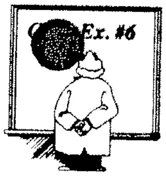

A radioactive isotope has a half-life of 7 years.

a) Use the formula $A = A_0 C^{\frac{t}{p}}$, with $C = \frac{1}{2}$, to determine how much of the isotope must initially be present to decay to 60 grams in 14 years.

$A = 60$
$A_0 = A_0$
$C = \frac{1}{2}$
$t = 14$ yrs
$p = 7$ yrs

$A = A_0 C^{t/p}$
$60 = A_0 \left(\frac{1}{2}\right)^{14/7}$
$60 = A_0 \left(\frac{1}{2}\right)^{2}$
$\rightarrow 60 = \frac{1}{4} A_0$
$240 = A_0$

→ must be 240grams.

b) Write an equivalent formula with $C = 2$ which would solve the problem in a).

$60 = A_0 2^{-2}$

-2 ½ lifes

c) In this particular example, explain how the solution to the problem in a) could be found without using any formula.

14yrs = 2(half life) → intial amount reducs by ½ + the by another ½ (¼) → The final amount is ¼ of the original so orginal 4X the final amount 4X60 = 240g

Complete Assignment Questions #8 - #15

Assignment

1. A truck bought for \$35 000 depreciates at a rate of 12% per year.

a) If the value of the truck is an exponential function of time, state the base of the exponential function.

$1 - 0.12 = 0.88$

b) Write an equation to represent the value, V, of the truck after t years.

$\rightarrow V = 35000(0.88)^4 = 20989.33$ Value = \$21000

c) Determine, to the nearest hundred dollars, the value of the truck after 4 years.

$\rightarrow V = 35000(0.88)^t$

d) How many years, to the nearest tenth, would it take for the value of the truck to reduce to one quarter of its purchase price?

$\frac{1}{4} 35000 = 8750$

$8750 = 35000(0.88)^t$

$0.25 = 0.88^t$

$\log_{0.88} 0.25 = t$

$t = \frac{\log 0.25}{\log 0.88}$

$t = 10.844...$

10.8yrs

2. In 2008 the world population was approximately 6.7 billion and was increasing at an annu: rate of 1.3%.

a) If the function representing the population, in billions, is of the form $y = ab^x$, state values for a and b.

$a = 6.7 \quad b = 1.013$

b) Write an equation to represent the world population, W billions, as a function of the number of years, n, since 2008.

$W = 6.7(1.013)^n$

c) Assuming the same growth rate, determine, to the nearest tenth of a billion, the expecte world population in the year 2025.

$n = 17 \quad W = 6.7(1.013)^{17} = 8.348 \quad$ 8.3 billion

d) If the population continues to grow at this rate, determine the number of years, to the nearest year, for the population to double from its 2008 size.

$6.7 \times 2 = 13.4$

$13.4 = 6.7(1.013)^n$

$2 = 1.013^n$

$\rightarrow \log_{1.013} 2 = n$

$n = \dfrac{\log 2}{\log 1.013} = 53.664$

$\boxed{54\text{ yrs}}$

e) Estimate the world population in 1950. State any assumptions you have made. How does your answer compare with the actual world population in 1950? Give a reason for any discrepancy.

$2008 - 1950 = 58$

$n = -58$

$W = 6.7(1.013)^{-58}$

$= 3.2$ billion assuming a growth rate of 1.3% since 1950

→ the actual population was 2.55 billion so average growth rate since 1950 must have been bigger than 1.3%.

3. The value of a type of robotic technology depreciates 25% per year.

a) Write an exponential function to represent the value of this robotic technology after t years.

$V = V_0(0.75)^t$ (maintains 75%)

b) How many years, to the nearest year, would it take for the value of the robotic technology, which initially cost \$575 000, to depreciate to \$25 000?

$25000 = 575000(0.75)^t$

$\dfrac{1}{23} = 0.75^t$

$\log_{0.75}\left(\dfrac{1}{23}\right) = t$

$\rightarrow t = \dfrac{\log\left(\frac{1}{23}\right)}{\log(0.75)}$

$= 10.899\ldots$

$\boxed{= 11 \text{ years}}$

4. A town in southern British Columbia is growing at a rate of 3.5% per annum. If the town continues to grow at this rate, it is projected that the population will reach 20 000 in 5 years.

a) Determine, to the nearest ten people, the current population of the town.

$A = P(1+i)^n$

$20000 = P(1.035)^5$

$P = \frac{20000}{(1.035)^5} = 16839.4$

→ current pop. is 16840 people

b) Assuming the same growth rate, determine how many years from now the population will reach 30 000. Answer to the nearest year.

