



A bacterium triples every six days. The number of bacteria n , present after x days, is given by the formula $n = 3^{\frac{x}{6}}$. After how many days are there 243 bacteria?

$$243 = 3^{\frac{x}{6}}$$

$$3^5 = 3^{\frac{x}{6}}$$

$$5 = \frac{x}{6}$$

$$30 = x$$

30 days



Solve the following exponential equations by converting each side to a common base.

a) $27^{x-2} = \frac{1}{81^{x+3}}$

$$(3^3)^{x-2} = \frac{1}{(3^4)^{x+3}}$$

$$3^{3x-6} = 3^{-4x-12}$$

$$3^{3x-6} = 3^{-4x-12}$$

$$3x-6 = -4x-12$$

$$7x = -6$$

$$x = -6/7$$

b) $\left(\frac{125}{216}\right)^{\frac{x}{4}} = \left(\frac{6}{5}\right)^{3x-3}$

$$\left(\frac{216}{125}\right)^{\frac{x}{4}} = \left(\frac{6}{5}\right)^{3x-3}$$

$$\left(\frac{6}{5}\right)^{\frac{3x}{4}} = \left(\frac{6}{5}\right)^{3x-3}$$

$$\frac{3}{4}x = 3x-3$$

$$3x = 12$$

$$12 = 9x$$

$$x = 12/9 = 4/3$$

Complete Assignment Questions #1 - #12

Assignment

1. Simplify.

a) $49^{x-1} \times 7^{2x-3}$

$$(7^2)^{x-1} \cdot 7^{2x-3}$$

$$= 7^{2x-2} \cdot 7^{2x-3}$$

$$= 7^{4x-5}$$

b) $216^x + (1296^{5x-4} \times 36^{x+5})$

$$= (6^3)^x + ((6^4)^{5x-4} \cdot (6^2)^{x+5})$$

$$= 6^{3x} + 6^{20x-16} \cdot 6^{2x+10}$$

$$6^{3x} + 6^{22x-6}$$

$$6^{-19x+6}$$

2. Solve the following exponential equations.

a) $2^{5x+2} = 2^{17}$

$$5x+2=17$$

$$5x=15$$

$$\boxed{x=3}$$

b) $2^{2-3x} = 2^{7x-8}$

$$2-3x=7x-8$$

$$10=10x$$

$$\boxed{x=1}$$

c) $9^{c+1} = 729$

$$9^{c+1} = 9^3$$

$$c+1=3$$

$$\boxed{c=2}$$

d) $5^{4a+2} = 25^a$

$$5^{4a+2} = 5^{2a}$$

$$4a+2=2a$$

$$2a=-2$$

$$\boxed{a=-1}$$

e) $2^{-t} = 64$

$$2^{-t} = 2^6$$

$$-t=6$$

$$\boxed{t=-6}$$

f) $10^{10x-2} = 1000^{20x-42}$

$$10^{10x-2} = (10^3)^{20x-42}$$

$$10x-2=60x-126$$

$$124=50x$$

$$\boxed{x=\frac{62}{25}}$$

3. Solve for x.

a) $2^x = 16\sqrt{2}$

$$2^x = 2^4 \cdot 2^{\frac{1}{2}}$$

$$2^x = 2^{9/2}$$

$$\boxed{x=\frac{9}{2}}$$

b) $8^{3x} = 4^{1-x}$

$$(2^3)^{3x} = (2^2)^{1-x}$$

$$2^{9x} = 2^{2-2x}$$

$$9x=2-2x$$

$$11x=2$$

$$\boxed{x=\frac{2}{11}}$$

c) $9^{3x+1} = 27^{3x}$

$$(3^2)^{3x+1} = (3^3)^{3x}$$

$$3^{6x+2} = 3^{9x}$$

$$6x+2=9x$$

$$2=3x$$

$$\boxed{\frac{2}{3}=x}$$

4. A bacterium doubles every 12 hours. The number of bacteria, N , present after H hours is given by the formula $N = 2^{\frac{H}{12}}$.

