$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\cos(A+B) = \cos A\cos B - \sin A\sin B$
$\sin(A-B) = \sin A\cos B - \cos A\sin B$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

Assignment

1. Simplify using the sum and difference identities.

a) $\cos(180 - B)°$

$= \cos 180° \cos B° + \sin 180° \sin B°$

$= (-1)\cos B + 0(\sin B)$

$= -\cos B°$

b) $\sin\left(\frac{\pi}{2} - x\right) = \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$

$= 1\cos x - 0\sin x$

$= \cos x$

c) $\cos(90 + t)°$

$= \cos 90° \cos t° - \sin 90° \sin t°$

$= 0(\cos t°) - (1)\sin t°$

$= -\sin t°$

d) $\sin(\pi + x) = \sin\pi\cos x + \cos\pi\sin x$

$= 0(\cos x) + (-1)\sin x$

$= \underline{-\sin x}$

2. Simplify and evaluate the following.

a) $\sin 70° \cos 20° + \cos 70° \sin 20°$

$\sin A\cos B + \cos A\sin B$

$= \sin(A+B) = \sin(70+20) = \sin 90 = 1$

b) $\cos 170° \cos 50° + \sin 170° \sin 50°$

$\cos A \cos B + \sin A\sin B$

$= \cos(A-B) = \cos(170-50) = \cos 120 = -\cos 60$

$= \underline{-\frac{1}{2}}$

c) $\sin\frac{\pi}{3}\cos\frac{\pi}{6} - \cos\frac{\pi}{3}\sin\frac{\pi}{6}$

$\sin A\cos B - \cos A\sin B$

$= \sin(A-B)$

$= \sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \underline{\frac{1}{2}}$

d) $\dfrac{\tan\frac{15\pi}{8} - \tan\frac{3\pi}{8}}{1 + \tan\frac{15\pi}{8}\tan\frac{3\pi}{8}} = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

$= \tan(A-B) = \tan\left(\frac{15\pi}{8} - \frac{3\pi}{8}\right)$

$= \tan\frac{3\pi}{2} = \underline{\text{undefined}}$

3. Use exact values to show that

a) $\sin 75° = \dfrac{\sqrt{6} + \sqrt{2}}{4}$

$\sin(45° + 30°) = \sin 45\cos 30 + \cos 45\sin 30$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} \cdot \frac{\sqrt{2}}{4}$

$= \dfrac{\sqrt{6} + \sqrt{2}}{4}$ ✓

b) $\cos\frac{\pi}{12} = \dfrac{\sqrt{6} + \sqrt{2}}{4}$

$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \dfrac{\sqrt{2} + \sqrt{6}}{4}$ ✓

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

4. Express $\frac{7\pi}{12}$ as a sum of two special angles and hence show that $\tan \frac{7\pi}{12} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$.

$$\frac{7\pi}{12} = \frac{3\pi}{12} + \frac{4\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$$

$$\tan\frac{7\pi}{12} = \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{3}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{3}} = \frac{1+\sqrt{3}}{1-(1)\sqrt{3}}$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \checkmark$$

5. Prove that the exact value of csc 105° is $\sqrt{6} - \sqrt{2}$.

$$\sin 105 = \sin(60° + 45°) = \sin 60 \cos 45° + \cos 60° \sin 45°$$

$$= \left(\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2} \cdot \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

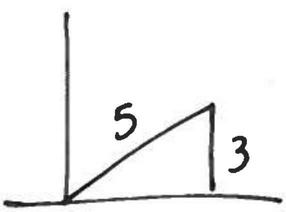
$$\csc 105° = \frac{1}{\sin 105} = \frac{4}{(\sqrt{6}+\sqrt{2})} \cdot \frac{(\sqrt{6}-\sqrt{2})}{(\sqrt{6}-\sqrt{2})} = \frac{4(\sqrt{6}-\sqrt{2})}{6-2} = \frac{\cancel{4}(\sqrt{6}-\sqrt{2})}{\cancel{4}}$$

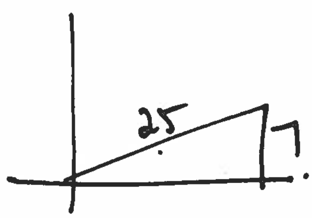
(rationalize denominator)

$$= \sqrt{6} - \sqrt{2}$$

6. Given $\sin x = \frac{3}{5}$ and $\sin y = \frac{7}{25}$, and x and y are both acute angles, show that $\tan(x+y) = \frac{4}{3}$.

