$\sin^2 x + \cos^2 x = 1$ $\quad 1+\tan^2 x = \sec^2 x$ $\quad 1+\cot^2 x = \csc^2 x$

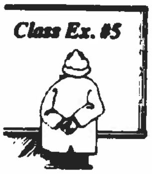
Using Identities to Simplify Trigonometric Expressions

Express each as a single trigonometric ratio. Use a graphing calculator to verify.

a) $\dfrac{\sin^2 x}{\cos^2 x} + 1$ $= \dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\cos^2 x}{\cos^2 x} = \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \sec^2 x$

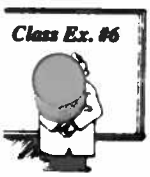
b) $\sin x + \cot x \cos x$ $= \sin x + \dfrac{\cos x}{\sin x} \cdot \cos x = \dfrac{\sin^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x} = \dfrac{\sin^2 x + \cos^2 x}{\sin x} = \dfrac{1}{\sin x}$

$= \csc x$



Express $\dfrac{2\tan A}{1+\tan^2 A}$ in terms of $\sin A$ and $\cos A$ and write in simplest form.

$= \dfrac{2\dfrac{\sin A}{\cos A}}{1+\dfrac{\sin^2 A}{\cos^2 A}} = \dfrac{\dfrac{2\sin A}{\cos A}}{\dfrac{\cos^2 A + \sin^2 A}{\cos^2 A}} = \dfrac{\dfrac{2\sin A}{\cos A}}{\dfrac{1}{\cos^2 A}} = \dfrac{2\sin A}{\cos A} \times \dfrac{\cos^2 A}{1} = 2\sin A\cos A$



Factor the following trigonometric expressions.

a) $3\cos^4\theta - 3\sin^4\theta$

$3(\cos^4\theta - \sin^4\theta)$

$= 3(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= 3(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)(1)$

$= 3(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$

b) $\sin^2\theta + \sin^2\theta\cot^2\theta$

$= \sin^2\theta(1+\cot^2\theta)$

$= \sin^2\theta(\csc^2\theta)$

$= \sin^2\theta\left(\dfrac{1}{\sin^2\theta}\right)$

$= 1$

Complete Assignment Questions #6 - #17

Assignment

1. Verify the following identities for the given value of the variable.

a) $\cot x = \dfrac{\cos x}{\sin x}$ for $x = 60°$

$\cot 60°$	$\dfrac{\cos 60°}{\sin 60°}$
$= \dfrac{1}{\tan 60°}$	$= \dfrac{1}{2} \div \dfrac{\sqrt{3}}{2}$
$= \dfrac{1}{\sqrt{3}}$	$= \dfrac{1}{2} \times \dfrac{2}{\sqrt{3}}$
	$= \dfrac{1}{\sqrt{3}}$ ✓

LS = RS

b) $\sin^2 x + \cos^2 x = 1$ for $x = \dfrac{\pi}{4}$

$\left(\sin\dfrac{\pi}{4}\right)^2 + \left(\cos\dfrac{\pi}{4}\right)^2$	1
$= \left(\dfrac{\sqrt{2}}{2}\right)^2 + \left(\dfrac{\sqrt{2}}{2}\right)^2$	
$= \dfrac{1}{2} + \dfrac{1}{2}$	
$= 1$	✓

LS = RS.

2. Verify the identity $1+\cot^2 x = \csc^2 x$ for the given values:

a) $x = \frac{\pi}{6}$

$1 + \left(\cot\frac{\pi}{6}\right)^2 \neq \left(\csc\frac{\pi}{6}\right)^2$

$= 1 + \frac{1}{\left(\tan\frac{\pi}{6}\right)^2}$

$= 1 + \frac{1}{(\sqrt{3}/3)^2}$

$= 1 + \frac{1}{1/3} = 1 + 3 = 4$ ✓

$= \frac{1}{(\sin\pi/6)^2} = \frac{1}{(1/2)^2}$

$= \frac{1}{\frac{1}{4}} = 4$ ✓

b) 120°

$LS = 1 + (\cot 120)^2 = 1 + \frac{1}{(\tan 120)^2} = 1 + \frac{1}{(-\tan 60°)^2}$

$= 1 + \frac{1}{(-\sqrt{3})^2} = 1 + \frac{1}{3} = \frac{4}{3}$

$RS = (\csc 120°)^2 = \frac{1}{(\sin 120°)^2} = \frac{1}{(\sin 60°)^2} = \frac{1}{(\sqrt{3}/2)^2}$

$= \frac{1}{3/4} = \frac{4}{3}$ $LS = RS$ ✓.

3. Explain why verifying that the two sides of a trigonometric identity are equal for given values (as in #1 and #2 above) is insufficient to conclude that the identity is valid.

- the left side + right side may be equal for some values, but may be unequal for other values. An identity is only valid if the left side + the right side are equal for all values for which the identity is defined.

