Trigonometry - Equations and Identities Lesson #9: Practice Test

ection A *No calculator may be used for this section of the test.*

1. The expression $\csc\left(\frac{\pi}{2}+x\right)$ is equivalent to

A. $\sec x$

B. $\csc x$

C. $-\csc x$

D. $-\sec x$

$\sin\left(\frac{\pi}{2}+x\right) = \sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x$

$= 1(\cos x) + 0(\sin x)$

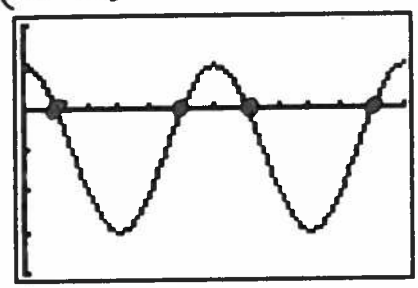
$= \cos x$

$\csc\left(\frac{\pi}{2}+x\right) = \frac{1}{\sin(\frac{\pi}{2}+x)} = \frac{1}{\cos x} = \sec x$

2. The partial graph of $y = 2\cos 2x - 1$, as represented on a graphing calculator screen, is shown.

The solution to the equation $2\cos 2x = 1$, $0 \le x \le 2\pi$, is

$2\cos 2x - 1 = 0$



Window

x: $[0, 2\pi, \pi/6$

y: $[-4, 2, 1]$

A. $0, \pi, 2\pi$ **B.** $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **C.** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ **D.** $\frac{\pi}{2}, \frac{3\pi}{2}$

3. The expression $3\cos^2 8A - 3\sin^2 8A$ is equal to

A. $\cos 64A$

B. $3\cos 16A$

C. $\cos 16A^3$

D. $3\cos 4A$

$= 3(\cos^2 8A - \sin^2 8A)$

$= 3\cos 2(8A)$

$= 3\cos 16A$

4. Assuming the appropriate restrictions on the value of a, the expression $\frac{\tan a + \cot a}{\sec a}$ is equivalent to

A. $\csc a$

B. $\sec a$

C. $\sin a$

D. $\cos a$

$= \dfrac{\frac{\sin a}{\cos a} + \frac{\cos a}{\sin a}}{\frac{1}{\cos a}}$

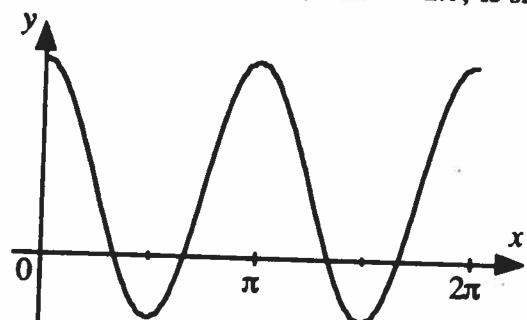
$= \dfrac{\frac{\sin^2 a + \cos^2 a}{\sin a\cos a}}{\frac{1}{\cos a}} = \frac{1}{\sin a\cos a} \cdot \frac{\cos a}{1}$

$= \frac{1}{\sin a}$

$= \csc a$

Use the following information to answer the next question.

The graph of $y = a\cos 2x + d$, $0 \le x \le 2\pi$, is shown.



4 solutions → 2 solutions.

Numerical Response **1.** The number of solutions of the equation $a\cos \underline{x} + d = 0$, $0 \le x \le 2\pi$, is _____.

(Record your answer in the numerical response box from left to right.)

$2x \to x$ (h.s. by a factor of 2)

2			

5. The exact value of $\cos\frac{\pi}{12}$ is

A. $\frac{\sqrt{2} - \sqrt{3}}{2}$

B. $\frac{\sqrt{2} - \sqrt{6}}{4}$

C. $\frac{\sqrt{6} - \sqrt{2}}{4}$

D. $\frac{\sqrt{6} + \sqrt{2}}{4}$

$\frac{\pi}{2} = \frac{\pi}{3} - \frac{\pi}{4}$

$\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$

ection B ***A graphing calculator may be used for the remainder of the test.***

6. $1 - (\sin A + \cos A)^2$ is equivalent to

FOIL

A. $\sin 2A + 2\cos^2 A$

B. $\sin 2A$

C. $-\sin 2A$

D. 0

$1 - (\sin^2 A + 2\sin A\cos A + \cos^2 A)$

$= 1 - (1 + \sin 2A)$

$= 1 - 1 - \sin 2A$

$= -\sin 2A$.

umerical esponse **2.** To the nearest degree, the smallest positive solution of the equation $\sin 5x = 0.75$ is $x =$ _____.

