Math 30-1 Review
Trigonometric Functions & Graphs

Name Key

Summarize key features & notation for domain + range of all functions.

y-int of $y = a\cos x + d$

$= \underline{\underline{d + a}}$

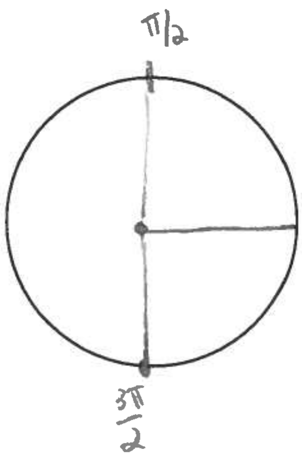
y-int of $y = a\sin x + d$

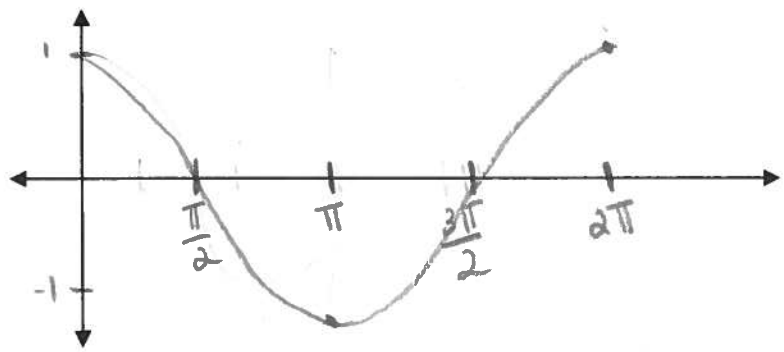
$= d.$

Math 30-1: Review Assignment
Trigonometric Functions & Graphs

Answer the following questions. Remember to show all your work.

1. Sketch the graph of $y = \cos\theta$ for the domain $[0, 2\pi]$. Explain how key points on your graph relate to the unit circle. **(T4.1)**





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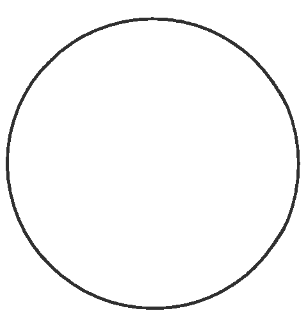
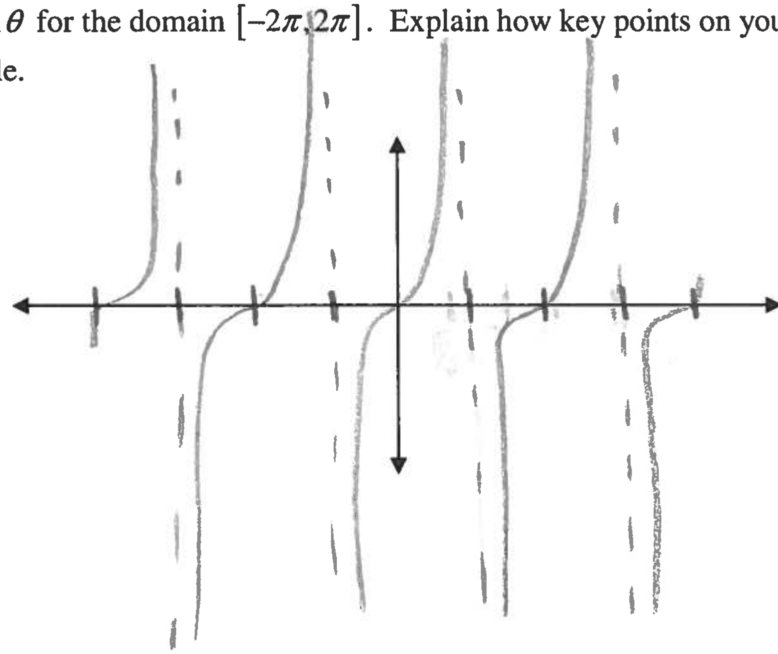
$\frac{\pi}{2}, \frac{3\pi}{2},$ x-int on graph

$0, \pi, 2\pi \rightarrow$ max/min.

