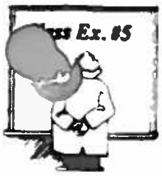


Determine the zeros of the following functions on the specified domain.

a) $f(x) = \csc^2 x - 3\csc x - 28$, domain $0° \le x \le 180°$
Answer to the nearest degree

$(\csc x - 7)(\csc x + 4) = 0$

$\csc x = 7$ or $\csc x = -4$

$\sin x = \frac{1}{7}$ or $\sin x = \frac{-1}{4}$

Q I/II — no solution on domain

ref $\angle = 8°$

$x = 8°, 180° - 8°$

$x = 8°, 172°$

zeros are $8°, 172°$

b) $g(\theta) = 2\cos^2\theta + 5\cos\theta - 3$, domain $-\pi \le \theta \le \pi$

$2\cos^2\theta - \cos\theta + 6\cos\theta - 3$

$= \cos\theta(2\cos\theta - 1) + 3(2\cos\theta - 1)$

$= (2\cos\theta - 1)(\cos\theta + 3)$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -3$ no solution

Q I/IV ref $\angle \frac{\pi}{3}$

on $0 \le \theta \le 2\pi$ $\theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

on $-\pi \le \theta \le \pi$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3} - 2\pi$

zeros are $-\frac{\pi}{3}, \frac{\pi}{3}$

Complete Assignment Questions #2 - #13

Assignment

1. Factor the following trigonometric expressions.

a) $4\sin^2\theta - \cos^2\theta$

$= (2\sin\theta - \cos\theta)(2\sin\theta + \cos\theta)$

b) $\cot^2 x - \cot x$

$= \cot x(\cot x - 1)$

c) $\sin^2\theta + 3\sin\theta + 2$

$= (\sin\theta + 1)(\sin\theta + 2)$

d) $\sec x\sin^2 x - 0.25\sec x$

$= \sec x(\sin^2 x - 0.25)$

$= \sec x(\sin x - 0.5)(\sin x + 0.5)$

e) $\cot^2\theta - 1$

$= (\cot\theta - 1)(\cot\theta + 1)$

f) $\sec^4\theta - 1$

$= (\sec^2\theta - 1)(\sec^2\theta + 1)$

$= (\sec\theta - 1)(\sec\theta + 1)(\sec^2\theta + 1)$

g) $4\cos^2 A - 4\cos A - 3$

$= 4\cos^2 A - 6\cos A + 2\cos A - 3$

$= 2\cos A(2\cos A - 3) + 1(2\cos A - 3)$

$= (2\cos A - 3)(2\cos A + 1)$

h) $2\sin^2 x - 7\sin x + 6$

$= 2\sin^2 x - 4\sin x - 3\sin x + 6$

$= 2\sin x(\sin x - 2) - 3(\sin x - 2)$

$= (\sin x - 2)(2\sin x - 3)$

2. Consider the equation $2\cos^2 x + 3\cos x + 1 = 0$.

a) Use a **graphical** approach to find the solution to the equation where $0 \le x \le 2\pi$. Give solutions as exact values.

window.

$X: [-\frac{\pi}{6}, 2\pi, \pi/6]$

$Y: [-1, 1, 0.1]$

$x = \frac{2\pi}{3}, \pi, 4\pi/3$

b) Use an **algebraic** approach to find the solution to the equation where $0 \le x \le 2\pi$. Give solutions as exact values.

$2\cos^2 x + 2\cos x + \cos x + 1 = 0$

$2\cos x(\cos x + 1) + 1(\cos x + 1) = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\cos x = \frac{-1}{2}$ or $\cos x = -1$

$\cos x = -1$

Q2|3

ref∠ = 0

$x = \pi - 0, \pi + 0$

$x = \pi$

$\cos x = \frac{-1}{2}$

Q2|3

ref∠ = $\pi/3$

$x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{2\pi}{3}, \pi, 4\pi/3$

c) State the general solution to the equation.

$x = \frac{2\pi}{3} + 2n\pi, \pi + 2n\pi, \frac{4\pi}{3} + 2n\pi, n \in I$

3. Algebraically find the solutions to the following trigonometric equations. Give solutions as exact values.

a) $2\sin^2\theta + \sin\theta = 0$ where $0 \le \theta \le 2\pi$

$\sin\theta(2\sin\theta + 1) = 0$

$\sin\theta = 0$ or $\sin\theta = \frac{-1}{2}$

$\theta = 0, \pi, 2\pi$

Q 3|4

ref∠ = $\pi/6$

$\theta = \pi + \frac{\pi}{6}, 2\pi - \pi/6$

$= \frac{7\pi}{6}, 11\pi/6$

$\theta = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

b) $2\sin^2 x - \sin x = 1$ where $0 \le x \le 2\pi$

$2\sin^2 x - \sin x - 1 = 0$

$2\sin^2 x - 2\sin x + \sin x - 1 = 0$

$2\sin x(\sin x - 1) + 1(\sin x - 1) = 0$

$(2\sin x + 1)(\sin x - 1)$

$\sin x = \frac{-1}{2}$

Q3|4, ref∠ = $\pi/6$

$x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$x = \frac{7\pi}{6}, 11\pi/6$

$\sin x = 1.$

Q1|2 ref∠ = $\frac{\pi}{2}$

$x = \frac{\pi}{2}, \pi - \frac{\pi}{2}$

$x = \pi/2$

$x = \frac{\pi}{2}, \frac{7\pi}{6}, 11\pi/6$

Use the following information to answer the next question.

