Assignment

1. Solve the following equations, expressing the solution

i) as an exact value in the form $\frac{\log M}{\log N}$ **ii)** to two decimal places

a) $4^x = 60$

$\log 4^x = \log 60$

$x \log 4 = \log 60$

$x = \frac{\log 60}{\log 4}$

$x \doteq 2.95$

b) $3^x = 90$

$\log 3^x = \log 90$

$x \log 3 = \log 90$

$x = \frac{\log 90}{\log 3}$

$x = 4.10$

c) $7^{x+2} + 3 = 444$ $7^{x+2} = 441$

$\log 7^{x+2} = \log 441$

$(x+2)\log 7 = \log 441$

$x \log 7 + 2\log 7 = \log 441$

$x \log 7 = \log 441 - 2\log 7$

$2\log 7 = \log 7^2 = \log 49$

$x \log 7 = \log\left(\frac{441}{49}\right)$

$x = \frac{\log 9}{\log 7} = 1.13$

2. Show that the solution to the equation $5^{x-2} = 4^{x+1}$ can be written as $\frac{2}{\log 1.25}$.

$\log 5^{x-2} = \log 4^{x+1}$

$(x-2)\log 5 = (x+1)\log 4$

$x\log 5 - 2\log 5 = x\log 4 + \log 4$

$x\log 5 - x\log 4 = \log 4 + 2\log 5$

$x(\log 5 - \log 4) = \log 4 + \log 5^2$

$x = \frac{\log 4 + \log 25}{\log 5 - \log 4} = \frac{\log(100)}{\log(5/4)}$

$= \frac{2}{\log 1.25}$

3. Determine the exact value of x in the form $\frac{\log M}{\log N}$.

a) $5^{x-3} = 40$

$\log 5^{x-3} = \log 40$

$(x-3)\log 5 = \log 40$

$x\log 5 - 3\log 5 = \log 40$

$x\log 5 = \log 40 + 3\log 5$

$x\log 5 = \log 40 + \log 5^3$

$x\log 5 = \log 40 + \log 125$

$x = \frac{\log 40 + \log 125}{\log 5}$

$= \frac{\log 5000}{\log 5}$

b) $0.5^{x+2} = 6^{x-1}$

$\log 0.5^{x+2} = \log 6^{x-1}$

$(x+2)\log 0.5 = (x-1)\log 6$

$x\log 0.5 + 2\log 0.5 = x\log 6 - \log 6$

$x\log 0.5 - x\log 6 = -\log 6 - 2\log 0.5$

$x(\log 0.5 - \log 6) = +\log 6^{-1} + \log 0.5^{-2}$

$x\left(\log\left(\frac{0.5}{6}\right)\right) = \log\left(6^{-1} \cdot 0.5^{-2}\right)$

$x = \frac{\log(2/3)}{\log 1/12}$ or $\frac{\log(3/2)}{\log 12}$

c) $2^{2x} = 6^{x-3}$

$2x\log 2 = (x-3)\log 6$

$2x\log 2 = x\log 6 - 3\log 6$

$2x\log 2 - x\log 6 = \log 6^{-3}$

$x(2\log 2 - \log 6) = \log 6^{-3}$

$x(\log 2^2 - \log 6) = \log 6^{-3}$

$x(\log 4 - \log 6) = \log 6^{-3}$

$x = \frac{\log 6^{-3}}{\log(4/6)} = \frac{\log(1/216)}{\log(2/3)}$ or $\frac{\log 216}{\log 3/2}$

4. Algebraically determine, to the nearest hundredth, the solution to the equation $3(2^x) = 6^{x-2}$

$\log 3(2)^x = \log 6^{x-2}$
$\log 3 + \log 2^x = (x-2)\log 6$
$\log 3 + x\log 2 = x\log 6 - 2\log 6$
$\log 3 + 2\log 6 = x\log 6 - x\log 2$
$\log 3 + \log 6^2 = x(\log 6 - \log 2)$

$x = \dfrac{\log 3 + \log 36}{\log 6 - \log 2} = \dfrac{\log(108)}{\log 3}$

$= \underline{4.26}$

5. A sports car priced at $60 000 depreciates at a rate of 14% per year. The value after n yea is given by $A = 60\,000(0.86)^n$ where A is the amount after depreciation. How long, to the nearest year, will it take for the value of the car to depreciate to $18 000?