2. Sketch the graph of $y = \tan\theta$ for the domain $[-2\pi, 2\pi]$. Explain how key points on your graph relate to the unit circle. **(T4.1)**





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x-int : $-2\pi, -\pi, 0, \pi, 2\pi$

VA $x = -\frac{3\pi}{2}, -\pi/2, \pi/2, 3\pi/2 \rightarrow$ $\tan\theta$ is undefined $\frac{\pi}{2}$

/6

3. Given $g(x) = a\cos(b(x-c)) + d$, describe how each of the following variables affects the graph of $f(x) = \cos x$.

a) a → vertical stretch by factor of a **(T4.3)**

b) b → horizontal stretch by factor of $\frac{1}{b}$ **(T4.6)**

c) c horizontal phase shift c units. **(T4.5)**

d) d vertical displacement d units **(T4.4)**

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4. State the following for the function $y = \sin x$ where x is in radian measure. **(T4.2)**

a) Domain: $x \mid x \in R$

b) Range: $y \mid -1 \leq y \leq 1, \in R$

c) Amplitude:

$= 1$

d) Period:

2π

e) x-intercept(s):

$\pi n, n \in I$ $(0, \pi, 2\pi \ldots)$

f) y-intercept(s):

0

$\frac{6}{}$

5. Determine the range of $y = -5\cos\pi\left(2\left(x - \frac{2\pi}{3}\right)\right) + 3$ **(T4.2)**

max: $-5(1) + 3 = -2$

min: $-5(-1) + 3 = 8$

$y \mid -2 \leq y \leq 8,\ \in R.$

/3

6. Determine the domain of $y = \tan\left(\frac{1}{2}x\right)$. **(T4.2)**

$x \mid x \neq \frac{\pi}{2} + n\pi,\ n \in I$

/2

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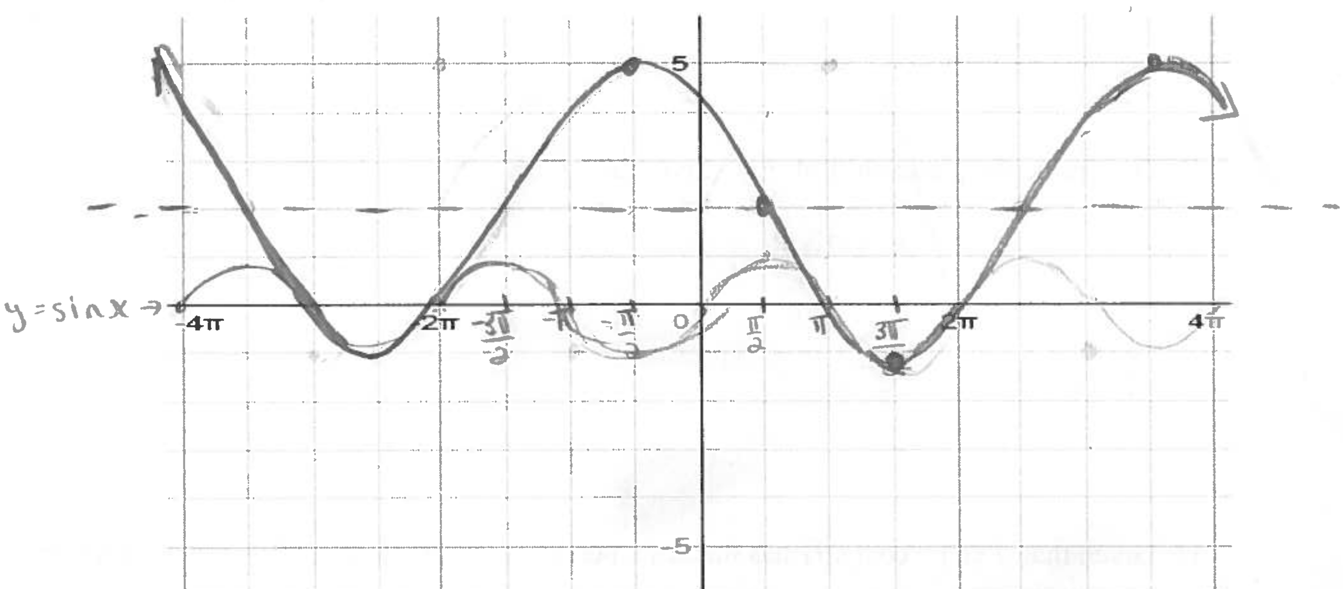
7. a) List the transformations that must be applied to the graph of $f(x) = \cos x$ to become $g(x) = -3\sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) + 2$. **(T4.7)**

vs by factor of 3
hs by factor of 2
reflection across x-axis.
hsp $\frac{\pi}{2}$ R
vertical displacement 2 u ↑

