** $\frac{4}{3}$

8. a) $\frac{4}{3} \log x$ **b)** none because each of these equals zero. **c)** $\frac{1}{3} \log x$

9. a) 7 **b)** 6 **c)** 4 **d)** t **e)** 7

10. B **11.** B **12.** A **13.** D **14.**

9	.	0	