4. Use the basic identities to prove the identities in questions #1 and #2.

1a) to prove $\cot x = \frac{\cos x}{\sin x}$

$LS = \frac{x}{y}$

$RS = \frac{x}{r} \div \frac{y}{r} = \frac{x}{r} \times \frac{r}{y} = \frac{x}{y}$

$LS = RS$.

1b) to prove $\sin^2 x + \cos^2 x = 1$

$LS = \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 = \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{x^2+y^2}{r^2} = \frac{r^2}{r^2} = 1 = RS$ ✓.

2) To prove $1 + \cot^2 x = \csc^2 x$

$LS = 1 + \left(\frac{x}{y}\right)^2 = 1 + \frac{x^2}{y^2} = \frac{y^2+x^2}{y^2} = \frac{r^2}{y^2}$

$RS = \left(\frac{r}{y}\right)^2 = \frac{r^2}{y^2}$ $LS = RS$ ✓.

5. Use the quotient identities or the Pythagorean identities to state whether the following are true or false.

a) $\cos^2 x = 1 + \sin^2 x$

false

$\cos^2 x = 1 - \sin^2 x$

b) $(\sin x)(\csc x) = 1$

true.

c) $\sin x = \pm\sqrt{1-\cos^2 x}$

true.

d) $(\tan x)(\cot x) = 1$

true

e) $\tan^2 x - \sec^2 x = 1$

false

$\tan^2 x - \sec^2 x = -1$

6. Write each expression as a single trigonometric ratio or as the number 1.

a) $\sin^2 x - 1$

$= -1(1-\sin^2 x)$

$= -\cos^2 x$

b) $\dfrac{\cos t}{\sin t}$

$= \cot t$

c) $\dfrac{1}{\sec \theta}$

$= \cos\theta$

d) $(\sec t)(\sin^2 t)(\csc t)$

$= \dfrac{1}{\cos t}\cdot \sin^2 t \cdot \dfrac{1}{\sin t}$

$= \dfrac{\sin t}{\cos t} = \tan t$

e) $\csc^2 x - \cot^2 x$

$= (1+\cot^2 x) - \cot^2 x$

$= 1$

f) $\sin\theta + (\cot\theta)(\cos\theta)$

$= \sin\theta + \dfrac{\cos\theta}{\sin\theta}\cdot\cos\theta$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \dfrac{1}{\sin\theta} = \csc\theta$

7. Factor to write each in a simpler form.

a) $\sec x \sin^2 x - \sec x$

$= \sec x(\sin^2 x - 1)$

$= \dfrac{1}{\cos x}(-\cos^2 x)$

$= -\cos x$

b) $\sin^4\theta - \cos^4\theta$

$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

$= (\sin\theta - \cos\theta)(\sin\theta + \cos\theta)(1)$

$= (\sin\theta - \cos\theta)(\sin\theta + \cos\theta)$

8. Very often in proving identities it is simpler to try to express each side in terms of only $\sin x$, $\cos x$, or both. Express each of the following in terms of only $\sin x$, $\cos x$, or both.

a) $\tan^2 x$

$= \dfrac{\sin^2 x}{\cos^2 x}$

b) $\dfrac{\tan x}{\sin x}$

$= \dfrac{\sin x}{\cos x} \div \sin x$

$= \dfrac{1}{\cos x}$

c) $\dfrac{\tan x}{\csc x} = \dfrac{\dfrac{\sin x}{\cos x}}{\dfrac{1}{\sin x}} = \dfrac{\sin x}{\cos x}\cdot\dfrac{\sin x}{1}$

$= \dfrac{\sin^2 x}{\cos x}$

d) $\dfrac{1}{1+\cot^2 x}$

$= \dfrac{1}{\csc^2 x} = \sin^2 x$

e) $\csc x - \sin x$

$= \dfrac{1}{\sin x} - \sin x$

$= \dfrac{1-\sin^2 x}{\sin x} = \dfrac{\cos^2 x}{\sin x}$

f) $1 - \csc^2 x$

$= -\cot^2 x$

$= -\dfrac{\cos^2 x}{\sin^2 x}$

g) $\dfrac{1+\cot^2 x}{\sec^2 x}$

$= \dfrac{\csc^2 x}{\sec^2 x}$

$= \dfrac{1}{\sin^2 x} \div \dfrac{1}{\cos^2 x}$

$= \dfrac{1}{\sin^2 x} \times \dfrac{\cos^2 x}{1} = \dfrac{\cos^2 x}{\sin^2 x}$

h) $\dfrac{\cos^2 x - 1}{\tan x}$

$= \dfrac{-\sin^2 x}{\dfrac{\sin x}{\cos x}}$

$= -\sin^2 x \cdot \dfrac{\cos x}{\sin x}$

$= -\sin x \cos x$

i) $\dfrac{1+\cot^2 x}{\cot^2 x}$

$= \dfrac{\csc^2 x}{\cot^2 x} = \dfrac{\dfrac{1}{\sin^2 x}}{\dfrac{\cos^2 x}{\sin^2 x}}$

$= \dfrac{1}{\sin^2 x}\cdot\dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{1}{\cos^2 x}$

Multiple Choice

In questions #9 - #14 assume the appropriate restrictions.