a) After how many hours are there 256 bacteria?

$$256 = 2^{\frac{H}{12}}$$

$$2^8 = 2^{\frac{H}{12}}$$

$$8 = \frac{H}{12}$$

$$H=96$$

$$\boxed{96 \text{ hours}}$$

b) Estimate how many hours it would take for the number of bacteria to reach 1000? (An algebraic technique to determine the exact answer to this problem will be shown later in this unit.)

$$1000 = 2^{\frac{H}{12}}$$

$$2^{10} = 1024 \text{ so approx } 10 = \frac{H}{12}$$

$$H \approx 120$$

$$\boxed{\sim 120 \text{ hrs}}$$

$$0 = 9 - \frac{m}{45}$$

168 Exponential and Logarithmic Functions Lesson #2: Solving Exponential Equations ... Common Base

5. A radioactive isotope has a mass of 512 grams, and has a half life of 45 minutes.

The number of grams, N , present after m minutes, is given by the formula $N = 512\left(\frac{1}{2}\right)^{\frac{m}{45}}$

How long would it take for the mass to reduce to one gram?

$$1 = 512\left(\frac{1}{2}\right)^{\frac{m}{45}} \quad \left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{\frac{m}{45}}$$

$$\frac{1}{512} = \left(\frac{1}{2}\right)^{\frac{m}{45}}$$

$$9 = \frac{m}{45}$$

$$m = 405$$

405 mins or 6 hrs. 45 mins

6. Solve for x .

a) $\left(\frac{4}{7}\right)^{5x} = \left(\frac{64}{343}\right)^{2x-1}$

$$\left(\frac{4}{7}\right)^{5x} = \left(\left(\frac{4}{7}\right)^3\right)^{2x-1}$$

$$5x = 6x - 3$$

$$3 = x$$

$$\boxed{x = 3}$$

c) $\left(\frac{125}{216}\right)^{-\frac{x}{2}} = \left(\frac{6}{5}\right)^{3x+2}$

$$\left(\frac{216}{125}\right)^{\frac{x}{2}} = \left(\frac{6}{5}\right)^{3x+2}$$

$$\left(\left(\frac{6}{5}\right)^3\right)^{\frac{x}{2}} = \left(\frac{6}{5}\right)^{3x+2}$$

$$\frac{3x}{2} = 3x + 2$$

$$3x = 6x + 4$$

$$-4 = 3x$$

$$\boxed{-\frac{4}{3} = x}$$

b) $49\left(\frac{7}{12}\right)^{2x} = 144$

$$\frac{49}{49} \quad \frac{144}{49} \rightarrow \left(\frac{12}{7}\right)^2$$

$$\left(\frac{7}{12}\right)^{2x} = \left(\frac{7}{12}\right)^{-2}$$

$$2x = -2$$

$$\boxed{x = -1}$$

d) $\left(\frac{9}{4}\right)^{x+3} = \left(\frac{8}{27}\right)^{-5}$

$$\left(\left(\frac{3}{2}\right)^2\right)^{x+3} = \left(\frac{27}{8}\right)^5$$

$$\left(\frac{3}{2}\right)^{2x+6} = \left(\left(\frac{3}{2}\right)^3\right)^5$$

$$2x + 6 = 15$$

$$2x = 9$$

$$\boxed{x = \frac{9}{2}}$$

7. Solve for x .

a) $2^{x-1} = (128^5)(2^x)$

$$2^{x-1} = (2^7)^5 (2^x)$$

$$x-1 = 8x$$

$$-1 = 7x$$

$$\boxed{x = -\frac{1}{7}}$$

b) $\left(\frac{1}{4}\right)^{x-12} = (2)(32)^{2x+1}$

$$4^{-x+12} = 2^1 (2^5)^{2x+1}$$

$$4^{-x+12} = 2^1 \cdot 2^{10x+5}$$

$$-x+12 = 10x+6$$

$$18 = 12x$$

$$\frac{18}{12} = x$$

$$\boxed{\frac{3}{2} = x}$$

c) $\left(\sqrt[3]{\frac{27^{2x-1}}{3^{x+1}}}\right)^3 = 9^3$

$$\frac{27^{2x-1}}{3^{x+1}} = (3^2)^3$$

$$\frac{(3^3)^{2x-1}}{3^{x+1}} = 3^6$$

$$\frac{3^{6x-3}}{3^{x+1}} = 3^6$$

$$3^{5x-4} = 3^6$$

8. Solve the equation $2(6^{2x}) - 74(6^x) + 72 = 0$.

(Hint: Write as a quadratic equation with the variable as 6^x .)

