Assignment

1. Solve for x as an exact value where $0° \le x \le 360°$. Degree mode.

a) $3 - 3\sin x - 2\cos^2 x = 0$

$3 - 3\sin x - 2(1 - \sin^2 x) = 0$

$3 - 3\sin x - 2 + 2\sin^2 x = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin^2 x - 2\sin x)(- \sin x + 1) = 0$

$2\sin x(\sin x - 1) - 1(\sin x - 1) = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = 1$

Q1|2 ref∠=90° Q1|2 ref∠=30°

$x = 90°$ $x = 30°, 150°$

$x = 30°, 90°, 150°$

b) $\tan^2 x - 1 = \sec x$

$(\sec^2 x - 1) - 1 = \sec x$

$\sec^2 x - \sec x - 2 = 0$

$(\sec x + 1)(\sec x - 2) = 0$

$\sec x = -1$ or $\sec x = 2$

$\cos x = -1$ or $\cos x = \frac{1}{2}$

Q2|3 Q1|4

ref∠=0° ref∠=60°

$x = 180°$ $x = 60°, 300°$

$x = 60°, 180°, 300°$

S | A
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2. Determine the roots of the equation $7\sec^2 x + 2\tan x - 6 = 2\sec^2 x + 2$, $0 \le x \le 2\pi$. Use exact values where possible; otherwise round the answers to the nearest tenth.

$7\sec^2 x + 2\tan x - 2\sec^2 x - 2 = 0$

$5\sec^2 x + 2\tan x - 2 = 0$

$5(1 + \tan^2 x) + 2\tan x - 2 = 0$

$5 + 5\tan^2 x + 2\tan x - 2 = 0$

$5\tan^2 x + 2\tan x + 3 = 0$

$(5\tan^2 x - 3\tan x)(+5\tan x - 3 = 0)$

$\tan x(5\tan x - 3) + 1(5\tan x - 3) = 0$

$(5\tan x - 3)(\tan x + 1) = 0$

$\tan x = \frac{3}{5}$ or $\tan x = -1$

Q1+3 ref∠=0.540 Q2+4 ref∠=$\frac{\pi}{4}$

$x = 0.540, \pi + 0.540$ $x = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$x = 0.5, 3.7$ $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}, 0.5, 3.7$

3. Consider the function $f(x) = 2\cos^2 \frac{1}{2}x - 1$. Express the function in terms of $\cos x$, and hence determine the zeros of the function where the domain is the set of real numbers. → general solution.

$2\cos^2 \frac{1}{2}x - 1 = \cos 2\left(\frac{1}{2}x\right) = \cos x$

$\cos x = 0$

$x = \frac{\pi}{2} + n\pi, n \in I$.

4. Consider the function $f(x) = \sin\left(\frac{\pi}{4} + x\right) - \sin\left(\frac{\pi}{4} - x\right)$.

a) Simplify $f(x)$.

$\sin\left(\frac{\pi}{4}+x\right) - \sin\left(\frac{\pi}{4}-x\right) = \left(\sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x\right) - \left(\sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x\right)$

$= \left(\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x\right) - \frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x$

$= \sqrt{2}\sin x$

b) Use the result in a) to solve the equation $f(x) = -1$, where $-2\pi \le x \le 2\pi$.

$\sqrt{2}\sin x = -1$

$\sin x = \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Q 3/4

ref ∠ = $\pi/4$

$x = \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{5\pi}{4} - 2\pi, \frac{7\pi}{4} - 2\pi$

$x = -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$

c) Verify the solutions in b) graphically.

5. Solve the following equations for $0 \le x \le 2\pi$.

a) $\cos 2x + \cos x = 0$

$2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos^2 x - \cos x)(+2\cos x - 1) = 0$

$\cos x(2\cos x - 1) + 1(2\cos x - 1) = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}$ or $\cos x = -1$

b) $\cos 2x = 1 - 2\sin x$

$1 - \sin^2 x = 1 - 2\sin x$

$0 = 2\sin^2 x - 2\sin x$

$0 = 2\sin x(\sin x - 1)$

$\sin x = 0$ or $\sin x = 1$

$x = 0, \pi, 2\pi$. $x = \frac{\pi}{2}$

$x = 0, \frac{\pi}{2}, \pi, 2\pi$

c) $\cos 2x - \sin x = 0$

$1 - 2\sin^2 x - \sin x = 0$

$0 = 2\sin^2 x + \sin x - 1$

$0 = 2\sin^2 x + 2\sin x - \sin x - 1$

$0 = 2\sin x(\sin x + 1) - 1(\sin x + 1)$

$0 = (2\sin x - 1)(\sin x + 1)$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

Q1/2 ref ∠ $= \frac{\pi}{6}$ $x = \frac{3\pi}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}.$

6. Consider the equations **i)** $\sin 2\theta = \sqrt{3}\sin\theta$ and **ii)** $\cos\theta = \sqrt{3}\sin\theta$.

a) In which of these equations is division by $\sin\theta$ a valid step?