$30\,000 = 16840(1.035)^n$

$\frac{30\,000}{16840} = 1.035^n$

$\frac{750}{421} = 1.035^n$

$\log_{1.035}\left(\frac{750}{421}\right) = n$

$n = 16.78\ldots$

17 years

5. A quantity of water contains 500 g of pollutants. Each time the water passes through a filter, 18% of the pollutants are removed. How many filters are needed to reduce the mass of pollutants to less than 150 g?

$A = P(1+i)^n$

$150 = 500(0.82)^n$

$0.3 = (0.82)^n$

$\log_{0.82} 0.3 = n$

$n = \frac{\log 0.3}{\log 0.82}$

$n = 6.066\ldots$

7 filters are needed to reduce the mass to less than 150g.

6. An x-ray beam of intensity, I_0, in passing through absorbing material x millimeters thick, merges with an intensity, I; given by $I = I_0 e^{-kx}$. When the material is 9 millimetres thick, 50% of the intensity is lost.

a) Calculate the value of the constant k to the three decimal places.

$I = 50\% I_0$

$x = 9$

$0.5 I_0 = I_0 e^{-9k}$

$0.5 = e^{-9k}$

$\ln(0.5) = \ln(e^{-9k})$

$\ln 0.5 = -9k$

$k = \frac{\ln(0.5)}{-9}$

$k = \boxed{0.077}$

b) What percentage intensity, to one decimal place, remains if the material is 20 millimetres thick?

$I = I_0 e^{-0.077x}$

let $I_0 = 100$ ← (100% of thickness)

$I = 100e^{-0.077x}$

$I = 100e^{-0.077(20)}$

$I = 21.438$

$\boxed{21.4\% \text{ remains}}$

7. A hot piece of metal loses heat according to the formula $T = T_0e^{-0.2t}$, where T is the temperature difference between the metal and the surrounding air after t minutes and T_0 is the initial temperature difference.

a) If the initial temperature of the metal was 330°C and of the air 30°C, find the temperature of the metal, to the nearest degree, after 5 minutes.

$T_0 = 330° - 30° = 300°$

$T = T_0 e^{-0.2t}$

$T = 300e^{-0.2(5)}$

$T = 110.36°$

T ← temp difference = 110°

air temp = 30°

temperature of metal = 110° + 30° = $\underline{140°}$

b) A different piece of hot metal cools to a temperature of 200°C after 8 minutes. What was the original temperature of the metal, to the nearest degree, if the air temperature was 27°C?

Temp diff = 200° − 27° = 173°

t = 8

$T = T_0 e^{-0.2t}$

$173 = T_0 e^{-0.2(8)}$

$T_0 = \dfrac{173}{e^{-1.6}}$

$T_0 = 857°$

original temp diff = 857°

orig. temp of metal

= 857° + 27°

$\boxed{884°}$

8. How much of a radioactive substance must be present to decay to 30 grams in 12 years if the half–life of the substance is 5.2 years? Round the answer to the nearest gram.

A = 30g

$A_0 = A_0$

$C = \frac{1}{2}$

t = 12yrs.

p = 5.2yrs.

$A = A_0 C^{t/p}$

$30 = A_0\left(\frac{1}{2}\right)^{12/5.2}$

$A_0 = \dfrac{30}{\left(\frac{1}{2}\right)^{12/5.2}}$

$A_0 = 148.527$

$\underline{149g}$ must be present

9. A radioactive isotope has a half–life of approximately 45 minutes. How long would it take for 480 mg of the isotope to decay to 15 mg?

A = 15mg

A_0 = 480 mg

$C = \frac{1}{2}$

t = t

p = 45min

$A = A_0 C^{t/p}$

$15 = 480\left(\frac{1}{2}\right)^{t/45}$

$\dfrac{15}{480} = \left(\dfrac{1}{2}\right)^{t/45}$

$\dfrac{1}{32} = \left(\dfrac{1}{2}\right)^{t/45}$

$\left(\dfrac{1}{2}\right)^{5} = \left(\dfrac{1}{2}\right)^{t/45}$

$5 = \dfrac{t}{45}$

t = 225

$\boxed{\text{time} = 225 \text{ mins}}$

10. A lab technician placed a bacterial cell into a vial at 5 a.m. The cells divide in such a way that the number of cells doubles every 4 minutes. The vial is full one hour later.

a) How long does it take for the cells to divide to produce 4096 cells?