/5

b) Using the transformations you listed above, sketch the graph of $g(x) = -3\sin\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) + 2$ for the domain $[-4\pi, 4\pi]$. *(Remember to label your axes)* **(T4.7)**



/4.

period $= \frac{2\pi}{\frac{1}{2}}$
$= 4\pi$

$x \to 2x + \frac{\pi}{2}$

$y \to -3y + 2$

$0, 0 \to \frac{\pi}{2}, 2$
$\frac{\pi}{2}, 1 \to \frac{3\pi}{2}, -1$
$\pi, 0 \to \frac{5\pi}{2}, 2$
$-\frac{3\pi}{2}, -1 \to \frac{7\pi}{2}, 5$
$2\pi, 0 \to \frac{9\pi}{2}, 2$

/9

8. For the function $y = a\cos\theta + d$, the range is [-4, 10]. What are the values of a and d? **(T4.9)**

$a = \frac{max - min}{2}$

$= \frac{10-(-4)}{2}$

$a = 7$

$a(1) + d = max$

$7(1) + d = 10$

$d = 3$

$a = 7,\ d = 3$

9. For the function $y = \sin(3x + \pi) + 7$, what is the phase shift and period of the corresponding graph? **(T4.8)**

$y = \sin\left[3\left(x + \frac{\pi}{3}\right)\right] + 7$

period $= \frac{2\pi}{3}$

h ps $= \frac{\pi}{3}$ u L

10. Determine the phase shift of $y = -2\cos(2\theta - \pi)$. **(T4.5)**

$y = -2\cos\left[2\left(\theta - \frac{\pi}{2}\right)\right]$

h.p.s $= \frac{\pi}{2}$ u R

11. Given that $f(\theta) = \cos(n\theta)$ has the same period as the graph of $g(\theta) = \tan\theta$, determine the value of n. **(T4.6)**

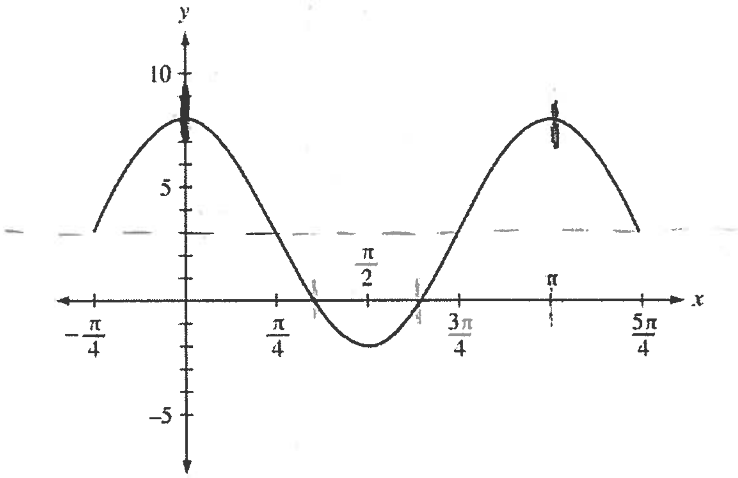
period $= \pi$

$b = \frac{2\pi}{\pi}$

$n = 2$

/8

12. The partial graph of the cosine function below has a minimum point at $\left(\frac{\pi}{2},-2\right)$ and a maximum point at $(\pi,8)$. The equation of the function can be expresses in the form $y=a\cos(bx)+d$. **(T4.9)**



What are the values of a, b, and d?