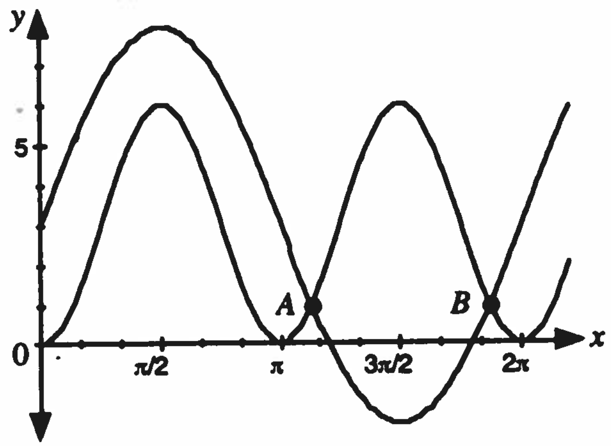
(Record your answer in the numerical response box from left to right.)

1	0		

$5x = 48.59...^\circ$ $x = 9.71^\circ$

Use the following information to answer the next question.

The partial graph of $y = 6\sin^2 x$ and $y = 3 + 5\sin x$ intersect at points A and B.



7. The x-coordinates of points A and B are solutions of

A. $6\sin^2 x(3 + 5\sin x) = 0$ B. $6\sin^2 x + 5\sin x + 3 = 0$

C. $6\sin^2 x - 5\sin x - 3 = 0$ D. $5\sin x + 3 = 0$

→ point of intersection

$6\sin^2 x = 3 + 5\sin x$
$6\sin^2 x - 3 - 5\sin x$.

8. The smallest positive solution of $\tan kx = c$ is $x = \frac{\pi}{8}$.
The general solution of the equation $\tan kx = c$ is

A. $x = \frac{\pi}{8} + 2nk\pi,\ n \in I$ B. $x = \frac{\pi}{8} + 2n\pi,\ n \in I$

C. $x = \frac{\pi}{8} + \frac{2n\pi}{k},\ n \in I$ D. $x = \frac{\pi}{8} + \frac{n\pi}{k},\ n \in I$

9. With the appropriate restrictions on the value of θ, the expression $\frac{\csc 2\theta}{\sec 2\theta}$ can be simplified to

A. $\sin 2\theta$

B. $\cos 2\theta$

C. $\tan 2\theta$

D. $\cot 2\theta$

$$\frac{\frac{1}{\sin 2\theta}}{\frac{1}{\cos 2\theta}} \quad \frac{1}{\sin 2\theta} \cdot \frac{\cos 2\theta}{1}$$
$$= \frac{\cos 2\theta}{\sin 2\theta} = \cot 2\theta$$

10. The complete solution to the equation $\sin x = \log x^2$, where x is in radian measure, is

A. 2.32 B. −0.55, 2.32

C. −0.52, 0.73 D. 0.73

x-int of $y = \sin x - \log x^2$.

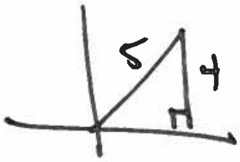
11. If angle A is acute such that $\sin A = \frac{4}{5}$, then $\cos\left(\frac{\pi}{2} + A\right)$ is equal to

A. $-\frac{4}{5}$

B. $-\frac{3}{5}$

C. $\frac{4}{5}$

D. $\frac{3}{5}$



$\cos\left(\frac{\pi}{2} + A\right) = \cos\frac{\pi}{2}\cos A - \sin\frac{\pi}{2}\sin A$

