

Applications of Exponential and Logarithmic Functions

Lesson #6: Practice Test

Section A

No calculator may be used for this section of the test.

Use the following information to answer the first question.

A teacher gave his class the following logarithmic equation to solve.

$$2\log_3 n - \log_3 2n = 2$$

The three most common solutions submitted by the students were

1. $n = 0$ 2. $n = 16$ 3. $n = 18$

1. The correct solution to the equation is

- A. 0 only
B. 16 only
☒ C. 18 only
D. none of the above

$$\log_3 n^2 - \log_3 2n = 2$$

$$\log_3 \left(\frac{n^2}{2n} \right) = 2$$

$$\log_3 \left(\frac{n}{2} \right) = 2$$

$$\frac{n}{2} = 3^2$$

$$\frac{n}{2} = 9$$

$$n = 18$$

Numerical Response

1. The root of the equation $5^{\log_3 x - \log_3 2} = 125$ is _____.

(Record your answer in the numerical response box from left to right.)

5	4		
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$$5^{\log_3 x - \log_3 2} = 5^3$$

$$\log_3 x - \log_3 2 = 3$$

$$\log_3 \left(\frac{x}{2} \right) = 3$$

$$\frac{x}{2} = 3^3$$

$$\frac{x}{2} = 27$$

$$x = 54$$

2. The magnitude of an earthquake that is 100 times as intense as an earthquake of magnitude 5.5 on the Richter scale is

- A. 5.7
☒ B. 7.5
C. 11
D. 550

$$100 = 10^2$$

$$5.5 + 2 = 7.5$$

3. If
- $\log_2[\log_x(x+12)] = 1$
- , then
- x
- is equal to

A. 3 only
 B. 4 only
 C. 3 or -4
 D. 4 or -3

$$\begin{aligned}\log_x(x+12) &= 2 \\ (x+12) &= x^2 \\ x+12 &= x^2 \\ x^2 - x - 12 &= 0 \\ (x-4)(x+3) &= 0\end{aligned}$$

$x = -3$ or $x = 4$
 reject $x = -3$ since base > 0
 $x = 4$

4. The population of an Alberta town is decreasing at the rate of 5% per annum. The current population is 25 000. Which equation can be used to determine the number of years,
- n
- , for the population to reduce to 15 000?

A. $25\,000 = 15\,000(0.95)^n$ B. $15\,000 = 25\,000(0.95)^n$
 C. $15\,000 = 25\,000(0.05)^n$ D. $25\,000 = 15\,000(0.05)^n$

$$1 - 0.05 = 0.95$$

5. If
- $\log_6(4a) = \log_6(a-2) + 2$
- , then
- a
- is equal to

A. -2
 B. 2
 C. $\frac{9}{4}$
 D. $1 + \sqrt{10}$

$$\begin{aligned}\log_6(4a) - \log_6(a-2) &= 2 \\ \log_6\left(\frac{4a}{a-2}\right) &= 2 \\ \frac{4a}{a-2} &= 6^2\end{aligned}$$

$$\frac{4a}{a-2} = 36$$

$$4a = 36(a-2)$$

$$4a = 36a - 72$$

$$72 = 32a$$

$$\frac{72}{32} = a$$

$$a = \frac{72}{32} = \frac{9}{4}$$

Section B

A graphing calculator may be used for the remainder of the test.

6. The exact solution to the equation
- $5(2^{4x}) = 3^{x+1}$
- is

A. $\frac{\log 3 - \log 5}{4 \log 2 - \log 3}$
 B. $\frac{\log 3}{4 \log 10 - \log 3}$
 C. $\frac{\log 3 - \log 5}{4 \log 2 - 1}$
 D. $\frac{\log 3}{4 \log 10 - 1}$

$$\begin{aligned}\log(5(2^{4x})) &= \log 3^{x+1} \\ \log 5 + \log 2^{4x} &= \log 3^{x+1}\end{aligned}$$

$$\begin{aligned}\log 5 + 4x \log 2 &= (x+1) \log 3 \\ \log 5 + 4x \log 2 &= x \log 3 + \log 3\end{aligned}$$

$$\begin{aligned}4x \log 2 - x \log 3 &= \log 3 - \log 5 \\ x(4 \log 2 - \log 3) &= \log 3 - \log 5\end{aligned}$$

$$x = \frac{\log 3 - \log 5}{4 \log 2 - \log 3}$$

Numerical
Response

2. A privately owned music store is selling all of its compact discs so it can focus on selling DVDs. During the first week of the sale all compact discs are priced at \$18 and each week the compact discs are reduced by 12% of the previous week's price.

During the 7th week of the sale, the price of compact discs, in dollars rounded to the nearest cent, will be ____.