$\frac{8-(-2)}{2} = 5$

$a = 5$

$d = \frac{8+(-2)}{2} = 3$

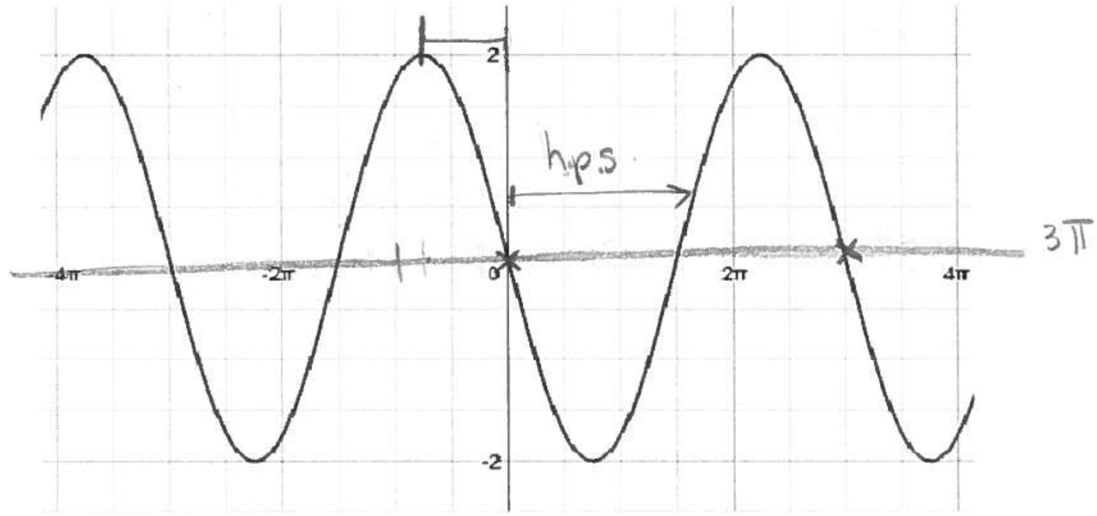
$d = 3$

$b = 2$

$y = 5\cos 2x + 3$

/3

13. Given the following graph of $y = f(x)$; a > 0 add **(T4.9)**



a) Determine an equation of the graph in the form $y = a\sin(b(x-c))+d$.

$a = \frac{2-(-2)}{2} = 2$

$b = \frac{2\pi}{3\pi} = 2/3$

→ could either reflect or be a h.p.s.

$d = \frac{2+(-2)}{2} = 0$

$C = \frac{3\pi}{2}$ u (R)

$y = 2\sin\left[\frac{2}{3}\left(x - \frac{3\pi}{2}\right)\right]$

/4

b) Determine an equation of the graph in the form $y = a\cos(b(x-c))+d$.

hps $\frac{3\pi}{4}$ u L

$y = 2\cos\left[\frac{2}{3}\left(x + \frac{3\pi}{4}\right)\right]$

/2

/6

14. For the graph of the function $f(x) = -3\sin[2(x-5)] + d$ the following statements were made. For each statement determine if the statement is true or false, and explain how you know.

(T4.8)

a) **Statement 1** The amplitude is 3.

true. $a = |-3| = 3$.

b) **Statement 2** The maximum value is $(d-3)$ False.

max $= (-3)(1) + d$.

c) **Statement 3** The period is 2π.

False $\frac{2\pi}{2} = \pi$ is period.

d) **Statement 4** Given that $g(x) = -3\sin(2x) + d$, the graph of $y = g(x)$ could be horizontally translated 5 units to the right to obtain the graph of $y = f(x)$.

