

Assignment

1. Solve for the variable in each equation. Remember to check for extraneous solutions.

a) $\log_3 x + \log_3 3 = \log_3 30$

$$\log_3 3x = \log_3 30$$

$$3x = 30$$

$$x = 10$$

verify $x = 10$ ✓

$$\boxed{x = 10}$$

2. Solve for x .

a) $\log_9 x - \log_9 3 = 1$

$$\log_9 \left(\frac{x}{3}\right) = 1$$

$$\frac{x}{3} = 9^1$$

$$x = 27 \text{, verify } x = 27 \text{ ✓}$$

$$\boxed{x = 27}$$

3. Solve for the variable in each equation.

a) $\log_5 x - \log_5(x-1) = \log_5 3$

$$\log_5 \left(\frac{x}{x-1}\right) = \log_5 3$$

$$\frac{x}{x-1} = 3 \quad x = 3(x-1)$$

$$x = 3x - 3$$

$$3 = 2x \quad x = 3/2$$

verify $x = 3/2$ ✓

$$\boxed{x = 3/2}$$

c) $\log(2x+3) + \log(x+2) - 1 = 0$

$$\log(2x+3)(x+2) = 1$$

$$\log(2x^2 + 7x + 6) = 1$$

$$2x^2 + 7x + 6 = 10^1$$

$$2x^2 + 7x - 4 = 0$$

● $2x^2 - x + 8x - 4 = 0$

$$x(2x-1) + (2x-1) = 0$$

$$(2x-1)(x+4) = 0$$

$$x = 1/2, -4$$

b) $\log_3 3y - \log_3 4 = \log_3 6$

$$\log_3 \left(\frac{3y}{4}\right) = \log_3 6$$

$$\frac{3y}{4} = 6$$

$$y = \frac{24}{3} = 8$$

verify $y = 8$ ✓

$$\boxed{y = 8}$$

c) $2 \log y = \log 25$

$$y^2 = 25 \quad y = \pm 5$$

verify $y = 5$ ✓

$$\boxed{y = 5}$$

b) $\log_4(x-5) + \log_4(x-2) = 1$

$$\log_4((x-5)(x-2)) = 1$$

$$(x-5)(x-2) = 4^1 \quad x^2 - 7x + 6 = 0$$

$$(x-6)(x-1) = 0 \quad x = 6, 1$$

verify $x = 1$ x

$$x = 6 \quad \checkmark$$

$$\boxed{x = 6}$$

b) $\log_3(3x-1) - \log_3(x-1) = 4$

$$\log_3 \left(\frac{3x-1}{x-1}\right) = 4$$

$$\frac{3x-1}{x-1} = 3^4 - 81$$

$$3x-1 = 81(x-1)$$

$$3x-1 = 81x-81$$

$$80 = 78x$$

$$\frac{80}{78} = x \quad x = 40/39$$

verify $x = \frac{40}{39}$ ✓

$$\boxed{x = \frac{40}{39}}$$

verify $x = \frac{1}{2}$ ✓

verify $x = -4$ x

$$\boxed{x = \frac{1}{2}}$$

264 Applications of Exponential and Logarithmic Functions Lesson #4: Solving Logarithmic Equations

4. Solve each logarithmic equation for the given variable.

State, and explain the reason for, any extraneous roots.

a) $\log_{49}(m+4) + \log_{49}(m-2) = \frac{1}{2}$

$$\log_{49}(m+4)(m-2) = \frac{1}{2}$$

$$\log_{49}(m^2 + 2m - 8) = \frac{1}{2} \rightarrow m^2 + 2m - 8 = 49^{\frac{1}{2}} = 7$$

$$m^2 + 2m - 15 = 0 \quad (m+5)(m-3) = 0$$

$$m = -5, 3$$

verify $m = -5$

$$\log_{49}(-5+4) + \log_{49}(-5-2) = \frac{1}{2}$$

no extraneous, \ominus argument

verify $m = 3$

c) $\log_5(7x-1) - \log_5 x = \log_5 4$

$$\log_5\left(\frac{7x-1}{x}\right) = \log_5 4$$

$$\frac{7x-1}{x} = 4 \quad 7x-1=4x$$

$$3x = 1 \quad x = \frac{1}{3}$$

Verify $x > \frac{1}{3}$

$$\log_5\left(\frac{4}{3}\right) - \log_5\left(\frac{1}{3}\right) = \log_5 4 \quad \checkmark$$

e) $\log_2 x = 2 + \frac{1}{2} \log_2(x-3)$

$$\log_2 x - \frac{1}{2} \log_2(x-3) = 2$$

$$\log_2 x - \log_2(x-3)^{\frac{1}{2}} = 2$$

$$\log_2\left(\frac{x}{(x-3)^{\frac{1}{2}}}\right) = 2$$

$$\frac{x}{(x-3)^{\frac{1}{2}}} = 2^2 = 4$$

$$x = 4\sqrt{x-3}$$

$$x^2 = 16(x-3)$$

$$x^2 = 16x - 48$$

b) $\log_8(-x) + \log_8(3-x) = \log_8 10$

$$\log_8(-x)(3-x) = \log_8 10 \quad (-x)(3-x) = 10$$

$$\log(-3x+x^2) = 10 \quad x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0 \quad x = 5, -2$$

Verify $x = 5 \rightarrow \log_8(-5) + \log_8(3-5) = \log_8 10$
extraneous, neg. argument.