(Record your answer in the numerical response box from left to right.)

8.36

from week 1 to week 7 there are 6 reductions in price

$$\text{Price} = 18(0.88)^6 = \$8.36$$

Use the following information to answer the next two questions.

A ball is dropped from a height of five metres. After each bounce, the ball rises to 75% of its previous height.

7. The maximum height of the ball, in metres, after it hits the ground for the sixth time is

A. 0.001

B. 0.667

C. 0.890

D. 1.187

$$\begin{aligned} &= 5(0.75)^6 \\ &= 0.8898 \\ &= 0.890 \end{aligned}$$

Numerical
Response

3. The number of bounces it would take for the ball to reach a maximum height of approximately 5 cm is ____.

(Record your answer in the numerical response box from left to right.)

16

$$\begin{aligned} 5\text{cm} &= 0.05\text{m} \\ 0.05 &= 5(0.75)^n \\ 0.01 &= 0.75^n \\ \log_{0.75} 0.01 &= n \end{aligned}$$

$$n = \frac{\log 0.01}{\log 0.75}$$

$$n = 16.007$$

8. Jane would like to visit her family in Australia. She figures that she will need at least \$9000 for the plane ticket and expenses. She goes to a local bank in Calgary to open a savings account. After some thought she decides to deposit \$7500 into an account that pays interest at 8% per annum compounded quarterly.

The number of compounding periods it will take her to reach her goal is

A. 9

B. 10

C. 11

D. 12

$$A = 9000$$

$$P = 7500$$

$$i = 0.08 \div 4 = 0.02$$

$$n = n$$

$$A = P(1+i)^n$$

$$9000 = 7500(1.02)^n$$

$$1.2 = 1.02^n$$

$$\log_{1.02} 1.2 = n$$

$$n = \frac{\log 1.2}{\log 1.02} = 9.2 \dots$$

round up

9. A population is growing continuously according to the formula $A = A_0 e^{kt}$, where A_0 is the initial population, A is the population at the end of t years, and k is the annual growth rate. If the initial population is 6000 and the population at the end of 6 years is 7000, the annual growth rate is

- A. 1.1%
 B. 2.4%
 C. 2.6%
 D. 2.8%

$$\begin{aligned}
 7000 &= 6000 e^{6k} \\
 \frac{7}{6} &= e^{6k} \\
 \ln \frac{7}{6} &= \ln(e^{6k}) \\
 \ln \frac{7}{6} &= 6k \\
 k &= \frac{\ln(\frac{7}{6})}{6} \\
 k &= 0.2569 \\
 &= 2.569\% \dots \\
 &\approx 2.6\%
 \end{aligned}$$

Use the following information to answer the next question.

A student is attempting to solve the equation $5^{4x-2} = 3^{3x+2}$. The student's work is shown below.

$$\begin{aligned}
 &\log 5^{4x-2} = \log 3^{3x+2} \\
 \text{Line A: } &(4x-2)\log 5 = 3x + 2\log 3 \\
 \text{Line B: } &4x - 3x = 2\log 3 + 2\log 5 \\
 \text{Line C: } &x = \log 3^2 + \log 5^2 = \log 9 + \log 25 = \log 225
 \end{aligned}$$

10. The student's work may contain one or more errors. In which line does the first error appear?

- A. Line A
 B. Line B
 C. Line C
 D. The student's solution is correct.

$4x - 2\log 5$ should be $(4x-2)\log 5$

11. In the process of solving $\log_5(2x+1) + \log_5(x-4) = 2$, a correct equation that can arise is

- A. $(2x+1)(x-4) = 2$
 B. $(2x+1)(x-4) = 10$
 C. $(2x+1)(x-4) = 25$
 D. $(2x+1)(x-4) = 32$

$$\begin{aligned}
 \log_5(2x+1)(x-4) &= 2 \\
 (2x+1)(x-4) &= 5^2 \\
 (2x+1)(x-4) &= 25
 \end{aligned}$$

12. The solution to the equation $\log_3(3-4y) - \log_3(y-2) = \log_3 3$ is

A. $y = \frac{9}{7}$

B. $y = 1$

C. $y = \frac{2}{5}$

D. no solution

$$\log_3 \left(\frac{3-4y}{y-2} \right) = \log_3 3$$

$$\frac{3-4y}{y-2} = 3$$

$$3-4y = 3(y-2)$$

$$3-4y = 3y-6$$

$$9 = 7y \quad y = 9/7$$

verify $y = \frac{9}{7}$

$$\log_3(3-4(\frac{9}{7})) - \log_3(\frac{9}{7}-2) = 1$$

$$\log_3(-\frac{15}{7}) - \log_3(-\frac{5}{7})$$

no (-) arguments.

