$2\times$ per yr

Class Ex. #4

Christine invested $2500 for 4 years compounded semi-annually and received $843.26 interest. What was the annual rate of interest?

$A = 2500 + 843.26 = 3343.26$

$P = 2500$

$i = i$

$n = 4 \times 2 = 8$

$$A = P(1+i)^n$$

$$3343.26 = 2500(1+i)^8$$

$$\frac{3343.26}{2500} = (1+i)^8$$

$$\left(\frac{3343.26}{2500}\right)^{1/8} = 1+i$$

$$1.037 = 1+i$$

$$0.037 = i$$

Annual Rate $= 2(3.7\%)$

$= 7.4\%$

Class Ex. #5

Barbara invests $8000 in an account which pays compound interest of 6% per annum compounded monthly. How long would it take, in years and months, for her investment to double in value ?

$A = 16000$

$P = 8000$

$i = 0.06 \div 12 = 0.005$

$n = n$

$$A = P(1+i)^n$$

$$\frac{16000}{8000} = (1.005)^n$$

$$2 = 1.005^n$$

$$\log_{1.005} 2 = n$$

$$n = \frac{\log 2}{\log 1.005}$$

$= 139$

(139 compounding periods)

$= 11$ years 7 months

Borrowing Money

When an amount of money is borrowed, interest is charged for the use of that money for a certain fixed period of time. If the loan is paid off in one payment at the end of the loan period, then the compound interest formula can be used.

If repayments are made on a regular basis during the period of the loan, the compound interest formula cannot be used.

Class Ex. #6

Andrea borrows $7500 from her parents to buy new car. Her parents charge her interest at the rate of 4% p.a. compounded quarterly. When she pays off the loan, she has to pay $785 interest. What was the length of the loan?

$A = 7500 + 785 = 8285$

$P = 7500$

$i = 0.04 \div 4 = 0.01$

$n = n$

$$8285 = 7500(1.01)^n$$

$$\frac{8285}{7500} = 1.01^n$$

$$\log_{1.01}\left(\frac{8285}{7500}\right) = n$$

$$n = \boxed{\frac{\log\left(\frac{8285}{7500}\right)}{\log 1.01}}$$

$n = 10$ quarters $= 30$ month

$= 2\tfrac{1}{2}$ years

Complete Assignment Questions #1 - #11

Assignment

1. Calculate the simple interest in each case.
   a) $740 is invested at 6% per annum for six months.

      $740 \times 0.06 \times 6/12 = \$22.20$

   b) $1500 is invested at 8%/a for 3 months.

      $\$1500 \times 0.08 \times 3/12 = \$30$

2. If the annual rate of interest is 9% per annum, state the interest rate per compounding peric and the total number of compounding periods in each case:

a) compounded semi-annually for 4 years

rate $= \frac{9}{2} = 4.5\%$, # periods $= 2 \times 4 = 8$

b) compounded quarterly for 3 years

rate $\frac{9}{4} = 2.25\%$, # periods $= 4 \times 3 = 12$

c) compounded monthly for $4\frac{1}{2}$ years

rate $\frac{9}{12} = 0.75\%$, # periods $= 12 \times 4.5 = 54$

d) compounded annually for 6 years

rate $= 9\%$, # periods $= 6$

3. $4000 is invested for 4 years at an annual interest rate of 7.2%.

a) Complete the table to calculate the final value of the investment if interest is compounded according to the period of time given in the table.

Compounding Period	*Number of Compounding Periods Per Year*	*Total Number of Compounding Periods*	*Interest Rate per Compounding Period*	*Formula* $A =$	*Amount*
Annually	1	$4 \times 1 = 4$	7.2%	$4000(1.072)^4$	\$ 5282.50
Semi-Annually	2	$4 \times 2 = 8$	$\frac{7.2}{2} = 3.6\%$	$4000(1.036)^8$	\$ 5308.09
Quarterly	4	$4 \times 4 = 16$	$\frac{7.2}{4} = 1.8\%$	$4000(1.018)^{16}$	\$ 5321.38
Monthly	12	$4 \times 12 = 48$	$\frac{7.2}{12} = 0.6\%$	$4000(1.006)^{48}$	\$ 5330.44

b) If the interest is compounded continuously, then the formula $A = Pe^{kt}$ can be used, where P is the initial amount invested, A is the final amount at the end of t years, and k is the annual interest rate.

i) Calculate the final amount if $4000 is invested for 4 years at an annual interest rate of 7.2% compounded continuously.

$A = 4000e^{(0.072)(4)} = 4000e^{0.288} = \$\ 5335.03$

ii) Use the formula in b) i) to determine the annual interest rate, to the nearest tenth of a percent, if an investment, compounded continuously, doubles in value in 11 years.

$8000 = 4000e^{11k}$ $\quad \ln(2) = \ln(e^{11k})$ $\quad k = \frac{\ln 2}{11} = 0.0630...$

$2 = e^{11k}$ $\quad \ln 2 = 11k$ $\quad$ annual rate $= 6.3\%$

4. The function represented by the compound interest formula is an exponential function.

a) What is the base of the exponential function represented by the compound interest formula? $1 + i$

b) How does the exponent in the compound interest formula compare with the exponent ii the exponential function $y = ab^x, x \in R$?

- exponent n in compound interest formula is = X
- different domains

5. If a student's summer job savings of \$3000 is invested at 12% per year compounded monthly, how many months will it take to earn at least \$600 in interest?