Verify $x = -2$

$$\log_8(2) + \log_8(3-2) = \log_8 10 \quad \checkmark$$

$x = -2$

d) $\log_2 3a + \log_2 2 = \log_2 8 - \log_2 4$

$$\log_2[(3a)(2)] = \log_2\left(\frac{8}{4}\right)$$

$$\log_2 6a = \log_2 2 \quad 6a = 2 \quad a = \frac{2}{6} = \frac{1}{3}$$

Verify $a = \frac{1}{3}$

$$\log_2 1 + \log_2 2 = \log_2 8 - \log_2 4$$

$a = \frac{1}{3}$

$\rightarrow x^2 - 16x + 48 = 0$

$$(x-4)(x-12) = 0 \quad x = 4, 12$$

Verify $x = 4$

$$\log_a 4 = 2 + \frac{1}{2} \log_a(1)$$

$$2 = 2 \quad \checkmark$$

Verify $x = 12$

$$\log_2 12 = 2 + \frac{1}{2} \log_2(9)$$

$x = 4, 12$

5. The number of students in a school t years after the school opens can be modelled by the equation $S = \underbrace{S_0}_{100}[\log_2(t+1) + 1]$, where S_0 is the original number of students in the school.

- a) If there were initially 100 students in the school, how many would be expected after 10 years?

$$S = 100[\log_2(11+1)] = 445.94\ldots$$

446 students after 10 years

- b) How many years will it take for the number of students to reach 800 if the original number of students in the school was 200?

$$800 = 200[\log_2(t+1) + 1] \quad (t+1) = 2^3$$

$$4 = \log_2(t+1) + 1$$

$$3 = \log_2(t+1)$$

$$\begin{aligned} t+1 &= 8 \\ t &= 7 \end{aligned}$$

7 years to reach 800

6. Determine the root(s) of the following equations.

a) $\frac{1}{2}\log_4(y+4) + \frac{1}{2}\log_4(y-4) = \log_4 3$

• multiply both sides by 2

$$\log_4(y+4) + \log_4(y-4) = 2\log_4 3$$

$$\log_4(y+4)(y-4) = 2\log_4 3$$

$$\log_4(y^2 - 16) = \log_4 9$$

$$y^2 - 16 = 9$$

$$y^2 = 25$$

$$y = \pm 5$$

$$\boxed{y=5}$$

verify $y = 5$ ✓
 $y = -5$ ✗

b) $\log_2(1-w) - \log_2(3-w) = -1$

$$\log_2\left(\frac{1-w}{3-w}\right) = -1$$

$$\frac{1-w}{3-w} = 2^{-1} = \frac{1}{2}$$

$$\frac{1-w}{3-w} = \frac{1}{2}$$

$$2(1-w) = 3-w$$

$$2-2w = 3-w$$

$$\text{Verify } w = -1 \quad \boxed{w = -1}$$

7. Solve for x . State, and explain the reason for, any extraneous roots.

a) $\log_5(\log_x(2x-3)) = 0$

$$\log_x(2x-3) = 5^0 = 1$$

$$2x-3 = x^1$$

$$\underline{x = 3}$$

$$\text{Verify } x = 3$$

$$\log_5(\log_3(3)) = \log_5(1)$$

$$\boxed{x=3}$$

b) $\log_2(\log_x(20-x)) = 1$

$$\log_x(20-x) = 2^1$$

$$20-x = x^2$$

$$0 = x^2 + x - 20$$

$$0 = (x+5)(x-4)$$

$$x = -5, 4$$

verify $x = -5$ ✗ verify $x = 4$ ✓

-5 is extraneous
(negative base)

c) $\log_3(\log_2(x^2 - 2x)) = 1$

$$\log_2(x^2 - 2x) = 3^1$$

$$x^2 - 2x = 2^3 = 8$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0 \quad x = 4, -2$$

