Assignment

1. Consider the statement $\dfrac{\cos x - \sin x}{\cos x} = \sin^2 x - \tan x + \cos^2 x$.

   a) Verify the statement is true for $x = \dfrac{\pi}{4}$.

$LS = \dfrac{\cos\frac{\pi}{4} - \sin\pi/4}{\cos\pi/4} = \dfrac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}}{\sqrt{2}/2} = 0$

$RS = \left(\sin\frac{\pi}{4}\right)^2 - \tan\frac{\pi}{4} + \left(\cos\frac{\pi}{4}\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 - 1 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} - 1 + \frac{1}{2} = 0$

$LS = RS$.

   b) Prove the statement is an identity.

$LS: \dfrac{\cos x - \sin x}{\cos x} = \dfrac{\cos x}{\cos x} - \dfrac{\sin x}{\cos x} = 1 - \tan x$

$RS = (\sin^2 x + \cos^2 x) - \tan x$
$= 1 - \tan x$
$LS = RS$.

   c) State, and give reasons for, any restrictions.

$\tan x = \dfrac{\sin x}{\cos x}$ so $\cos x \neq 0$, denominator $\cos x \neq 0$ $\quad x \neq \frac{\pi}{2} + n\pi, n \in I$

2. In each of the following
   i) verify the possibility of an identity using a graphing calculator (you should get the exact same graph).
   ii) prove the identity using an algebraic approach and state any restrictions.

   a) $\dfrac{\tan\theta\cos\theta}{\sin\theta} = 1$

$LS = \dfrac{\frac{\sin\theta}{\cos\theta} \cdot \cos\theta}{\sin\theta} = \dfrac{\sin\theta}{\sin\theta} = 1 = RS$ ✓

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ so $\cos\theta \neq 0$
denominator $\sin\theta \neq 0$
$\theta = n\pi, \frac{\pi}{2} + n\pi, n \in I$

   b) $\sec^2 x - \sin^2 x = \cos^2 x + \tan^2 x$

LS
$= (1 + \tan^2 x) - (1 - \cos^2 x)$
$= 1 + \tan^2 x - 1 + \cos^2 x$
$= \cos^2 x + \tan^2 x$.
$= RS$ ✓

~~RS~~.

$\sec x = \dfrac{1}{\cos x}$ so $\cos x \neq 0$

$\tan x = \dfrac{\sin x}{\cos x}$ so $\cos x \neq 0$

$x = \frac{\pi}{2} + n\pi, n \in I$

3. Consider the statement $\dfrac{\cot x - 1}{\tan x - 1} = -\dfrac{1}{\tan x}$.

a) Verify the statement is true for $x = \dfrac{\pi}{3}$

$$\frac{\cot\frac{\pi}{3}-1}{\tan\frac{\pi}{3}-1} = \frac{\frac{1}{\tan\frac{\pi}{3}}-1}{\tan\frac{\pi}{3}-1} \qquad -\frac{1}{\tan x} = \frac{-1}{\tan\pi/3} = \frac{-1}{\sqrt{3}}$$

$$= \frac{\frac{1}{\sqrt{3}}-1}{\sqrt{3}-1} = \frac{\frac{1-\sqrt{3}}{\sqrt{3}}}{\sqrt{3}-1}$$

$LS = RS$.

$$= \frac{1-\sqrt{3}}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}-1}$$

$$= \frac{-(\sqrt{3}-1)}{\sqrt{3}(\sqrt{3}-1)} = -\frac{1}{\sqrt{3}}$$

b) Prove the statement is an identity.

$$LS = \frac{\frac{\cos x}{\sin x}-1}{\frac{\sin x}{\cos x}-1} = \frac{\frac{\cos x}{\sin x}-\frac{\sin x}{\sin x}}{\frac{\sin x}{\cos x}-\frac{\cos x}{\cos x}}$$

$$= \frac{\frac{\cos x-\sin x}{\sin x}}{\frac{\sin x-\cos x}{\cos x}} = \frac{\cos x-\sin x}{\sin x}\cdot\frac{\cos x}{\sin x-\cos x}$$

$$= \frac{-(\sin x-\cos x)(\cos x)}{\sin x(\sin x-\cos x)} = -\frac{\cos x}{\sin x} = -\cot$$

$$= -\frac{1}{\tan x} = RS.$$

c) Determine the non-permissible values of the identity.