A student is solving the equation $8\cos^2 x + 2\cos x - 3 = 0$ on the interval $0° \le x \le 360°$. The student's work is shown below.

$$8\cos^2 x + 2\cos x - 3 = 0$$

$$(2\cos x - 1)(4\cos x + 3) = 0$$

$\cos x = \frac{1}{2}$ or $\cos x = -\frac{3}{4}$

✓ quadrant 1/4	quadrant 2/3
reference angle = 60°	reference angle = 139°
✓ $x = 60°$	$x = 180° - 139°$
$x = 360° - 60°$	$x = 180° + 139°$
$x = 60°, 300°$	~~$x = 41°, 319°$~~

$x = 41°, 60°, 300°, 319°$

correct.

must be acute.

Degree mode

4. a) Verify algebraically that $x = 60°$ is a solution to the equation. → sub $x = 60°$ into equation.

$8(\cos 60°)^2 + 2(\cos 60°) - 3 = 0$

$8(\frac{1}{2})^2 + 2(\frac{1}{2}) - 3 = 0$

$2 + 1 - 3 = 0$ ✓

$x = 60°$ is a solution.

b) Show that $x = 41°$ does not satisfy the equation.

$8(\cos 41°)^2 + 2(\cos 41°) - 3 = 0$

$4.55669 + 1.5094 - 3 = 0$

$3.0661\ldots \neq 0$ → $x = 41°$ is not a solution

c) Explain the error in the student's work and provide a correct solution to the problem.

→ there is an incorrect reference angle for $\cos x = \frac{-3}{4}$

The correct reference angle is 41°

$8\cos^2 x + 2\cos x - 3 = 0$

$(2\cos x - 1)(4\cos x + 3) = 0$

$\cos x = \frac{1}{2}$ or $\cos x = -\frac{3}{4}$

$x = 60°, 300°$

→ $\cos x = \frac{-3}{4}$, Q 2/3

ref∠ = 41°

$x = 180° - 41°, 180° + 41°$

$x = 139°, 221°$

$x = 60°, 139°, 221°, 300°$

Use the following information to answer the next question.

Christine is determining the roots of the equation $2\sin x\cos x = 3\sin x$ on the domain $0 \le x \le 2\pi$. Her work is shown at the side.	$2\sin x\cos x = 3\sin x$ $\dfrac{2\sin x\cos x}{\sin x} = \dfrac{3\sin x}{\sin x}$ $2\cos x = 3$ $\cos x = \dfrac{3}{2}$ no solution

NO

5. a) Is Christine correct in stating that $\cos x = \dfrac{3}{2}$ has no solution? Explain.

Yes → the range of the graph $y = \cos x$ is $-1 \le x \le 1$ and $\frac{3}{2} > 1$.

b) Use a graphical approach to show that the equation $2 \sin x \cos x = 3 \cos x$ on the domain $0 \le x \le 2\pi$ does have roots. Give solutions as exact values.

$x = 0, \pi, 2\pi$

c) Identify Christine's error and provide a correct algebraic solution to the problem.

$2\sin x \cdot \cos x = 3\sin x$

$2\sin x\cos x - 3\sin x = 0$

$\sin x(2\cos x - 3) = 0$

$\sin x = 0$ or $\cos x = 3/2$

$\sin x = 0$

$x = 0, \pi, 2\pi$

$\cos x = \frac{3}{2}$

no solution

$x = 0, \pi, 2\pi$

6. A trigonometric function, $f(x)$, has a period of 2π radians.

a) If the roots of the equation $f(x) = 0$ on the domain $0 \le x \le 2\pi$ are $x = a$, $x = b$, and $x = c$ state the general solution to the equation $f(x) = 0$.

$x = a + 2n\pi,\ b + 2n\pi,\ c + 2n\pi,\ n \in I$

b) Use the generalization in 6a) and the solution in 5c) to state the general solution to the equation $2 \sin x \cos x = 3 \cos x$.

$x = 2n\pi,\ \pi + 2n\pi,\ 2\pi + 2n\pi,\ n \in I$

c) The three sets of answers in b) can be simplified to a single set of answers. Write the general solution to the equation $2 \sin x \cos x = 3 \cos x$ in simplest form.