$= 0\cos A - 1\sin A$

$= -\sin A$

$= -\frac{4}{5}$

12. The equation $\sin x = \sin 2x$ has the same solutions as which of the following equations?

A. $2\cos x - 1 = 0$

B. $\sin x\,(2\cos x - 1) = 0$

C. $\sin x = 0$

D. $2\sin x\,(\cos x - 1) = 0$

$\sin x = 2\sin x\cos x$

$0 = 2\sin x\cos x - \sin x$

$0 = \sin x(2\cos x - 1)$

13. If b is a positive integer greater than 1, then the number of solutions in the interval $0 \le x \le 2\pi$ to the equation $\sin bx = \frac{3}{4}$ is

A. 2 **B.** $2b$

C. b **D.** $\frac{1}{2}b$

$\sin x = \frac{3}{4}$ → has 2 solutions for $0 \le x \le 2\pi$

period of $\sin bx = \frac{2\pi}{b}$ → # of solutions = $2(b)$ =

$2b$

14. If $\theta \neq n\pi, n \in I$, then $\cot\theta - \frac{\cos\theta + 1}{\sin\theta}$ is equal to

A. $\csc\theta$

B. $\sec\theta$

C. $-\sec\theta$

D. $-\csc\theta$

$\frac{\cos\theta}{\sin\theta} - \frac{\cos\theta + 1}{\sin\theta}$

$= \frac{\cos\theta - (\cos\theta + 1)}{\sin\theta} = \frac{-1}{\sin\theta} = -\csc\theta$

Numerical Response **3.** If $4\cos^2\theta - 11\cos\theta - 3 = 0$, $0° < \theta < 180°$, then the measure of θ, to the nearest degree, is ______ .

(Record your answer in the numerical response box from left to right.)

1	0	4	

$4\cos^2\theta - 12\cos\theta + \cos\theta - 3 = 0$

$4\cos\theta(\cos\theta - 3) + 1(\cos\theta - 3) = 0$

$(4\cos\theta + 1)(\cos\theta - 3)$ Q2 ref∠ = 76°

$\theta = 180° - 76° = 104°$

$\cos\theta = -\frac{1}{4}$ or $\cos\theta = 3$ no solution

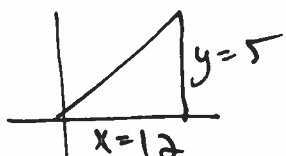
15. If $\tan P = \frac{5}{12}$, $0 \le P \le \frac{\pi}{2}$, Q1 and $\tan Q = \frac{4}{3}$, $\frac{\pi}{2} \le Q \le \frac{3\pi}{2}$, Q3 then the exact value of $\sin(P - Q)$ is

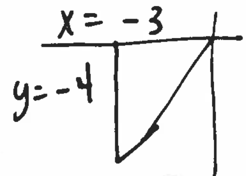
A. -33

B. $-\frac{33}{65}$

C. $\frac{33}{65}$

D. $-\frac{63}{65}$





$r^2 = 12^2 + 5^2$
$r = 13$
$\sin P = \frac{5}{13}$
$\cos P = \frac{12}{13}$

$r = (-3)^2 + (-4)^2$
$r = 5$
$\sin Q = \frac{-4}{5}$
$\cos Q = \frac{-3}{5}$

$\sin(P-Q) = \sin P\cos Q - \cos P\sin Q = \left(\frac{5}{13}\right)\left(\frac{-3}{5}\right) - \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right) = \frac{33}{65}$

Numerical Response **4.** To the nearest tenth of a radian, the solution to the equation $\cos\left(x - \frac{\pi}{4}\right) = 2x - 5$ is ______ .

Rad mode.

(Record your answer in the numerical response box from left to right.)