$A = 60\,000(0.86)^n$

$\dfrac{18\,000}{60\,000} = \dfrac{60\,000(0.86)^n}{60\,000}$

$0.3 = 0.86^n$

$\log_{0.86} 0.3 = n$

$n = \dfrac{\log 0.3}{\log 0.86}$

$= 7.982$

$\boxed{18 \text{ years}}$

6. The number, N, of throat swab bacteria being grown in a culture after t hours, is given by formula $N = N_0(10^{0.43t})$, where N_0 is the original number of bacteria. If there are initiall 500 bacteria in the culture, determine how long it would take, to the nearest tenth of an ho for the number of bacteria to grow to 1 million.

$N = N_0(10^{0.43t})$

$\dfrac{1\,000\,000}{500} = \dfrac{500(10^{0.43t})}{500}$

$2000 = 10^{0.43t}$

$\log_{10} 2000 = 0.43t$

$t = \dfrac{\log_{10} 2000}{0.43} = 7.6768\ldots$

time = 7.7 hours.

7. Solve the following equations expressing the solution
i) as an exact value ii) to three decimal places

a) $e^x = 5$

$\ln(e^x) = \ln 5$
$x = \ln 5$
$\underline{x = 1.609}$

b) $6 + 0.5e^{2x} = 13$

$0.5e^{2x} = 7$
$e^{2x} = 14$
$\ln(e^{2x}) = \ln 14$
$2x = \ln 14$
$x = \frac{1}{2}\ln 14$
$\underline{x = 1.320}$

c) $5e^{-x} - 4 = 6$

$5e^{-x} = 10$
$e^{-x} = 2$
$\ln(e^{-x}) = \ln 2$
$-x = \ln 2$
$x = -\ln 2$
$\underline{x = -0.693}$

8. Over the last ten years, the amount of money, M (in billions of dollars), spent in North America by car dealerships advertising their product can be modelled by the equatic $M = 0.15e^{0.3t} + 0.78$, where $t = 0$ represents the year 2000.
In what year was about 3 billion dollars spent by car dealerships on advertising?

$3 = 0.15e^{0.3t} + 0.78$

$2.22 = 0.15e^{0.3t}$

$\frac{2.22}{0.15} = e^{0.3t}$

$14.8 = e^{0.3t}$

$\ln 14.8 = \ln(e^{0.3t})$

$\ln 14.8 = 0.3t$

$\frac{\ln 14.8}{0.3} = t$

$8.982 = t$

$2000 + 8.982$

≈ 2009

9. Describe **two** graphing methods to determine the value of x if $5^x = 30$. State an appropriat window. Determine the solution graphically to the nearest hundredth and verify algebraically.

Intersect

Graph $y_1 = 5^x$
$y_2 = 30$

- determine x-coordinate of point of intersection

zero method

rearrange original equation →
$y = 5^x - 30$
graph $y_1 = 5^x - 30$
- determine x-intercept using zero function on calculator.

verify: $5^x = 30$
$\log_5 30 = x$

$x = \frac{\log 30}{\log 5}$

$x = 2.11$.

10. Use a graphing calculator to solve each equation to the nearest hundredth.

a) $4^{x+2} = 7^{2x+5}$

$x = -2.78$

b) $2^{x-5} + 10 = 9^{x-3} - 15$

$x = 4.48$.

(I used intersect feature).

Multiple Choice

11. If $21 = 6^{3x}$, then the value of x is

A. $\frac{\log 21}{\log 216}$

B. $\frac{\log 21}{3}$

C. $\frac{\log 3}{3}$

D. $\frac{\log 21}{\log \sqrt[3]{6}}$

$21 = 6^{3x}$

$\log_6 21 = 3x$

$x = \frac{\log_6 21}{3} = \frac{1}{3} \cdot \log_6 21$

$= \frac{\log 21}{3 \log 6}$

$x = \frac{\log 21}{\log 6^3} = \frac{\log 21}{\log 216}$.

Numerical Response

12. A current, I_0 amperes, falls to I amperes after t seconds according to the formula $I = I_0e^{-}$
The value of the constant, k, to the nearest whole number, if a current of 25 amperes falls to 2.5 amperes in 0.01 seconds is ______ .

(Record your answer in the numerical response box from left to right.)