$A = 4096$, $A_0 = 1$, $c = 2$, $t = t$, $p = 4\text{min}$

$A = A_0 c^{t/p}$

$4096 = 1(2)^{t/4}$

$2^{12} = 2^{t/4}$

$12 = \frac{t}{4}$

$48 = t$

time = 48mins

b) At what time is the vial half full?

since doubling time is 4 mins, it will be half full 4 mins before it is full - 4 mins before 6 am is 5:56 am

c) At what time is the vial $\frac{1}{16}$ full?

$\frac{1}{16} = \left(\frac{1}{2}\right)^4 \rightarrow$ 4 doubling periods are required

$6\text{am} - \underbrace{4(4\text{mins})}_{(16\text{mins})} = 5{:}44\text{am}.$

11. The population of germs in a dirty bathtub doubles every 20 minutes. How long, to the nearest minute, would it take for the population to triple?

$A = 3n$, $A_0 = n$, $c = 2$, $t = t$, $p = 20\text{min}$

$3n = n(2)^{t/20}$

$3 = 2^{t/20}$

$\log_2 3 = \frac{t}{20}$

$t = 20\log_2 3$

$\rightarrow t = 20\left(\frac{\log 3}{\log 2}\right) = 31.699$

$\rightarrow$ 32 mins to triple

12. A radioactive isotope has a half-life of approximately 25 weeks. How much of a sample of 50 grams of the isotope would remain after 630 days? (Round the answer to the nearest hundredth of a gram.)

$A = A$, $A_0 = 50\text{g}$, $c = \frac{1}{2}$, $t = 630\text{days}$, $p = 25 \times 7 = 175$ days.

$A = A_0 c^{t/p}$

$A = 50\left(\frac{1}{2}\right)^{630/175}$

$\boxed{A = 4.12\text{g will remain}}$

13. What is the half-life, to the nearest month, of a radioactive isotope if it takes 7 years for 560 grams to decay to 35 grams?

$A = 35\text{g}$, $A_0 = 560\text{g}$, $c = 1/2$, $t = 7$, $p = p$

$A = A_0 c^{t/p}$

$35 = 560\left(\frac{1}{2}\right)^{7/p}$

$\frac{35}{560} = \left(\frac{1}{2}\right)^{7/p}$

$\rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^{7/p}$

$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{7/p}$

$4 = \frac{7}{p}$

$p = \frac{7}{4}$ yrs

$= 21$ months

Numerical Response

14. The tripling period, to the nearest tenth of an hour, of a bacterial culture which grows fro[m] 500 cells to 64 000 cells in 50 hours is _____ .

(Record your answer in the numerical response box from left to right.)

1	1	.	3

$A = 64\,000$
$A_0 = 500$
$C = 3$
$t = 50\text{h}$
$p = p$

$A = A_0 C^{t/p}$
$64000 = 500(3)^{50/p}$
$128 = 3^{50/p}$
$\log_3 128 = \frac{50}{p}$

$p = \frac{50}{\log_3 128}$
$= \frac{50}{(\log 128 / \log 3)}$
$= 11.321$

15. Radioactive material decays to 40% of its original mass in 5 years. The half-life of the radioactive material, to the nearest hundredth of a year, is _____ .

(Record your answer in the numerical response box from left to right.)

3	.	7	8

Let $A_0 = 100\%$
$A = 40$
$C = \frac{1}{2}$
$t = 5\text{yrs}$
$p = p$

$A = A_0 C^{t/p}$
$40 = 100\left(\frac{1}{2}\right)^{5/p}$
$0.4 = \left(\frac{1}{2}\right)^{5/p}$
$\log_{\frac{1}{2}} 0.4 = \frac{5}{p}$

$p = \frac{5}{[\log 0.4 / \log(1/2)]}$
$= 3.78$

Answer Key

1. a) 0.88 **b)** $V = 35\,000(0.88)^t$ **c)** \$21 000 **d)** 10.8

2. a) $a = 6.7$, $b = 1.013$ **b)** $W = 6.7(1.013)^n$ **c)** 8.3 billion **d)** 54 years
e) 3.2 billion assuming a growth rate of 1.3% since 1950. The actual population was 2.55 billion, so the average growth rate since 1950 must have been greater than 1.3%.

3. a) $V_t = V_0(0.75)^t$ **b)** 11 **4. a)** 16 840 **b)** 17 years **5.** 7 filters

6. a) 0.077 **b)** 21.4% **7. a)** 140°C **b)** 884°C **8.** 149 grams

9 225 min **10. a)** 48 min **b)** 5:56 a.m. **c)** 5:44 a.m.

11. 32 min **12.** 4.12 g **13.** 21 months

17.

1	1	.	3

18.

3	.	7	8