9. $\dfrac{\cos x}{1-\sin^2 x}$ is equal to

A. $\sec x$

B. $\csc x$

C. $\sin x$

D. $\tan x$

$\dfrac{\cos x}{\cos^2 x} = \dfrac{1}{\cos x} = \sec x$

10. $\dfrac{\tan^2 x + 1}{\sec x}$ is equal to

A. $\sec x$

B. $\csc x$

C. $\sin x$

D. $\tan x$

$\dfrac{\sec^2 x}{\sec x} = \sec x$

11. $\dfrac{\csc x}{\cot x}$ is equal to

A. $\cos x$

B. $\sin x$

C. $\sec x$

D. $\tan x$

$\dfrac{1}{\sin x} \div \dfrac{\cos x}{\sin x}$

$\dfrac{1}{\cancel{\sin x}} \cdot \dfrac{\cancel{\sin x}}{\cos x}$

$\dfrac{1}{\cos x} = \sec x$

12. Which is **not** an identity?

A. $\cos^2 x + \sin^2 x = 1$

B. $\sin x + \cos x = 1$

C. $\sec^2 x - \tan^2 x = 1$

D. $\tan x \cot x = 1$

13. $\sec x - \cos x$ is equal to

A. $\dfrac{1-\cos x}{\cos x}$

B. $\dfrac{1-2\cos x}{\cos x}$

C. $\sin^2 x$

D. $\sin x \tan x$

$\dfrac{1}{\cos x} - \cos x = \dfrac{1}{\cos x} - \dfrac{\cos^2 x}{\cos x}$

$= \dfrac{1-\cos^2 x}{\cos x}$

$= \dfrac{\sin^2 x}{\cos x} = \dfrac{\sin x \cdot \sin x}{\cos x} = \sin x \tan x$

14. The expression $\dfrac{\tan A \cos^2 A}{\sec A}$, expressed in terms of $\sin A$, is

A. $\dfrac{\sin A}{1-\sin^2 A}$

B. $\dfrac{1-\sin^2 A}{\sin A}$

C. $\sin^2 A$

D. $\sin A - \sin^3 A$

$\dfrac{\dfrac{\sin A}{\cos A} \cdot \cos^2 A}{\dfrac{1}{\cos A}} =$

$\sin A \cos A \div \dfrac{1}{\cos A}$

$= \sin A \cos^2 A$

$= \sin A (1 - \sin^2 A) = \sin A - \sin^3 A$

15. If $\tan x \neq 0, \cos x \neq 0, \cot x \neq 0$, then $\dfrac{1}{\tan x \cos x \cot x}$ is equal to

A. $\dfrac{1}{\sin x}$

B. $\sin x$

C. $\dfrac{1}{\cos x}$

D. $\cos x$

$$\frac{1}{\cancel{\frac{\sin x}{\cos x}} \cdot \cos x \cdot \cancel{\frac{\cos x}{\sin x}}}$$

16. If $\sin x \neq 0, \cos x \neq 0$, then $\dfrac{\tan x \cos x}{3 \sec x \cot x}$ is equal to

A. $\dfrac{1}{3}$

B. 3

C. $\dfrac{1}{3}\sin^2 x$

D. $\dfrac{1}{3}\csc^2 x$

$$\frac{\frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x}}{3 \cdot \frac{1}{\cancel{\cos x}} \cdot \frac{\cos x}{\sin x}} = \frac{\sin x}{\frac{3}{\sin x}} = \sin x \cdot \frac{\sin x}{3} = \frac{1}{3}\sin^2 x.$$

Numerical Response

17. When verifying the identity $\cot^2 x + 1 = \csc^2 x$ for $x = \frac{\pi}{7}$, the value on each side of the identity, to the nearest tenth, is _____ .

(Record your answer in the numerical response box from left to right.)

5	.	3	

$$\left(\csc \frac{\pi}{7}\right)^2 = \frac{1}{\left(\sin\frac{\pi}{7}\right)^2} = 5.3$$

Answer Key

3. The left side and right side may be equal for some values, but may be unequal for other values. An identit is only valid if the left side and the right side are equal for all values for which the identity is defined.

5. b), c) and d) are true.

6. a) $-\cos^2 x$ b) $\cot t$ c) $\cos \theta$ d) $\tan t$ e) 1 f) $\csc \theta$

7. a) $-\cos x$ b) $(\sin \theta - \cos \theta)(\sin \theta + \cos \theta)$

8. a) $\dfrac{\sin^2 x}{\cos^2 x}$ b) $\dfrac{1}{\cos x}$ c) $\dfrac{\sin^2 x}{\cos x}$ d) $\sin^2 x$ e) $\dfrac{\cos^2 x}{\sin x}$

f) $-\dfrac{\cos^2 x}{\sin^2 x}$ g) $\dfrac{\cos^2 x}{\sin^2 x}$ h) $-\sin x \cos x$ i) $\dfrac{1}{\cos^2 x}$

9. A **10.** A **11.** C **12.** B **13.** D **14.** D

15. C **16.** C **17.**

5	.	3	