2	3	0	

$I = 2.5$
$I_0 = 25$
$t = 0.01$

$I = I_0 e^{-kt}$
$2.5 = 25e^{-0.01k}$
$\frac{2.5}{25} = e^{-0.01k}$
$0.1 = e^{-0.01k}$

$\ln(0.1) = \ln(e^{-0.01k})$
$\ln(0.1) = -0.01k$
$\frac{\ln 0.1}{-0.01} = k$
$230.25... = k$

13. The price of a famous brand name camera lens can be found by the equation $P = 14(1.1)^c$, where c is the circumference of the lens in centimetres and P is the price o the lens in dollars. The diameter, to the nearest tenth of a centimetre, of a camera lens which costs \$2500 is ______ .

(Record your answer in the numerical response box from left to right.)

1	7	.	3

$P = 14(1.1)^c$
$2500 = 14(1.1)^c$
$\frac{2500}{14} = (1.1)^c$
$\frac{1250}{7} = 1.1^c$

$\log_{1.1}\left(\frac{1250}{7}\right) = c$
$\frac{\log(1250/7)}{\log 1.1} = c$
$54.401 = c$

circumference.
$C = \pi d$
$\frac{54.401}{\pi} = \frac{\pi d}{\pi}$
$17.316... = d$

14. Kyle determined the exact solution to the exponential equation $5^{2-x} = 2$. He wrote the answer as the quotient of two logarithms in the form $\frac{\log M}{\log 5}$.
The value of M is ______ .

(Record your answer in the numerical response box from left to right.)

1	2	.	5

$\log 5^{2-x} = \log 2$
$(2-x)\log 5 = \log 2$
$2\log 5 - x\log 5 = \log 2$
$2\log 5 - \log 2 = x\log 5$
$\frac{\log 5^2 - \log 2}{\log 5} = x$

$x = \frac{\log 25 - \log 2}{\log 5} = \frac{\log(25/2)}{\log 5}$

$x = \frac{\log 12.5}{\log 5}$

15. The solution to the equation $4^{x-3} = 5(2)^{x+1}$ can be written as $\frac{\log P}{\log 2}$, where P is a whol number. The value of P is ______ .

(Record your answer in the numerical response box from left to right.)

6	4	0	

$\log 4^{x-3} = \log 5 + \log 2^{x+1}$

$(x-3)\log 4 = \log 5 + x\log 2 + \log 2$

$x\log 4 - 3\log 4 = \log 5 + x\log 2 + \log 2$

$x\log 4 - x\log 2 = \log 5 + \log 2 + \log 4^3$

$x(\log 4 - \log 2) = \log 5 + \log 2 + \log 64$

$x = \frac{\log(5\cdot 2\cdot 64)}{\log(4/2)} = \frac{\log 640}{\log 2}.$

Answer Key

1. **a)** **i)** $\frac{\log 60}{\log 4}$ **ii)** 2.95 **b)** **i)** $\frac{\log 90}{\log 3}$ **ii)** 4.10 **c)** **i)** $\frac{\log 9}{\log 7}$ **ii)** 1.13

3. **a)** $\frac{\log 5000}{\log 5}$ **b)** $\frac{\log 1.5}{\log 12}$ **c)** $\frac{\log 216}{\log 1.5}$

4. 4.26 **5.** 8 years **6.** 7.7 hours

7. **a)** **i)** ln 5 **ii)** 1.609 **b)** **i)** $\frac{1}{2}\ln 14$ or $\ln\sqrt{14}$ **ii)** 1.320 **c)** **i)** $-\ln 2$ or $\ln 0.5$ **ii)** -0.693

8. 2009

9. **Intersect Method**: Insert 5^x into Y_1. Insert the 13 into Y_2. Graph Y_1 and Y_2 and determine the x-coordinate of the point of intersection by using the intersect feature of a graphing calculator.
x:[−4, 4, 1] y:[−10, 40, 10]
Zero Method: Rearrange the original equation into $5^x - 30 = 0$. Insert $5^x - 30$ into Y_1, graph and determine the x-intercept(s) of the graph using the zero feature of a graphing calculator.
x:[−4, 4, 1] y:[−30, 20, 10]
The solution is $x = 2.11$.

10. **a)** −2.78 **b)** 4.48 **11.** A

12.

2	3	0	

13.

1	7	.	3

14.

1	2	.	5

15.

6	4	0	