2	.	5	

graphs intersect at $x = 2.452$

16. The general solution to the equation $\sin 3x = 1$, $x \in R$ is $x =$

A. $\frac{\pi}{2} + 2n\pi$, $n \in I$ **B.** $\frac{\pi}{2} + \frac{2}{3}n\pi$, $n \in I$

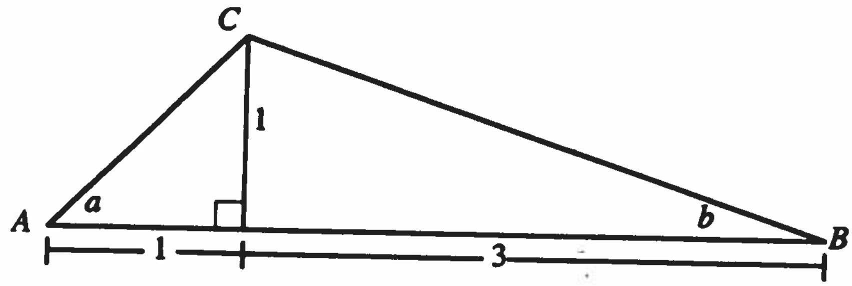
C. $\frac{\pi}{6} + \frac{2}{3}n\pi$, $n \in I$ **D.** $\frac{\pi}{6} + 2n\pi$, $n \in I$

$\sin x = 1$ $x = \frac{\pi}{2} + 2n\pi$

$\sin 3x = 1$ $x = \frac{\pi}{6}, \frac{2}{3}n\pi$

Use the following information to answer the next question.

In triangle *ABC* shown, the exact value of $\sin(a+b)$ can be written in the form $k\sqrt{5}$.



Numerical Response **5.** To the nearest tenth, the value of k is _____.

(Record your answer in the numerical response box from left to right.)

0 . 4

$AC^2 = 1^2 + 1^2 = 2$

$AC = \sqrt{2}$

$BC^2 = 1^2 + 3^2 = 10$

$BC = \sqrt{10}$

$\sin a = \frac{1}{\sqrt{2}}$

$\cos a = \frac{1}{\sqrt{2}}$

$\sin b = \frac{1}{\sqrt{10}}$

$\cos b = \frac{3}{\sqrt{10}}$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$= \frac{1}{\sqrt{2}} \cdot \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{10}} = \frac{3}{\sqrt{20}} + \frac{1}{\sqrt{20}}$

$= \frac{4}{\sqrt{20}} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$k = 2/5 = 0.4$

17. The expression $\dfrac{1-\cos^2 A}{1+\tan^2 A}$ is equivalent to

A. $\sin^2 A \cos^2 A$

B. $\sin^4 A$

C. $-\dfrac{\cos^4 A}{\sin^2 A}$

D. $\sin^2 A$

$\frac{1-\cos^2 A}{1+\tan^2 A} = \frac{\sin^2 A}{\sec^2 A} = \frac{\sin^2 A}{\frac{1}{\cos^2 A}} = \sin^2 A \cdot \frac{\cos^2 A}{1}$

$= \sin^2 A \cos^2 A$

18. A student uses the angle $x = \frac{\pi}{6}$ to verify the identity $\dfrac{\sin^2 x}{\sec x + 1} = \dfrac{1-\cos x}{\sec x}$.

Using $x = \frac{\pi}{6}$, the exact value of each side of the identity is

A. $\frac{1}{4}$

B. 1

C. $\dfrac{2\sqrt{3}-3}{3}$

D. $\dfrac{2\sqrt{3}-3}{4}$

$\sec\frac{\pi}{6} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\sqrt{3}/2} = \ldots$

$\frac{\sin^2 \frac{\pi}{6}}{\sec\frac{\pi}{6}+1} = \frac{\left(\frac{1}{2}\right)^2}{\frac{2}{\sqrt{3}}+1} = \frac{\frac{1}{4}}{\frac{2+\sqrt{3}}{\sqrt{3}}}$

$= \frac{1}{4} \cdot \frac{\sqrt{3}}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2\sqrt{3}-3}{4(4-3)} = \frac{2\sqrt{3}-3}{4}$

19. The expression $\dfrac{\cos^2 x - \cos^4 x}{\sin^4 x}$, with $\sin x \neq 0$, simplifies to

A. $\tan^2 x$

B. $\cot^2 x$

C. $\cos^2 x \sin^2 x$

D. $-\cot^2 x \csc^2 x$

$$\frac{\cos^2 x\,(1-\cos^2 x)}{\sin^4 x} = \frac{\cos^2 x \sin^2 x}{\sin^4 x} = \frac{\cos^2 x}{\sin^2 x} = \cot$$

