Assignment

1. Use a graphical approach to determine the roots of the equation $\sin 2x = \frac{\sqrt{3}}{2}$

a) on the domain $0 \le x \le 2\pi$

$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

b) on the domain of real numbers.

period $= \frac{2\pi}{2} = \pi$

$x = \frac{\pi}{6} + n\pi, \frac{\pi}{3} + n\pi, n \in I.$

2. Consider the function $f(x) = 1 - \cot 2x$.
Using a graphical approach, determine the zeros of the function

$\cot 2x = 1$
$\tan 2x = 1$

a) for $0 \le x \le 2\pi$

zeros are:
$\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

b) for $x \in R$.

period $= \frac{\pi}{2}$

zeros are: $\frac{\pi}{8} + \frac{n\pi}{2}, n \in I$

3. Determine the general solution to $\cos \frac{1}{2}x = \frac{\sqrt{3}}{2}$ using a graphical approach.

period $= \frac{2\pi}{1/2} = 4\pi$

$x = \frac{\pi}{3} + 4n\pi, \frac{11\pi}{3} + 4n\pi, n \in I$

$y_1 = \cos \frac{1}{2}x$
$y_2 = \frac{\sqrt{3}}{2}.$

4. a) Use an algebraic approach to solve the equation $\sin 2x = \frac{\sqrt{2}}{2}$, $0 \le x \le 2\pi$.

$\sin 2x = \frac{\sqrt{2}}{2}, 0 \le 2x \le 4\pi$

Q 1 & 2 ref $\angle = \pi/4$

$2x = \frac{\pi}{4}, \pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}, 2\pi + \left(\pi - \frac{\pi}{4}\right)$

$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

b) State the general solution to the equation $\sin 2x = \frac{\sqrt{2}}{2}$.

period $= \frac{2\pi}{2} = \pi$

$x = \frac{\pi}{8} + n\pi, \frac{3\pi}{8} + n\pi, n \in I$

Degree mode

5. a) Use an algebraic approach to solve the equation $\sec 3x - \sqrt{2} = 0$, $0° \le x \le 360°$.

$\sec 3x = \sqrt{2}$ $\quad \cos 3x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad 0° \le 3x \le 1080°$

Q 1 + 4 ref $\angle = 45°$

$3x = 45°, 360° - 45°, 360° + 45°, 360° + (360 - 45), 720 + 45, 720 + (360 - 45)$

$3x = 45°, 315°, 405°, 675°, 765°, 1035°$

$x = 15°, 105°, 135°, 225°, 255°, 345°$

b) State the general solution to the equation $\sec 3x - \sqrt{2} = 0$, where x is expressed in degrees.

period $= \frac{360°}{3} = 120°$

$x = 15° + 120n°, 105° + 120n°, n \in I$

6. Use an algebraic approach to determine the general solution to the equations

a) $\tan 4x = 1$

period $= \frac{\pi}{4}$ Q1, ref∠ $= \frac{\pi}{4}$

$4x = \frac{\pi}{4}$ $x = \pi/16$

general solution

$x = \frac{\pi}{16} + n\frac{\pi}{4}, n \in I$

b) $\tan \frac{1}{4}x = 1$

period $= \frac{\pi}{1/4} = 4\pi$ Q1

ref∠ $= \frac{\pi}{4}$

$\frac{1}{4}x = \frac{\pi}{4}$ $x = \pi$

general solution:

$x = \pi + 4n\pi, n \in I$

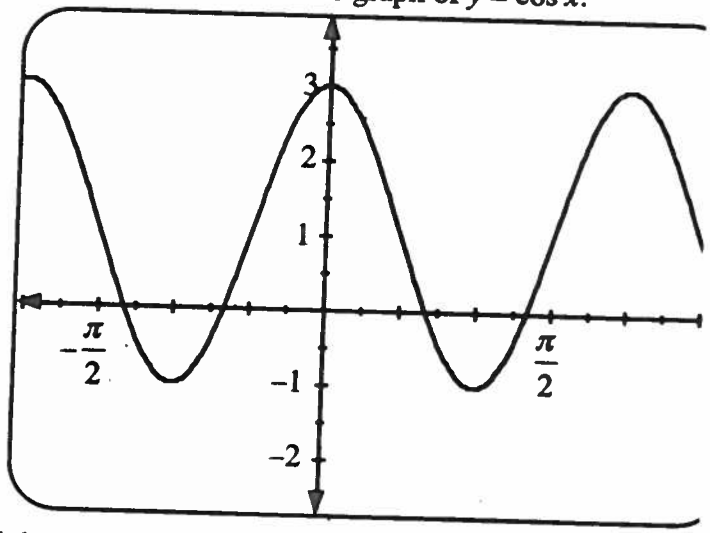
7. The graph of $y = 2\cos 3x + 1$ is displayed on a graphing calculator.

a) Describe the effects of the parameters 2, 3, and 1 on the graph of $y = \cos x$.

The amp is ↑ed to 2,

The period is reduced to $\frac{2\pi}{3}$

- vertical displacement is 1 unit ↑



b) A student was asked to find all the values of θ which satisfy the equation $\cos 3x = -\frac{1}{2}, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Explain how the student can find these values from the graph above and mark these points on the grid.

$\cos 3x = -\frac{1}{2}$

$2\cos 3x + 1 = 0$

Find the x-int on the graph of $y = 2\cos 3x + 1$ on domain $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

c) Show how to find these values by solving algebraically $\cos 3x = -\frac{1}{2}, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

$\cos 3x = -\frac{1}{2}$ $-\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$.

Q2/3, ref∠ $= \pi/3$

$3x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}, (\pi - \frac{\pi}{3}) - 2\pi, (\pi + \pi/3) - 2\pi$

$3x = \frac{2\pi}{3}, 4\pi/3, -4\pi/3, -2\pi/3$

$x = -4\pi/9, -2\pi/9, 2\pi/9, 4\pi/9.$

Multiple Choice

10. Which of the following is NOT a solution to the equation $2 \sin 3x = 0$?

A. $\frac{\pi}{3}$ B. $\frac{\pi}{2}$

C. $\frac{4\pi}{3}$ D. 2π

$\sin 3x = 0$
$3x = 0, \pi, 2\pi$
$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}$

general solution
$x = \frac{n\pi}{3}, n \in I$.

11. Which of the following equations does not have a solution in the interval $\pi \le x \le \frac{3\pi}{2}$. Q3.

A. $\csc x = -2$ Q 3/4 ✓ $(\sin x = -\frac{1}{2})$

B. $\cos 2x = 1$ ref ∠ = 0, $2x = 0, 2\pi, 4\pi$, $x = 0, \pi, 2\pi$ ✓

C. $\tan \frac{1}{2}x = 1$ ref ∠ = $\frac{\pi}{4}$, Q 1/3 $\frac{1}{2}x = \frac{\pi}{4}, \frac{5\pi}{4}$ $x = \frac{\pi}{2}, \frac{5\pi}{2}$ X

D. $\sin 3x = 1$ ref ∠ = $\frac{\pi}{2}$, $3x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$ → $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ ✓

Numerical Response

12. The smallest positive solution to the equation $\sin 4x = 0.48$, correct to the nearest hundredth of a radian, is $x =$ _____ .
(Record your answer in the numerical response box from left to right.)

0	.	1	3

$\sin 4x = 0.48$ $4x = 0.50065$
$x = 0.12516$

Answer Key

1. a) $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$ b) $x = \frac{\pi}{6} + n\pi, \frac{\pi}{3} + n\pi, n \in I$

2. a) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$ b) $x = \frac{\pi}{8} + \frac{n\pi}{2}, n \in I$

3. $x = \frac{\pi}{3} + 4n\pi, \frac{11\pi}{3} + 4n\pi, n \in I$

4. a) $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$ b) $x = \frac{\pi}{8} + n\pi, \frac{3\pi}{8} + n\pi, n \in I$

5. a) 15°, 105°, 135°, 225°, 255°, 345° b) $x = 15° + 120n°, 105° + 120n°, n \in I$