$A = 3600$

$P = 3000$

$i = \frac{0.12}{12} = 0.01$

$n = n$

$A = P(1+i)^n$

$3600 = 3000(1.01)^n$

$\frac{3600}{3000} = 1.01^n$

$1.2 = 1.01^n$

$\log_{1.01} 1.2 = n$

$\rightarrow n = \frac{\log 1.2}{\log 1.01} = 18.32\ldots$

– it will take 19 months to earn @ least \$1600 in interest.

6. The value of an investment is given by $f(x) = 2600(1.062)^x$, where x is the number of years for which the investment is held.

a) What is the annual interest rate if interest is compounded annually? 6.2%

b) Determine the number of complete years until the investment is worth at least \$6000.

$6000 = 2600(1.062)^x$

$\frac{6000}{2600} = 1.062^x$

$\frac{30}{13} = 1.062^x$

$\log_{1.062}(30/13) = x$

$x = \left[\frac{\log(30/13)}{\log 1.062}\right] = 13.90\ldots$

– it will take 14 years for the investment to be worth at least \$6000.

7. A student borrows \$6000. Interest is charged at 5% per year compounded semi-annuall. The loan is paid off in one final payment of \$6958. What was the length of the loan?

$A = 6958$

$P = 6000$

$i = 0.05 \div 2 = 0.025$

$n = n$

$A = P(1+i)^n$

$6958 = 6000(1.025)^n$

$\frac{6958}{6000} = 1.025^n$

$\rightarrow \log_{1.025}\left(\frac{6958}{6000}\right) = n$

$\left[\frac{\log\left(\frac{6958}{6000}\right)}{\log 1.025}\right] = n$

$n = 6.0$

$\rightarrow$ 6 compounding periods $\therefore$ 3 year loan

8. Find the interest rate per annum (to one decimal place) at which a \$3000 investment, compounded quarterly, will double in value over a period of 5 years.

$A = 6000$

$P = 3000$

$i = i$

$n = 4 \times 5 = 20$

$6000 = 3000(1+i)^{20}$

$2 = (1+i)^{20}$

$2^{\frac{1}{20}} = 1 + i$

$1.03526 = 1 + i$

$0.03526 = i$

Annual Rate $= 4(0.03526)$

$= 0.14104$

$= 14.1\%$

9. Mary-Ann invested $3500 in a Canada Savings Bond at an interest rate of 5.4% per year compounded monthly. Carlos invested $3000 in a G.I.C. at an interest rate of 6.8% per year, compounded annually. After how many years will the two investments be approximately equal in value?

let number of years = x.

mary-Ann
$P = 3500$
$i = \frac{0.054}{12} = 0.045$
$n = 12x$

Carlos
$P = 3000$
$i = 0.068$
$n = x$

$3500(1.045)^{12x} = 3000(1.068)^x$

$\frac{7}{6}(1.045)^{12x} = 1.068^x$

$\log\frac{7}{6} + \log(1.045)^{12x} = \log(1.068)^x$

$\log\frac{7}{6} + 12x\log(1.045) = x\log(1.068)$

$\log\frac{7}{6} = x\log(1.068) - 12x\log 1.045$

$\log 7/6 = x(\log 1.068 - 12\log 1.045)$

$x = \frac{\log(7/6)}{\log 1.068 - 12\log 1.045}$

$x = 12.944$

After 13yrs the 2 investment will be approx equal.

Multiple Choice **10.** A student invests $100 @ 8% per year compounded semi-annually. The amount of money that the student will have at the end of each year is increased from the amount at the end of the previous year by a factor of

A. 1.04 **B.** 1.08
C. 1.0816 **D.** 1.16

$A = 100(1.04)^2 = 108.16$

factor $\frac{108.16}{100} = 1.0816$.

Numerical Response **11.** George invests $2500 in an account which pays compound interest of 8.1% per annum compounded quarterly. The number of quarters it will take George's investment to at least double in value is _____ .

(Record your answer in the numerical response box from left to right.)

3	5		

$A = 5000$
$P = 2500$
$i = \frac{0.081}{4} = 0.02025$
$n = n$

$5000 = 2500(1.02025)^n$

$2 = 1.02025^n$

$\log_{1.02025} 2 = n$

$n = 34.57$

$= 35$ quarters.

Answer Key

1. a) $22.20 **b)** $30 **2. a)** 4.5%, 8 **b)** 2.25%, 12 **c)** 0.75%, 54 **d)** 9%, 6

3.a) see table below **b) i)** $5335.03 **ii)** 6.3%

Compounding Period	*Number of Compounding Periods Per Year*	*Total Number of Compounding Periods*	*Interest Rate per Compounding Period*	*Formula*	*Amount*
Annually	1	4	7.2%	$A = 4000(1.072)^4$	$5282.50
Semi-Annually	2	8	3.6%	$A = 4000(1.036)^8$	$5308.09
Quarterly	4	16	1.8%	$A = 4000(1.018)^{16}$	$5321.38
Monthly	12	48	0.6%	$A = 4000(1.006)^{48}$	$5330.44

4. a) $1 + i$
b) The exponent n in the compound interest formula is equal to x, but the set of values for n is discrete, whereas the set of values for x is continuous, i.e. the domains of the functions are not the same.

5. 19 **6. a)** 6.2% **b)** 14 **7.** 3 years **8.** 14.1% **9.** 13

10. C **11.**

3	5		