13. Josh borrowed \$2000. Interest was charged at the rate of 3% per annum compounded semi-annually. He did not make any interim payments and paid off the loan in full at the end of the term of the loan.

If the interest he was charged was \$253, then the length of the loan was

A. 2 years

B. 4 years

C. 6 years

D. 8 years

$$A = 2253$$

$$P = 2000$$

$$i = 0.03 \div 2 = 0.015$$

$$n = n$$

$$2253 = 2000(1.015)^n$$

$$1.1265 = 1.015^n$$

$$\log_{1.015} 1.1265 = n$$

$$n = \frac{\log 1.1265}{\log 1.015} = 8.000 \text{ (4 yrs)}$$

Numerical Response

4. An earthquake in Tokyo measured 5.6 on the Richter scale. An aftershock was one-quarter as intense as the original earthquake. To the nearest tenth, the magnitude of the aftershock on the Richter scale was _____.

(Record your answer in the numerical response box from left to right.)

5.0

$$\frac{1}{4} = 10^{x-5.6}$$

$$\log_{10} \frac{1}{4} = x - 5.6$$

$$\log_{10} \frac{1}{4} + 5.6 = x$$

$$4.997... = x$$

$$5.0 = x$$

14. The formula $\frac{I_1}{I_2} = 10^{\frac{dB_1 - dB_2}{10}}$ can be used to compare the intensity (I) of two sounds measured in decibels (dB).

How many times more intense is the sound of a student talking (60 dB) than a student whispering (30 dB)?

A. 2

B. 3

C. 100

D. 1000

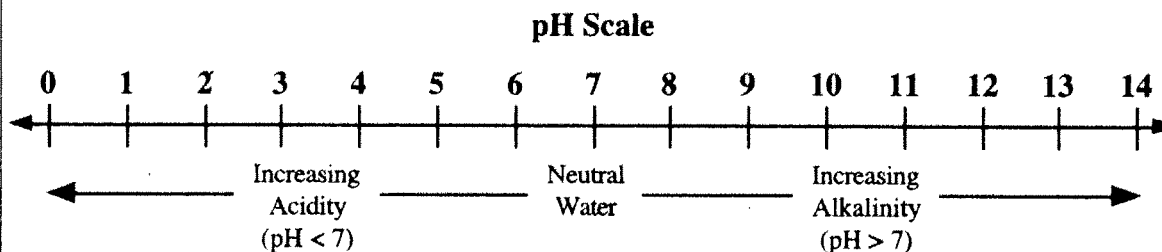
$$\frac{I_1}{I_2} = 10^{\frac{60-30}{10}} = 10^3 = 1000$$

compounding periods

Use the following information to answer the next two questions.

The pH scale is used to measure the acidity or alkalinity of a solution.

The scale is logarithmic in base 10. Thus, a difference of 1 unit in pH corresponds to a factor of 10 difference in intensity. Vinegar (pH of 3) is 10 times as acidic as tomato juice (pH of 4) and 100 times as acidic as rain (pH of 5).



Numerical Response

5. If lemon juice has a pH value of 2.5, then, to the nearest whole number, how many times more acidic is lemon juice than tomato juice?

(Record your answer in the numerical response box from left to right.)

3	2		
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$$10^{4-2.5} = 10^{1.5} = \underline{\underline{31.62}}$$

15. Solution X has a pH of 9.2. Solution Y is twenty times more alkaline than solution X. The pH of solution Y is

A. 7.2

B. 7.9

(C) 10.5

D. 11.2

$$20 = 10^{y-9.2}$$

$$\log_{10} 20 = y - 9.2$$

$$\log_{10} 20 + 9.2 = y$$

$$\underline{\underline{10.5}} = y$$

16. University tuition will be approximately \$25 000 in six years' time. Money invested now will earn interest at 8% per annum compounded semi-annually. The amount of money that should be invested now to have a value of \$25 000 in six years' time is approximately

A. \$19 758

B. \$16 016

C. \$15 754

(D) \$15 615

$$A = 25\,000$$

$$p = p$$

$$r = 0.08 \div 2 = 0.04$$

$$n = 6 \times 2 = 12$$

$$25\,000 = P(1.04)^{12}$$

$$P = \frac{25\,000}{1.04^{12}}$$

$$= \$15\,615$$

17. A radioactive substance loses 90% of its radioactivity in 200 days. An equation which can be used to determine the half-life, p days, of the substance is

A. $10 = 100\left(\frac{1}{2}\right)^{\frac{200}{p}}$

B. $10 = 100\left(\frac{1}{2}\right)^{\frac{p}{200}}$

C. $90 = 100\left(\frac{1}{2}\right)^{\frac{200}{p}}$

D. $90 = 100\left(\frac{1}{2}\right)^{\frac{p}{200}}$

$$100 - 90\% \text{ of } 100 = 10$$

$$A = A_0 C^{t/p}$$

$$10 = 100\left(\frac{1}{2}\right)^{\frac{200}{p}}$$

Numerical Response

6. In a population of insects 32 insects increase to 500 insects in 25 weeks. The doubling time for this population of insects, to the nearest tenth of a week, is _____.