$\cot x = \dfrac{\cos x}{\sin x}$, $\sin x \neq 0$

$\tan x = \dfrac{\sin x}{\cos x}$, $\cos x \neq 0$

denominator $\tan x - 1 \neq 0$ so $\tan x \neq 1$

$\tan x \neq 0$ so $\tan x \neq 0$

$\sin x \neq 0$, $x \neq n\pi$, $n \in I$

$\cos x \neq 0$, $x \neq \frac{\pi}{2} + n\pi, n \in I$

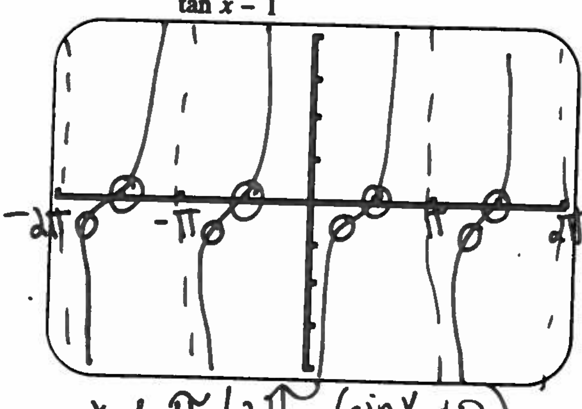
$\tan x \neq 1$ $x \neq \frac{\pi}{4}, n\pi, n \in I$

$x \neq n\pi, \frac{\pi}{4}+n\pi, \frac{\pi}{2}+n\pi, n \in I$.

d) Show the non-permissible values on a graph of each side of the identity for the domain $0 \leq x \leq 2\pi$.

$\sin x \neq 0$, $\cos x \neq 0$
$\tan x \neq 1$

$y = \dfrac{\cot x - 1}{\tan x - 1}$



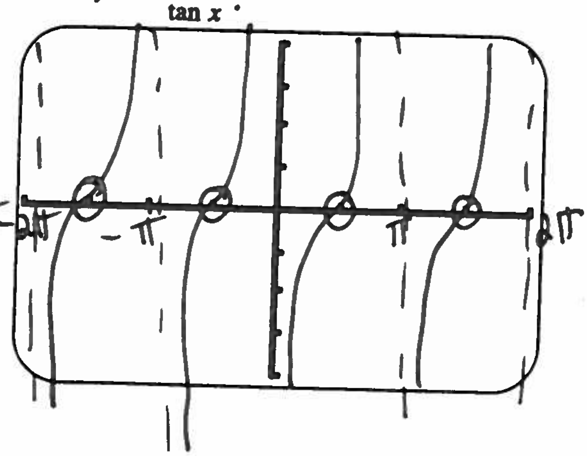
$x \neq \pi/2\pi$ ($\sin x \neq 0$)

$x \neq \frac{\pi}{2}$

$x \neq \pi/4$

$\cos x \neq 0, \sin x \neq 0$

$y = -\dfrac{1}{\tan x}$.



$x \neq \pi/2\pi$

$x \neq \pi/2$.

4. Prove the following identities using an algebraic approach.

a) $(1 - \cos^2 x)(\csc x) = \sin x$

$LS = \sin^2 x \cdot \frac{1}{\sin x}$

$= \sin x = RS.$

b) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

FOIL

$LS = \sin^2 x + 2\sin x\cos x + \cos^2 x$

$= (\sin^2 x + \cos^2 x) + 2\sin x\cos x$

$= 1 + 2\sin x\cos x = \underline{\underline{RS}}$

c) $\frac{1 - \cos x}{\sin x} = \frac{\tan x - \sin x}{\tan x \sin x}$

$RS = \frac{\frac{\sin x}{\cos x} - \sin x}{\frac{\sin x}{\cos x} \cdot \sin x} = \frac{\frac{\sin x}{\cos x} - \frac{\sin x\cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}}$

$= \frac{\sin x - \sin x\cos x}{\cos x} \cdot \frac{\cos x}{\sin^2 x}$

$= \frac{\sin x(1 - \cos x)}{\cos x} \cdot \frac{\cos x}{\sin^2 x}$

$= \frac{1 - \cos x}{\sin x} = \underline{\underline{LS}}$

d) $\frac{2}{1 - \sin x} + \frac{2}{1 + \sin x} = 4\sec^2 x$

$LS = \frac{2(1+\sin x) + 2(1-\sin x)}{(1-\sin x)(1+\sin x)}$

$= \frac{2 + 2\sin x + 2 - 2\sin x}{(1 - \sin x^2)}$

$= \frac{4}{\cos^2 x} = 4\sec^2 x = \underline{\underline{RS}}$

e) $\frac{1 + \cos x}{\tan x + \sin x} = \cot x$

$LS = \frac{1+\cos x}{\frac{\sin x}{\cos x} + \sin x} = \frac{1+\cos x}{\frac{\sin x}{\cos x} + \frac{\sin x\cos x}{\cos x}}$