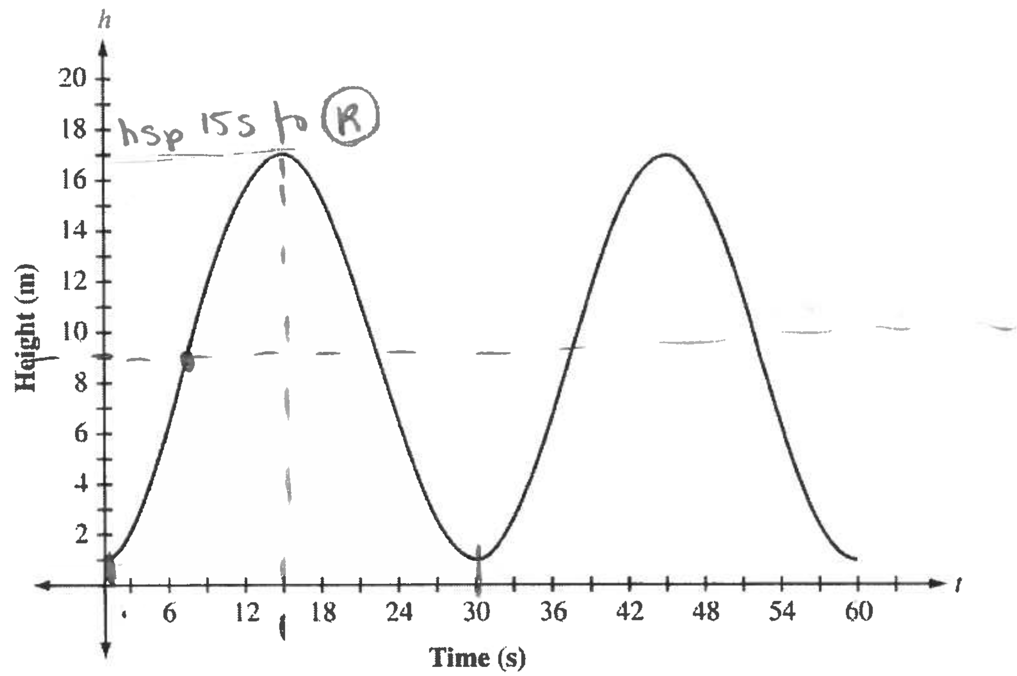
true

e) **Statement 5** If $d > 3$, then the graph of $y = f(x)$ will have no x-intercepts.

true, if vert. disp. is greater than amplilitude the graph will not touch x-axis.

/5

15. The graph below shows the height of a point on a Ferris wheel, h, in metres above the ground, as a function of time, t, in seconds. The maximum height of the Ferris wheel is 17 m and the minimum height is 1 m. **(T4.9)**



Write an equation for the height of the particular point on the Ferris wheel, h, as a function of time, t, in the form $y = a\cos\left[b(t-c)\right] + d$.

$\text{amp} = \frac{17-1}{2} = 8 = a$

$\frac{17+1}{2} = 9$ } midline + vd

$b = \frac{2\pi}{30} = \frac{\pi}{15}$ or $\frac{360}{30} = 12$

$h = 8\cos\left[\frac{\pi}{15}(t-15)\right] + 9$

$c = 15s$ R / L

or

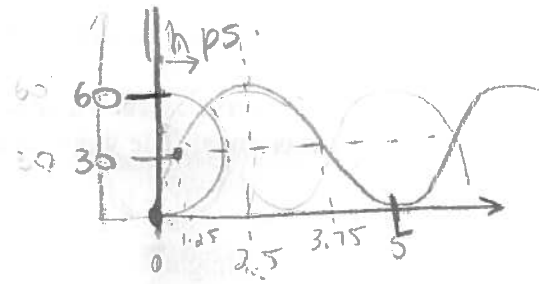
$h = 8\cos\left[\frac{\pi}{15}(t+15)\right] + 9.$

$d = \frac{17+1}{2} = 9$

poorly worded

16. A tire, with a nail caught in the tread, has a diameter of 60 cm and rotates once every 5 seconds. Assume the nail is at the bottom of the tire at time zero. **(T4.9)**

a) Determine an equation to represent the height, h cm, of the nail as a function of time, t seconds, in the form $h(t) = a\sin b(t-c)+d$.



period = 5 sec

$a = \frac{60}{2} = 30$

$b = \frac{2\pi}{5}$ or $\frac{360}{5} = 72$.

c = hps 1.25 s (R)

$h(t) = 30\sin\left[\frac{2\pi}{5}(t-1.25)\right] + 30$

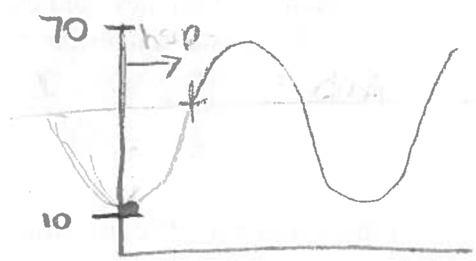
d = 30

or

$h(t) = 30\sin[72(t-1.25)] + 30$

$\frac{}{5}$

b) The car was hoisted so the bottom of the tire was 10 cm above the ground and the tire was spun at a rate of one rotation every 4 seconds. Determine the new equation to represent the height of the nail in the form $h(t) = a\sin b(t-c)+d$.