6. b) Determine the solution to each equation in a) on the domain $0° \leq \theta \leq 360°$.

7. Consider the equation $\sin^2 x - \cos^2 x = 0,\ 0 \leq x \leq 2\pi$.

a) Solve the equation using a trigonometric identity.

b) Solve the equation by dividing by $\cos^2 x$.

c) Provide a third algebraic method which can be used to solve the equation.

8. Determine the roots of the following equations for $-180° \leq x \leq 180°$.

a) $\sin 2x + \cos x = 0$

$2\sin x \cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0 \quad \sin x = -\frac{1}{2}$

$x = 90, 270°$ Q3/4

$x = 90°, -90°$ ref∠ $= 30° \Rightarrow 210° + 330°$

$x = 210 - 360,\ 330 - 360$

$x = -150°, -30°$

$x = -150°, -90°, -30°, 90°$

b) $\sin 2x - \cos 2x = 0$

$\frac{\sin 2x}{\cos 2x} = \frac{\cos 2x}{\cos 2x} \quad \cos 2x \neq 0$

$\tan 2x = 1 \quad -360° \leq 2x \leq 360°$

~~tan~~ $2x = 45°, 225°, 45 - 360, 225 - 36$

$2x = 45, 225°, -315°, -135°$

$x = -157.5°, -67.5°, 22.5°, 112.5°$

Use the following information to answer the next question.

Consider the equation $2\cos^2 x + \sin x = 1,\ 0 \le x \le 2\pi$. Olive and Jacob both incorrectly determined that the equation had only one root. Their work is shown below.

Olive's Solution

Line 1	$2\cos^2 x + \sin x = 1$	
Line 2	$2(\sin^2 x - 1) + \sin x = 1$	
Line 3	$2\sin^2 x - 2 + \sin x = 1$	
Line 4	$2\sin x^2 x + \sin x - 3 = 0$	
Line 5	$(2\sin x + 3)(\sin x - 1) = 0$	
Line 6	$2\sin x + 3 = 0$	$\sin x - 1 = 0$
Line 7	$\sin x = -\frac{3}{2}$	$\sin x = 1$
Line 8	no solution	$x = \frac{\pi}{2}$

Solution $x = \frac{\pi}{2}$

Jacob's Solution

Line 1	$2\cos^2 x + \sin x = 1$	
Line 2	$2(1 - \sin^2 x) + \sin x = 1$	
Line 3	$2 - 2\sin^2 x + \sin x = 1$	
Line 4	$2 - 1 = 2\sin x^2 x - \sin x$	
Line 5	$1 = \sin x(2\sin x - 1)$	
Line 6	$\sin x = 1$	$2\sin x - 1 = 1$
Line 7	$\sin x = 1$	$\sin x = 1$
Line 8	$x = \frac{\pi}{2}$	$x = \frac{\pi}{2}$