Q1 or tanx + tany.





$$\sin x = \frac{y}{r} = \frac{3}{5}$$
$$x^2 + 3^2 = 5^2$$
$$x^2 = 16 \quad x = 4$$
$$\cos x = \frac{x}{r} = \frac{4}{5}$$
$$\tan x = \frac{y}{x} = \frac{3}{4}$$

$$\sin y = \frac{y}{r} = \frac{7}{25}$$
$$x^2 + 7^2 = 25^2$$
$$x^2 = 576$$
$$x = 24$$
$$\cos y = \frac{x}{r} = \frac{24}{25}$$
$$\tan y = \frac{y}{x} = \frac{7}{24}$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$= \frac{\frac{3}{4} + \frac{7}{24}}{1 - \frac{3}{4} \cdot \frac{7}{24}}$$

$$= \frac{\frac{25}{24}}{1 - \frac{7}{32}} = \frac{25}{24} \div \frac{25}{32}$$

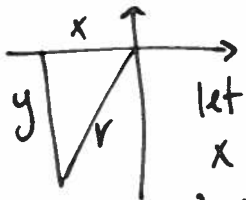
$$= \frac{25}{24} \times \frac{32}{25} = \frac{32}{24}$$

$$\boxed{= \frac{4}{3}}$$

$\sin \pi/6 = \frac{1}{2}$ $\quad \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}.$

7. Determine exact values for $\cos\left(\theta + \frac{\pi}{6}\right)$ and $\sin\left(\theta + \frac{\pi}{6}\right)$ if $\tan \theta = \frac{4}{3}$, and $\pi \le \theta \le \frac{3\pi}{2}$. Q3

$\tan\theta = \frac{4}{3}$ in Q3



let $y = -4$
$x = -3$

$r^2 = (-3)^2 + (-4)^2$
$= 25$
$r = 25$

$\sin\theta = \frac{y}{r} = \frac{-4}{5}$

$\cos\theta = \frac{x}{r} = \frac{-3}{5}$

$\cos\left(\theta + \frac{\pi}{6}\right) = \cos\theta \cos\frac{\pi}{6} - \sin\theta \sin \pi/6$

$= \left(\frac{-3}{5}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{-4}{5}\right)$

$= \frac{-3\sqrt{3}}{10} + \frac{4}{10} = \frac{4 - 3\sqrt{3}}{10}$

$\sin\left(\theta + \frac{\pi}{6}\right) = \sin\theta \cos\frac{\pi}{6} + \cos\theta \sin \pi/6$

$= \left(\frac{-4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{-3}{5}\right)\left(\frac{1}{2}\right)$

$= \frac{-4\sqrt{3}}{10} - \frac{3}{10} = \frac{-4\sqrt{3} - 3}{10}$

8. Prove the following identities.

a) $\tan\left(x + \frac{\pi}{4}\right) = \frac{1 + \tan x}{1 - \tan x}$

$LS = \tan\left(x + \frac{\pi}{4}\right) = \frac{\tan x + \tan\frac{\pi}{4}}{1 - \tan x \tan \pi/4}$

$= \frac{\tan x + 1}{1 - \tan x(1)} = \frac{1 + \tan x}{1 - \tan x} = RS$

$\left(\tan\frac{\pi}{4} = 1\right)$

b) $\frac{\sin(A - B)}{\cos A \cos B} = \tan A - \tan B$

$LS = \frac{\sin A - B}{\cos A \cos B} = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}$

$= \frac{\sin A \cancel{\cos B}}{\cos A \cancel{\cos B}} - \frac{\cancel{\cos A} \sin B}{\cancel{\cos A} \cos B} = \tan A - \tan B$

$LS = RS$ ✓

c) $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 2[1 + \cos(A - B)]$

FOIL FOIL

$LS = \cos^2 A + 2\cos A \cos B + \cos^2 B + \sin^2 A + 2\sin A \sin B + \sin^2 B$

$= (\cos^2 A + \sin^2 A) + (\cos^2 B + \sin^2 B) + 2(\cos A \cos B + \sin A \sin B)$

$= 1 + 1 + 2(\cos A - B)$

$= 2 + 2\cos(A - B)$

$= 2[1 + \cos(A - B)]$

$= RS$ ✓

. . At this point you are thinking - seriously ... more - Smile with a jo... ...end day is in sight ... like BEIT. !!

d) $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$

$LS = (\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$ FOIL!