$a = \frac{70-10}{2} = 30$

$d = \frac{70+10}{2} = 40$

$b = \frac{2\pi}{4} = \frac{\pi}{2}$ or $\frac{360}{4} = 90°$

c = hps = 1.25

$h(t) = 30\sin\left[\frac{\pi}{2}(t-1.25)\right] + 40$

or

$h(t) = 30\sin[90(t-1.25)] + 40$

$\frac{}{4}$

$\frac{}{9}$

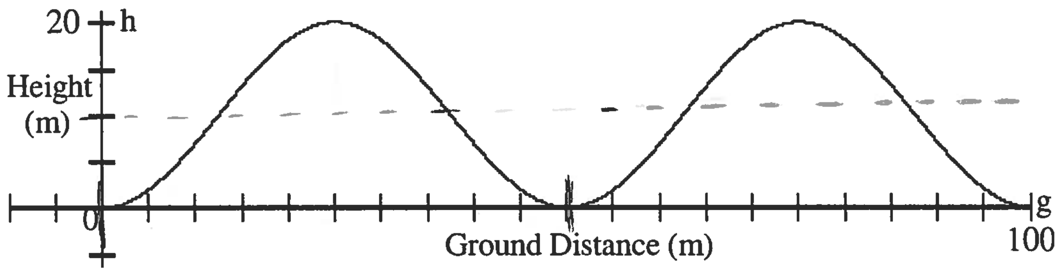
Use the following information to answer the next four questions.

The function that describes the path of the current children's rollercoaster at an amusement park is

$$h = -5\cos\left(\frac{2\pi}{100}x\right) + 5$$

where h is the height above the ground in metres and x is the distance along the ground in metres.

Engineers designed a new rollercoaster for the park that provides riders with a more thrilling experience. The graph that describes the path of the new rollercoaster is shown below.



17. From the graph of the new rollercoaster state the **(T4.2)**

a) amplitude 10

b) period 50

1/2

18. The periods of the new and current rollercoasters are different. **Explain** how the **period** of the new rollercoaster makes the ride more thrilling for passengers. **(T4.11)**

→ shorter distance travelled from a higher height = steeper + faster ride

1

19. Find a trigonometric function of the form $h = a\cos(bx) + d$, that describes the path of the new rollercoaster. Use exact values for *a, b,* and *d* in your equation. **(T4.9)**

$h = -10\cos\left(\frac{\pi}{25}x\right) + 10$ $\qquad b = \frac{2\pi}{50} = \frac{\pi}{25}$

1/2

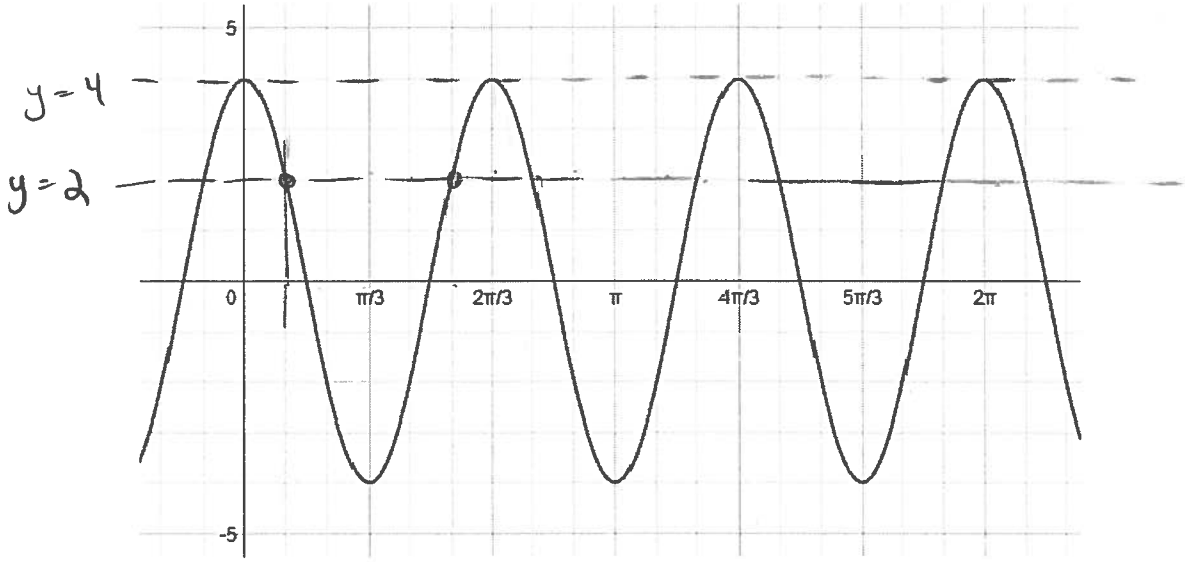
20. A future rollercoaster considered by the engineers would follow the path defined by the function $h = -20\cos\left(\frac{2\pi}{25}x\right) + 20$. **Describe** how the period and amplitude of the future rollercoaster would be different from the new rollercoaster they designed. **(T4.11)**