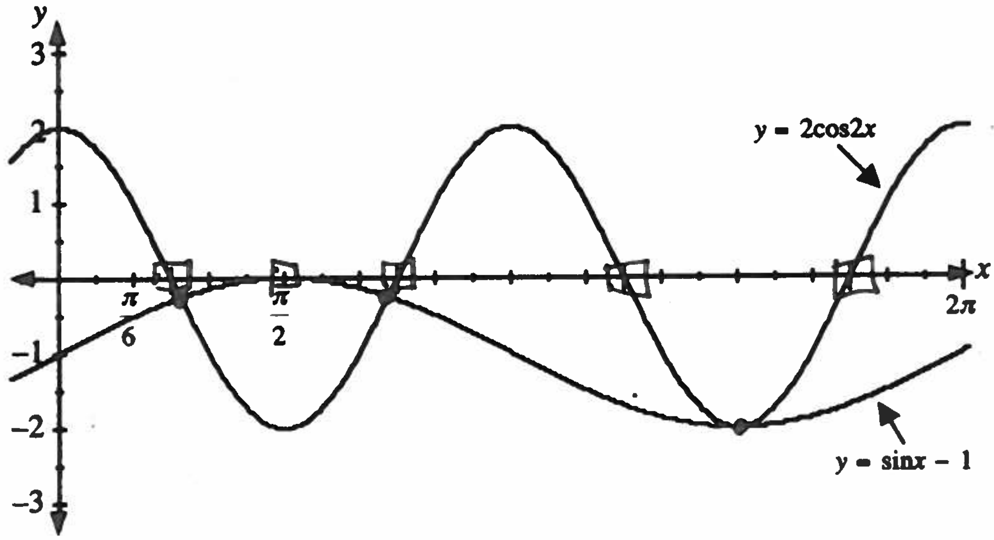
Solution $x = \frac{\pi}{2}$

9. a) Olive made one error in her work which led to the incorrect answer. In which line did the error occur? Describe the error she made.

b) Jacob adopted the wrong method in trying to solve the problem, but in which line did he first write a statement which does not follow mathematically from the previous statement. Describe the mathematical error.

c) Algebraically determine the roots of the equation $2\cos^2 x + \sin x = 1,\ 0 \le x \le 2\pi$.

10. The diagram shows the graphs of two trigonometric functions $y = 2\cos 2x$ and $y = \sin x - 1$ for $0 \le x \le 2\pi$.



a) Describe how to use the graphs to solve the equation $2\cos 2x - \sin x + 1 = 0$, where $0 \le x \le 2\pi$. Mark these points with **DOTS** on the grid.

$2\cos 2x = \sin x - 1$ → Find x- coordinates of point of intersection.

b) Use a graphing calculator to determine the roots of the equation in a). Use exact values where possible; otherwise round the answers to the nearest tenth.

$0.85,\ 2.29,\ 3\pi/2.$

c) Verify the roots in b) algebraically.

$2(1-2\sin^2 x) - \sin x + 1 = 0$

$2 - 4\sin^2 x - \sin x + 1 = 0$

$0 = 4\sin^2 x + \sin x - 3$

$0 = 4\sin^2 x - 3\sin x + 4\sin x - 3$

$0 = \sin x(4\sin x - 3) + 1(4\sin x - 3)$

$0 = (\sin x + 1)(4\sin x - 3)$

$\sin x = -1 \qquad \sin x = \frac{3}{4}$

$\sin x = 3/4$ Q 1,2

ref $\angle = 0.8480\ldots$

$x = 0.8480, \pi - 0.8480$

$x = 0.8480, 2.29$

$\sin x = -1 \quad x = 3\pi/2$

$x = 0.85, 2.29, 3\pi/2.$

d) Describe how to use the graphs to solve the equation $2\cos 2x(\sin x - 1) = 0$, where $0 \le x \le 2\pi$. Mark these points with a **SQUARE** on the grid.

Find x-int. of each graph.

e) Use a graphing calculator to determine the exact roots of the equation in d).

$\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

f) Verify the roots in e) algebraically.

$2\cos 2x(\sin x - 1) = 0$ $\qquad$ $\sin x = 1$ or $\cos 2x = 0$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ or $x = \frac{\pi}{2}$.

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \pi/2$

Answer Key

1. a) 30°, 90°, 150° b) 60°, 180°, 300° 2. $\frac{3\pi}{4}, \frac{7\pi}{4}$, 0.5, 3.7 3. $x = \frac{\pi}{2} + n\pi$
4. a) $\sqrt{2}\sin x$ b) $-\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
5. a) $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ b) $x = 0, \frac{\pi}{2}, \pi, 2\pi$ c) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$
6. a) in ii) b) i) 0°, 30°, 180°, 330°, 360° ii) 30°, 210°
7. a), b) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ c) Factor using a difference of squares, and solve each factor equal to zero.
8. a) $x = -150, -90°, -30°, 90°$ b) −157.5°, −67.5°, 22.5°, 112.5°
9. a) In Line 2, $\cos^2 x = 1 - \sin^2 x$, not $\sin^2 x - 1$.
   b) In Line 6. If the product of two quantities is equal to 1, it does not follow that each of the quantities has to equal 1.
   c) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$
10. a) Find the x-coordinates of the points of intersection of the two graphs.
   b), c) 0.85, 2.29, $\frac{3\pi}{2}$ d) Find the x-intercepts of each graph. e), f) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$