$= \sin^2 x\cos^2 y - \sin x\cos y\cos x\sin y + \cos x\sin y\sin x\cos y - \cos^2 x\sin^2 y$

$= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y$

$\cos^2 y = 1 - \sin^2 y$

multiply $= \sin^2 x(1-\sin^2 y) - (1-\sin^2 x)\sin^2 y$

$= \sin^2 x - \sin^2 x\sin^2 y - \sin^2 y + \sin^2 x\sin^2 y$

$= \sin^2 x - \sin^2 y$

$= RS$

Multiple Choice **9.** If $\cos(A+B) = 0.8320$ and $\cos(A-B) = 0.4358$, then the value of $\cos A\cos B$ is

A. 1.2678
B. 0.6339
C. 0.3962
D. 0.1981

$\cos A\cos B - \sin A\sin B = 0.8320$
$\cos A\cos B + \sin A\sin B = 0.4358$

Add. $2\cos A\cos B = 1.2678$

$\cos A\cos B = 0.6339$.

10. The value of $\cos(\pi + y) - \cos(\pi - y)$ is

A. 0
B. 2
C. -2
D. dependent on the value of y

$(\cos\pi\cos y - \sin\pi\sin y) - (\cos\pi\cos y + \sin\pi\sin y)$

$= \cos\pi\cos y - \sin\pi\sin y - \cos\pi\cos y - \sin\pi\sin y$

$= -2\sin\pi\sin y$

$= -2(0)\sin y$

$= 0$.

11. Given $\csc x = \frac{-17}{15}$, where $\frac{3\pi}{2} \le x \le 2\pi$, and $\cot y = -\frac{3}{4}$, where $\frac{\pi}{2} \le y \le \pi$, the value of $\cos(x - y)$ is

Q4 Q2.

A. $-\frac{84}{85}$

B. $-\frac{36}{35}$

C. $\frac{84}{85}$

D. $\frac{36}{35}$

$\csc = \frac{r}{y} = \frac{-17}{15}$

$r = 17,\ y = -15$

$x^2 + (-15)^2 = 17^2$

$x^2 = 64$

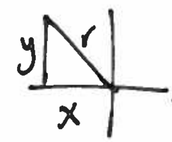
$x = 8$

$\sin x = \frac{y}{r} = \frac{-15}{17}$

$\cos x = \frac{x}{r} = \frac{8}{17}$

$\cot y = \frac{x}{y} = \frac{-3}{4}$

let $x = -3$ $y = 4$



$(-3)^2 + (4)^2 = r^2$

$r^2 = 25,\ r = 5$

$\sin y = \frac{y}{r} = \frac{4}{5}$ $\cos y = \frac{x}{r} = \frac{-3}{5}$

$\cos(x - y) = \cos x \cos y + \sin x \sin y$

$= \left(\frac{8}{17}\right)\left(\frac{-3}{5}\right) + \left(\frac{-15}{17}\right)\left(\frac{4}{5}\right) = -\frac{84}{85}$

Numerical Response **12.** If $\sin(A + B) = 0.75$ and $\sin(A - B) = 0.43$, then the value of $\cos A \sin B$, to the nearest hundredth, is ______.

(Record your answer in the numerical response box from left to right.)

0	.	1	6

$\sin A \cos B + \cos A \sin B = 0.75$

$\sin A \cos B - \cos A \sin B = 0.43$

subtract

$2 \cos A \sin B = 0.32$

$\cos A \sin B = 0.16$

Answer Key

1. **a)** $-\cos B°$ **b)** $\cos x$ **c)** $-\sin t°$ **d)** $-\sin x$

2. **a)** 1 **b)** $-\frac{1}{2}$ **c)** $\frac{1}{2}$ **d)** undefined

7. $\cos\left(\theta + \frac{\pi}{6}\right) = \frac{4 - 3\sqrt{3}}{10}$, $\sin\left(\theta + \frac{\pi}{6}\right) = \frac{-4\sqrt{3} - 3}{10}$

9. B **10.** A **11.** A **12.**

0	.	1	6