$= \frac{1+\cos x}{\frac{\sin x + \sin x\cos x}{\cos x}} = \frac{1+\cos x}{\frac{\sin x(1+\cos x)}{\cos x}}$

$= (1+\cos x) \cdot \frac{\cos x}{\sin x(1+\cos x)}$

$= \frac{\cos x}{\sin x} = \cot x = \underline{\underline{RS}}$

f) $\sec x - \cos x = \frac{\sin x}{\cot x}$

$LS = \frac{1}{\cos x} - \cos x = \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}$

$= \frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$

$RS = \frac{\sin x}{\frac{\cos x}{\sin x}} = \sin x \cdot \frac{\sin x}{\cos x}$

$= \frac{\sin^2 x}{\cos x}$

$\underline{LS = RS}$ ✓

Multiple Choice

5. The identity $\frac{\sec x + 1}{\sec x - 1} + \frac{\cos x + 1}{\cos x - 1} = 0$ has restrictions

A. $x \neq 2n\pi, \frac{\pi}{2} + 2n\pi, n \in I$

B. $x \neq 2n\pi, \frac{\pi}{2} + n\pi, n \in I$

C. $x \neq \pi + 2n\pi, \frac{\pi}{2} + 2n\pi, n \in I$

D. $x \neq 2n\pi, n \in I$

$\sec x = \frac{1}{\cos x}$ so $\cos x \neq 0$

denominator $\sec x - 1 \neq 0$ $\sec x \neq 1$ so $\cos x \neq$

denominator $\cos x - 1 \neq 0$ $\cos x \neq 1$

$\cos x \neq 0$ $x \neq \frac{\pi}{2} + n\pi$

$\cos x \neq 1$ $x \neq 2n\pi$

Numerical Response

6. The value of n, to the nearest tenth, for which the statement below is an identity, is ______.

$$\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \frac{n}{\cos x}.$$

(Record your answer in the numerical response box from left to right.)

FOIL

2	.	0	

$$LS = \frac{(1+\sin x)(1+\sin x) + (\cos x)(\cos x)}{\cos x(1+\sin x)} = \frac{1 + 2\sin x + \overbrace{(\sin^2 x + \cos^2 x)}^{=1}}{\cos x(1+\sin x)}$$

$$= \frac{1 + 2\sin x + 1}{\cos x(1+\sin x)} = \frac{2 + 2\sin x}{\cos x(1+\sin x)} = \frac{2(1+\sin x)}{\cos x(1+\sin x)} = \frac{2}{\cos x} \Rightarrow \frac{n}{\cos x}$$

so $n = 2$

7. If $\frac{p}{2}\cos^2\frac{\pi}{5} + \frac{p}{2}\sin^2\frac{\pi}{5} = 4$, the value of p, to the nearest tenth, is ______.

(Record your answer in the numerical response box from left to right.)

8	.	0	

$$LS = \frac{p}{2}\left(\cos^2\frac{\pi}{5} + \sin^2\frac{\pi}{5}\right) = \frac{p}{2}(1) = \frac{p}{2}$$

$RS = 4$

so $\frac{p}{2} = 4$

$p = 8$

does matter what degree of angle $\cos^2 A + \sin^2 A = 1$

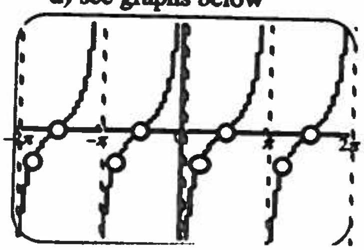
Answer Key

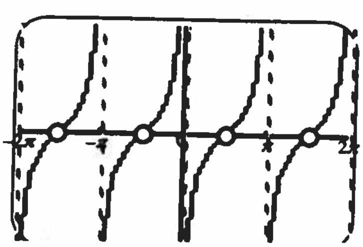
1. a) both sides equal 0 c) $x \neq \frac{\pi}{2} + n\pi, n \in I$

2. a) $\theta \neq n\pi, \frac{\pi}{2} + n\pi, n \in I$ or $\theta \neq n\frac{\pi}{2}, n \in I$ b) $x \neq \frac{\pi}{2} + n\pi, n \in I$

3. a) both sides equal $-\frac{\sqrt{3}}{3}$ c) $x \neq n\pi, \frac{\pi}{4} + n\pi, \frac{\pi}{2} + n\pi, n \in I$ or $x \neq \frac{\pi}{4} + n\pi, n\frac{\pi}{2}, n \in I$

d) see graphs below





5. B

6.

2	.	0	

7.

8	.	0	