(Record your answer in the numerical response box from left to right.)

6	.	3	
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$$A = A_0 C^{t/p}$$

$$500 = 32(2)^{25/p}$$

$$15.625 = 2^{25/p}$$

$$\log_2 15.625 = 25/p$$

$$p = \frac{25}{\log_2 15.625} = 6.3039$$

18. The value of a painting doubles every 20 years. If the value was \$40 000 on July 1, 2011, then the approximate value on July 1, 1999 was

A. \$12 599

B. \$24 000

C. \$26 390

D. \$60 629

$$40\,000 = A_0 2^{t/20}$$

$$A_0 = \frac{40\,000}{2^{0.6}} = 26\,390$$

19. The complete solution to the equation $\log_5(x-2) = 1 - \log_5(x+2)$ is $x =$

A. 3

B. ± 3

C. $\sqrt{5}$

D. $\pm\sqrt{5}$

$$\log_5(x-2) + \log_5(x-2) = 1$$

$$\log_5(x-2)(x-2) = 1$$

$$(x-2)(x-2) = 5$$

$$x^2 - 4 = 5$$

$$x^2 = 9$$

$$x = \pm 3$$

verify $x = 3$ ✓

$x = -3$ ✗
(negative #)

20. The solution to the equation $p^{x+1} = q^x$ is

A. $\frac{1}{\log q - \log p}$

B. $\frac{\log p}{\log q - \log p}$

C. $\frac{1}{\log p - \log q}$

D. $\frac{\log p}{\log p - \log q}$

$$\begin{aligned}
 p^{x+1} &= q^x \\
 (x+1)\log p &= x\log q \\
 x\log p + \log p &= x\log q \\
 \log p &= x\log q - x\log p \\
 \log p &= x(\log q - \log p) \\
 x &= \frac{\log p}{\log q - \log p}
 \end{aligned}$$

Written Response

A Pre-Calculus student correctly solves each of the following exponential equation problems as a review for an examination.

Problem #1	Problem #2	Problem #3
$2(4^x) = 32$	$9^{x+2} = \left(\frac{1}{27}\right)^x$	$2^{x-3} = 27$
Only solution: $x = 2$	Only solution: $x = -\frac{4}{5}$	Only Solution: $x = 7.75$ (correct to the nearest hundredth)

- In the student's original attempt to solve Problem #1, **one error** was made. The student's original solution is shown below. State the step in which the error was made and explain the error.

Problem #1: $2(4^x) = 32$

Step #1	$8^x = 32$
Step #2	$x\log 8 = \log 32$
Step #3	$x = \frac{\log 32}{\log 8}$
Step #4	$x = 1.67$ (correct to the nearest hundredth)

→ error in Step 1
 $2(4^x) \neq 8^x$

- Algebraically, **using logarithms**, show that $x = 2$ is the **only** correct solution to Problem #1.

$$2(4^x) = 32$$

$$4^x = 16$$

$$\log_4 16 = x$$

or

(We know $x=2$, however

$$\begin{aligned} x &= \frac{\log 16}{\log 4} \\ &= \frac{\log 4^2}{\log 4} \\ &= \frac{2 \log 4}{\log 4} \\ &= 2. \end{aligned}$$

- Algebraically, **without the use of logarithms**, show that $x = -\frac{4}{5}$ is the **only** correct solution to Problem #2.

$$9^{x+2} = \frac{1}{27} x$$

$$3^{2(x+2)} = \left(\frac{1}{3^3}\right)^x$$

$$3^{2x+4} = 3^{-3x}$$

$$2x+4 = -3x$$

$$5x = -4$$

$$x = -\frac{4}{5}$$

- Problem #3 was solved graphically by the student. **Explain** clearly how this problem can be solved using a graphical technique. Include an appropriate graphical window setting for the problem in the form $x: [x_{\min}, x_{\max}, x_{\text{sc1}}], y: [y_{\min}, y_{\max}, y_{\text{sc1}}]$.

Graph $y_1 = 2^{x-3}$

$y_2 = 27$

- use intersect feature to determine x-coordinate of point of intersection of 2 graphs

poss. window: $x: [0, 10, 2] \quad y: [0, 40, 10]$