Let $A = 6^x$ then $A^2 = (6^x)^2 = 6^{2x}$

$$2A^2 - 74A + 72 = 0$$

$$2(A^2 - 37A + 36) = 0$$

$$2(A-1)(A-36) = 0$$

$$A = 1 \text{ or } A = 36$$

$$6^x = 1 \text{ or } 6^x = 36$$

$$x = 0 \text{ or } x = 2$$

$$\boxed{x = 0, 2}$$

$$5^{x-4} = 6$$

$$5x = 10$$

$$\boxed{x = 2}$$

Multiple Choice

9. If $4^{2x-7} = \frac{1}{64}$, then the value of \sqrt{x} is

- A. 2
- ☒ B. $\sqrt{2}$
- C. $\sqrt{5}$
- D. $\frac{3}{2}$

$$4^{2x-7} = \frac{1}{4^3}$$

$$4^{2x-7} = 4^{-3}$$

$$2x-7 = -3$$

$$2x = 4$$

$$x = 2$$

$$\sqrt{x} = \sqrt{2}$$

Numerical Response

10. The solution to the equation $25^{x+1} = 5^{3(x-1)}$, to the nearest tenth, is $x = \underline{\hspace{1cm}}$.

(Record your answer in the numerical response box from left to right.)

5.0

$$(5^2)^{x+1} = 5^{3x-3}$$

$$5^{2x+2} = 5^{3x-3}$$

$$2x+2 = 3x-3$$

$$5 = x$$

11. The solution to the equation $\left(\frac{1}{8}\right)^{x-3} = (2)(16)^{2x+1}$, to the nearest hundredth is $x =$ _____.

(Record your answer in the numerical response box from left to right.)

0	.	3	6
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$$\begin{aligned} \left(\frac{1}{2^3}\right)^{x-3} &= 2 \cdot (2^4)^{2x+1} \\ (2^3)^{-x+3} &= 2^1 \cdot 2^{8x+4} \\ -3x+9 &= 8x+5 \end{aligned}$$

$$\begin{aligned} -3x+9 &= 8x+5 \\ -3x-8x &= 5-9 \\ -11x &= -4 \\ x &= \frac{4}{11} \\ x &= 0.3636 \end{aligned}$$

12. The solution to the equation $8^{2x-1} = 16$, to the nearest tenth, is $x =$ _____.

(Record your answer in the numerical response box from left to right.)

1	.	2	
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$$\begin{aligned} (2^3)^{2x-1} &= 2^4 \\ 2^{6x-3} &= 2^4 \\ 6x-3 &= 4 \\ 6x &= 7 \\ x &= \frac{7}{6} \\ x &= 1.1666... = 1.2 \end{aligned}$$

Answer Key

1. a) 7^{4x-5} b) 6^{-19x+6}

2. a) 3 b) 1 c) 2 d) -1 e) -6 f) $\frac{62}{25}$ or 2.48

3. a) $x = \frac{9}{2}$ b) $x = \frac{2}{11}$ c) $x = \frac{2}{3}$

4. a) 96 hours b) 120 hours

5. 6 hours and 45 minutes

6. a) $x = 3$ b) $x = -1$ c) $x = -\frac{4}{3}$ d) $x = \frac{9}{2}$

7. a) $x = -\frac{1}{7}$ b) $x = \frac{3}{2}$ c) $x = 2$

8. $x = 0, x = 2$

9. B

10.

5	.	0	
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11.

0	.	3	6
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12.

1	.	2	
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