20. The expression $\dfrac{\sin A}{\cos A + \sin A \cot A}$ is equivalent to

A. $2 \tan A$

B. $\tan A + \cot A$

C. $\tan A \sec A$

D. $\dfrac{1}{2} \tan A$

$$\frac{\sin A}{\cos A + \cancel{\sin A} \cdot \frac{\cos A}{\cancel{\sin A}}} = \frac{\sin A}{2\cos A} = \frac{1}{2}\tan A$$

Numerical Response **6.** The smallest positive solution to the equation $\sec x - 5 = 0$, correct to the nearest tenth of a radian, is $x =$ _____.

(Record your answer in the numerical response box from left to right.)

1	.	4	

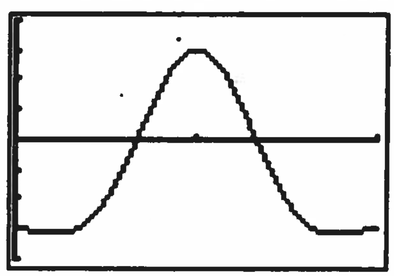
$\sec X = 5$ $\quad \cos X = \frac{1}{5}$ $\quad X = 1.369$

Written Response

Use the following information to answer this question.

A student graphs the trigonometric function $f(x) = \cos(2x) - 3\cos x - 1$ on a graphing calculator, with window x:[0, 2, 1] y:[–4, 4, 1].

The graphing calculator screenshot is shown.



- How can the student use the calculator graph to determine the roots of the equation $0 = \cos(2x) - 3\cos x - 1$ in the domain $0 \leq x \leq 2\pi$?

- the roots of the equation are the x-int of the graph → use zero feature of calculator.

- Use a calculator graph to determine the roots to the equation $0 = \cos(2x) - 3\cos x - 1$ correct to the nearest hundredth of a radian, in the domain $0 \le x \le 2\pi$.

roots are 2.09 + 4.19

- Use an addition identity to prove the identity $\cos(2x) = 2\cos^2 x - 1$.

$$\cos(2x) = \cos(x+x) = \cos x \cos x - \sin x \sin x$$
$$= \cos^2 x - \sin^2 x \to \cos^2 x - (1 - \cos^2 x)$$
$$= \cos^2 x - 1 + \cos^2 x$$
$$= 2\cos^2 x - 1 = \underline{\underline{RS}}$$

- By using the identity in bullet 3, **algebraically** determine the roots of the equation $0 = \cos(2x) - 3\cos x - 1$ in the domain $0 \le x \le 2\pi$. Give the answer in radians as exact values in terms of π.

$$0 = \cos(2x) - 3\cos x - 1$$
$$0 = 2\cos^2 x - 1 - 3\cos x - 1 \to 2\cos^2 x - 3\cos x - 2$$
$$0 = 2\cos^2 x - 4\cos x + \cos x - 2$$
$$0 = 2\cos x(\cos x - 2) + 1(\cos x - 2)$$
$$= (2\cos x + 1)(\cos x - 2)$$
$$\cos x = \frac{-1}{2} \text{ or } \cos x = 2$$
no solution

$\cos x = \frac{-1}{2}$ Q 2|3

ref $\angle = \frac{\pi}{3}$

$x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$.

Answer Key

Multiple Choice

1. A	2. B	3. B	4. A	5. D	6. C	7. C	8. D
9. D	10. B	11. A	12. B	13. B	14. D	15. C	16. C
17. A	18. D	19. B	20. D				

Numerical Response

1.	2			
2.	1	0		
3.	1	0	4	
4.	2	.	5	
5.	0	.	4	
6.	1	.	4	

Written Response

- The roots of the equation are the x-intercepts of the graph which are found using the zero feature of the calculator.
- 2.09, 4.19
- Use the identity for $\cos(x + x)$.
- $\frac{2\pi}{3}, \frac{4\pi}{3}$