6. a) $x = \frac{\pi}{16} + \frac{n\pi}{4}$ $n \in I$ b) $x = \pi + 4n\pi, n \in I$

7. a) The amplitude is increased to 2, the period is reduced to $\frac{2\pi}{3}$, and the vertical displacement is 1 unit u

b) Find the x-intercepts of the graph of $y = 2 \cos 3x + 1$ on the given domain. c) $-\frac{4\pi}{9}, -\frac{2\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}$

8. a) 4 cycles b) 30° C
c) 3:30 am, 5:30 am, 9:30 am, 11:30 am, 3:30 pm, 5:30 pm, 9:30 pm, 11:30 pm

9. Since x is replaced by $\frac{1}{2}x$, the roots will be doubled and any value outside the domain $0 \le x \le 2\pi$ will be disregarded. The only root is π.

10. B **11.** C **12.**

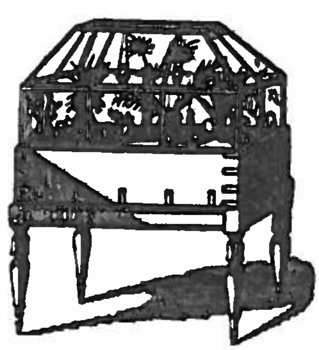
0	.	1	3

Use the following information to answer the next question.

In a controlled laboratory experiment, the temperature in a greenhouse is controlled by an electronic thermostat. The temperatures vary according to the sinusoidal function

$$C(t) = 28 + 4\sin\frac{\pi}{3}t$$

where C is the temperature in degrees Celsius and t is the time in hours past midnight.



8. a) How many temperature cycles are there in one day?

$b = \frac{\pi}{3}$ period $= \frac{2\pi}{\pi/3} = 6$ hours $\quad \frac{24}{6} = 4 \quad$ 4 cycles/day

b) Algebraically determine the temperature at 2:30 pm. — 14.5 hrs.

$t = 14.5 \quad C(14.5) = 28 + 4\sin\left(\frac{\pi}{3} \cdot 14.5\right) = 30°$

c) Algebraically determine the times during the day when the temperature is 26°.

solve $26 = 28 + 4\sin\frac{\pi}{3}t$ in the 1st 6 hours. $\quad 0 \le t \le 6$

$0 \le \frac{\pi}{3}t \le 2\pi$

$-2 = 4\sin\frac{\pi}{3}t$

$-\frac{1}{2} = \sin\frac{\pi}{3}t \quad$ Q 3,4 ref $\angle = \pi/6$.

$\frac{\pi}{3}t = \pi + \frac{\pi}{6}, \ 2\pi - \frac{\pi}{6}$

$\frac{\pi}{3}t = \frac{7\pi}{6}, \ 11\pi/6 \quad$ add periods of 6

$t = \frac{7}{2}, \frac{11}{2}, \frac{19}{2}, \frac{23}{2}, \frac{31}{2}, \frac{35}{2}, 43/2, 4$

time: 3:30am, 5:30am, 9:30am, 11:30am, 3:30pm, 5:30pm, 9:30pm, 11:30pm.

9. The roots of the equation $2\sin^2 x - \sin x - 1 = 0$, for $0 \le x \le 2\pi$, are $\frac{\pi}{2}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

Describe how the roots of the equation $2\sin^2\left(\frac{1}{2}x\right) - \sin\left(\frac{1}{2}x\right) - 1 = 0$, $0 \le x \le 2\pi$, relate to the roots of the equation $2\sin^2 x - \sin x - 1 = 0$, $0 \le x \le 2\pi$. Determine the roots.

- since x is replaced by $\frac{1}{2}x$, the roots will be doubled & any value outside the domain 0 to 2π we ignore.

$2\left(\frac{\pi}{2}\right) = \pi \quad 2\left(\frac{7\pi}{6}\right) = \frac{7\pi}{3}$

$2\left(\frac{11\pi}{6}\right) = \frac{11\pi}{3}$

- The only root is π