Assignment

1. Use the sum identity for cosine to prove the following double angle identities.

a) $\cos 2x = 2\cos^2 x - 1$

$\cos(x+x) = \cos x\cos x - \sin x\sin x$

$= \cos^2 x - \sin^2 x = \cos^2 x - (1-\cos^2 x)$

$= \cos^2 x - 1 + \cos^2 x = 2\cos^2 x - 1 = RS$

b) $\cos 2x = 1 - 2\sin^2 x$.

$LS = \cos(x+x) = \cos x\cos x - \sin x\sin x$

$= \cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x$

$= 1 - 2\sin^2 x = RS$

2. Write double angle identities for

a) $\tan 6x = \tan 2(3x)$

$= \frac{2\tan 3x}{1 - \tan^2 3x}$

b) $\cos 3\theta = \cos 2\left(\frac{3}{2}\theta\right)$

$= \cos^2\frac{3}{2}\theta - \sin^2\frac{3}{2}\theta$

c) $\sin\frac{1}{2}A = \sin 2\left(\frac{1}{4}A\right)$

$= 2\sin\frac{1}{4}A\cos\frac{1}{4}A$

3. Express each of the following in terms of a single trigonometric function.

a) $2\sin\frac{1}{2}x\cos\frac{1}{2}x$

$= \sin 2\left(\frac{1}{2}x\right) = \sin x$

b) $\cos^2 2A - \sin^2 2A$

$= \cos 2(2A) = \cos 4A$

c) $1 - 2\sin^2 3x$

$= \cos 2(3x) = \cos 6x$

4. Use a double angle identity to simplify and evaluate

a) $\dfrac{2\tan\frac{\pi}{8}}{1 - \tan^2\frac{\pi}{8}}$

$= \tan 2\left(\frac{\pi}{8}\right) = \tan \pi/4$

$= 1$

b) $\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}$

$= \cos 2\frac{\pi}{12} = \cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

c) $\sin\frac{5\pi}{12}\cos\frac{5\pi}{12}$

$= \frac{1}{2}\sin 2\left(\frac{5\pi}{12}\right) = \frac{1}{2}\sin\frac{5\pi}{6}$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$.

5. Prove the identity $\dfrac{\sin 2x}{1 - \cos 2x} = \cot x$ and state the restrictions on x.

$LS = \frac{2\sin x\cos x}{1 - (1 - 2\sin^2 x)} = \frac{2\sin x\cos x}{1 - 1 + 2\sin^2 x} = \frac{2\sin x\cos x}{2\sin^2 x} = \frac{\cos x}{\sin x} = \cot x = RS$

$\cot x \neq \frac{\cos x}{\sin x}$ so $\sin x \neq 0$, $x \neq n\pi$, $n \in I$

denominator $1 - \cos 2x \neq 0$

$\cos 2x \neq 1$

$2x \neq 2n\pi$

$x \neq n\pi$, $n \in I$

$x = n\pi$, $n \in I$

6. Prove the identities a) $\dfrac{1-\tan^2 x}{1+\tan^2 x} = \cos 2x$ b) $\sin 4x = 4\sin x \cos^3 x - 4\sin^3 x \cos$

a) $LS = \dfrac{1-\frac{\sin^2 x}{\cos^2 x}}{\sec^2 x} = \dfrac{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}{1/\cos^2 x}$

$= \dfrac{\cos^2 x-\sin^2 x}{\cos^2 x} \cdot \dfrac{\cos^2 x}{1}$

$= \cos^2 x - \sin^2 x = \cos 2x$

$= RS$ ✓

b) $LS = \sin 2(2x) = 2\sin 2x \cos 2x$

$= 2(2\sin x \cos x)(\cos^2 x - \sin^2 x)$

$= 4\sin x \cos x(\cos^2 x - \sin^2 x)$

$= 4\sin x \cos^3 x - 4\sin^3 x \cos x$

$= RS$

Multiple Choice

7. The expression $\dfrac{\cos^2 \frac{3}{2}x - \sin^2 \frac{3}{2}x}{\sin \frac{3}{2}x \cos \frac{3}{2}x}$ is equivalent to

A. $\cos \frac{3}{2}x - \sin \frac{3}{2}x$ B. $\cot 3x$

C. $2\cot 3x$ D. $2\csc 3x$

$\dfrac{\cos 2(\frac{3}{2}x)}{\frac{1}{2}\sin 2(\frac{3}{2}x)} = \dfrac{\cos 3x}{\frac{1}{2}\sin 3x} = \dfrac{2\cos 3x}{\sin 3x}$

$= 2\cot 3x$

Numerical Response

8. If $a\cos^2 \frac{\pi}{8} - a\sin^2 \frac{\pi}{8} = 4\sqrt{2}$, the value of a, to the nearest tenth, is _____.

(Record your answer in the numerical response box from left to right.)

8.	0	

$a(\cos^2 \frac{\pi}{8} - \sin^2 \frac{\pi}{8}) = a\cos 2(\frac{\pi}{8}) = a\cos \frac{\pi}{4} + a\frac{\sqrt{2}}{2} - \frac{1}{2}a\sqrt{2}$

$\frac{1}{2}a = 4 \quad a = 8$

9. If $\sin 6\theta \cos 2\theta - \cos 6\theta \sin 2\theta = 2\sin a\theta \cos a\theta$, then the value of a is _____.

(Record your answer in the numerical response box from left to right.)

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$\sin 60\cos 20 - \cos 60 \sin 20$

$= \sin(60-20) = \sin 40 = \sin 2(20) = 2\sin 20 \cos 20$

Answer Key

2. a) $\dfrac{2\tan 3x}{1-\tan^2 3x}$ b) $\cos^2 \frac{3\theta}{2} - \sin^2 \frac{3\theta}{2}$ c) $2\sin \frac{1}{4}A \cos \frac{1}{4}$

3. a) $\sin x$ b) $\cos 4A$ c) $\cos 6x$ 4. a) $\tan \frac{\pi}{4} = 1$ b) $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ c) $\frac{1}{2}\sin \frac{5\pi}{6} = \frac{1}{4}$

5. $x = n\pi, n \in I$ 7. C 8. 8.0 a. 2