- amplitude ↑ed by factor of 2, period ↓s from 50 → 25, less time, more height, steeper roller coaster

$2\pi \times \frac{25}{2\pi}$ period = 25

2.

21. Use the graph of $y = 4\cos(3x)$ to solve each trigonometric equation. **(T5.3)**



a) $4\cos(3x) - 4,\ 0 \le x \le 2\pi$

$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$

b) $4\cos(3x) = 2$, general solution in radians

$\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}, \frac{19\pi}{9}$

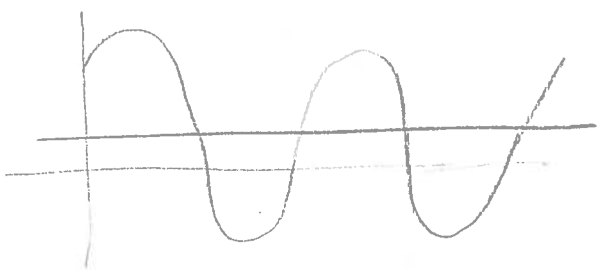
$x = \frac{\pi}{9} + \frac{2\pi}{3}n, \frac{5\pi}{9} + \frac{2\pi}{3}n, \quad n \in I$

22. Graphically solve $10\sin\left(2(x-3)\right)+5=4$, $0\le x\le 2\pi$. Round your answers to two decimal places. Sketch the graph you used. State your window settings. **(T5.3)**

max $10(1)+5=15$
min $(10)(-1)+5=-5$

$x:[0,2\pi,\pi/6]$

$y:[-10,20,2]$



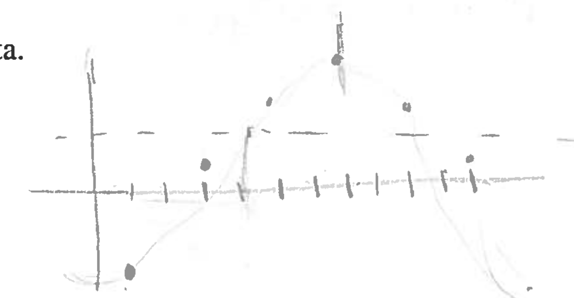
$x=1.48, 2.95, 4.62, 6.09$

23. The following chart shows the monthly average high temperature, in degrees Celsius, for Edmonton, Alberta. **(T4.9)**

Month	1 (Jan)	3 (Mar)	5 (May)	7 (July)	9 (Sept)	11 (Nov)
Average High	-8	1	17	23	16	-1

a) Create a trigonometric equation to represent the data.

$amp=\frac{23-(-8)}{2}=15.5=a$



$b=\frac{2\pi}{12}$ or $\frac{360}{12}=\frac{\pi}{6}$ or $30°$

$c=$ hps 4 mths R.
$d=\frac{23+(-8)}{2}=7.5$

$y=15.5\sin\left[\frac{\pi}{6}(x-4)\right]+7.5$

b) Use your equation to predict the average high for the month of August.

$x=8$ $y=20.9°=21°$