$$\text{Verify } x = 4 \quad \boxed{x = 4}$$

$$\log_3(\log_2 8) = 1$$

$$\log_3(\log_2(8)) = 1$$

$$\boxed{x = -2}$$

$$\boxed{x = 4, -2}$$

8. Solve.

a) $\log x + (\log x)^2 = 0$

let $\log x = A$

$$A + A^2 = 0 \quad A(1+A) = 0$$

$$A = 0 \text{ or } -1$$

If $\log x = 0$ or $\log x = -1$

$$x = 10^0 \quad x = 10^{-1}$$

$$x = 1 \quad x = \frac{1}{10}$$

Verify $x = 1$ ✓

$$x = \frac{1}{10} \checkmark$$

$$\boxed{x = 1, \frac{1}{10}}$$

b) $(\log x)^2 - \log x^5 = 14$

let $A = \log x \quad A^2 - 5A - 14 = 0$

$$(A-7)(A+2) = 0$$

$$A = 7 \text{ or } A = -2$$

$$\log x = 7$$

$$\log x = -2$$

$$x = 10^7$$

$$x = 10^{-2} = \frac{1}{100}$$

Verify $x = 10^7$ ✓

$$x = \frac{1}{100} \checkmark$$

$$\boxed{x = \frac{1}{100}, 1000000}$$

9. Solve each logarithmic equation.

a) $(\log x)^2 + \log x^{-1} - 12 = 0$

$$(\log x)^2 - \log x - 12 = 0$$

let $\log x = A$

$$A^2 - A - 12 = 0 \quad (A-4)(A+3) = 0$$

$$A = 4 \quad A = -3$$

$$\log x = 4 \quad \log x = -3$$

$$x = 10^4 \checkmark \quad x = 10^{-3} = \frac{1}{1000}$$

$$\boxed{x = 10000, \frac{1}{1000}}$$

c) $3(\log_3 x)^2 - 36 = \log_3 x^{23}$

let $A = \log_3 x$

$$3A^2 - 23A - 36 = 0$$

$$3A^2 - 27A + 4A - 36 = 0$$

$$3A(A-9) + 4(A-9) = 0$$

$$(3A+4)(A-9) = 0$$

$$A = -\frac{4}{3}, 9$$

b) $2(\log_3 n)^3 - (\log_3 n)^2 = 0$

let $\log_3 n = A$

$$2A^3 - A^2 = 0$$

$$A^2(2A-1) = 0 \quad A = 0 \text{ or } \frac{1}{2}$$

$$A = 0$$

$$A = \frac{1}{2}$$

$$\log_3 n = 0$$

$$\log_3 n = \frac{1}{2}$$

$$n = 3^0$$

$$n = 3^{\frac{1}{2}}$$

Verify $n = 3$ ✓

$$n = \sqrt{3} \checkmark \quad \boxed{n = 1, \sqrt{3}}$$

$$\log_3 x = 9$$

$$\log_3 x = -\frac{4}{3}$$

$$x = 3^9$$

~~$$x = 3^{-\frac{4}{3}}$$~~

Verify $x = 3^9$ ✓

$$x = 3^{-\frac{4}{3}}$$

$$\boxed{x = 3^{-\frac{4}{3}}, 3^9 \text{ or } \sqrt[3]{\frac{1}{81}}, 19683}$$



10. If $\log_4(2x+1) + \log_4(x-1) = \frac{1}{2}$, then the value(s) of x is/are

A. $\frac{3}{2}, -1$

B. $\frac{3}{2}$ only

C. 3 only

D. $\frac{1}{2}, 3$

$$\log_4(2x+1)(x-3) = \frac{1}{2}$$

$$(2x+1)(x-3) = 4^{\frac{1}{2}} = 2$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - x - 3 = 0$$

$$2x^2 - 3x + 2x - 3 = 0$$

$$2(x-3) + 1(2x-3) = 0$$

$$(x+1)(2x-3) = 0$$

$$x = \frac{3}{2} \text{ or } -1$$

Verify $x = \frac{3}{2}$ ✓

$$x = -1$$

leads to neg.
argument

- Numerical Response 11. The equation $2 \log x - \log 25 = \log 9 + \log x, x > 0$, has an integral solution.

The value of x is _____.
(Record your answer in the numerical response box from left to right.)

2	2	5
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$$2 \log x - \log 25 = \log 9 + \log x \rightarrow x^2 = 225x$$

$$\log x^2 - \log 25 = \log 9x \rightarrow x^2 - 225x = 0$$

$$\log\left(\frac{x^2}{25}\right) = \log 9x \rightarrow x(x-225) = 0$$

$$\frac{x^2}{25} = 9x \rightarrow x = 0, 225$$

$$x > 0 \text{ so } 225 \checkmark$$

Verify $x = 225$ ✓

12. If $\frac{1}{3} \log \sqrt{x} + \log x^3 = n \log x$ for all values of x , then the value of n , to the nearest tenth, is _____.
(Record your answer in the numerical response box from left to right.)

3	1	2
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$$\frac{1}{3} \log \sqrt{x} + \log x^3 = n \log x$$

$$\frac{1}{3} \log x^{\frac{1}{2}} + 3 \log x = n \log x$$

$$\frac{1}{2} \left(\frac{1}{3}\right) \log x + 3 \log x = n \log x$$

$$\frac{1}{6} \log x + 3 \log x = n \log x$$

$$\frac{19}{6} \log x = n \log x$$

$$\frac{19}{6} = n$$

$$n = 3.166 \dots$$