$x = n\pi,\ n \in I$

7. Algebraically find the solutions to the following trigonometric equations. Give solutions as exact values.

a) $\cot^2 A + \cot A = 0$ where $-\pi \le A \le \pi$

$\cot A(\cot A + 1)$

$\cot A = 0$ or $\cot A = -1$

$\tan A$ is undefined or $\tan A = -1$

ref $\angle = \frac{\pi}{2}$ ref $\angle = \frac{\pi}{4}$ Q2/4

on domain $0 \le A \le 2\pi$

$A = \frac{\pi}{2}, \frac{3\pi}{2}$ $A = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$A = \frac{3\pi}{4}, \frac{7\pi}{4}$

on domain $-\pi \le A \le \pi$

$A = \frac{\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{4}, -\pi/4$

b) $2\cos^2 x = \sqrt{3}\cos x$ where $-2\pi \le x \le 0$

$2\cos^2 x - \sqrt{3}\cos x = 0$

$\cos x(2\cos x - \sqrt{3}) = 0$

$\cos x = 0$ or $\cos x = \frac{\sqrt{3}}{2}$

ref $\angle = \frac{\pi}{2}$ ref $\angle = \frac{\pi}{6}$

Q1/4

on domain $0 \le x \le 2\pi$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $x = \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

on domain $-2\pi \le x \le 0$ $x = \frac{\pi}{6}, \frac{11\pi}{6}$

$x = -\frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{11\pi}{6}, -\frac{\pi}{6}$

$-\frac{11\pi}{6}, -\frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{6}$.

8. Algebraically find the general solutions to the following trigonometric equations. Give solutions as exact values.

a) $2\csc^2\theta - 2 = 3\csc\theta$

$2\csc^2\theta - 3\csc\theta - 2 = 0$

$2\csc^2\theta - 4\csc\theta + \csc\theta - 2 = 0$

$2\csc\theta(\csc\theta - 2) + 1(\csc\theta - 2) = 0$

$(2\csc\theta + 1)(\csc\theta - 2) = 0$

$\csc\theta = -\frac{1}{2}$ or $\csc\theta = 2$

$\sin\theta = -2$ or $\sin\theta = \frac{1}{2}$

no solution. Q1/2 ref $\angle \pi/6$

on $0 \le \theta \le 2\pi$

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

general solution: $\theta = \frac{\pi}{6} + 2n\pi, \frac{5\pi}{6} + 2n\pi$

b) $3\sec\theta = 2 + \sec^2\theta$

$\sec^2\theta - 3\sec\theta + 2 = 0$

$(\sec\theta - 1)(\sec\theta - 2) = 0$

$\sec\theta = 1$ or $\sec\theta = 2$

$\cos\theta = 1$ or $\cos\theta = \frac{1}{2}$

ref $\angle 0$ ref $\angle = \pi/3$

Q1/4

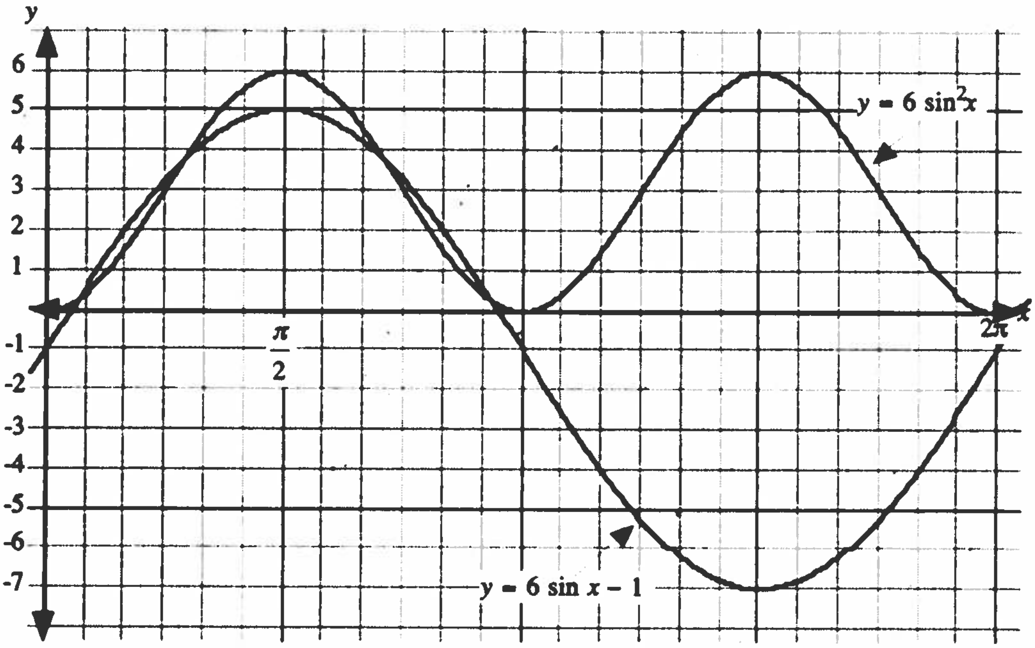
on $0 \le \theta \le 2\pi$

$\theta = 0, 2\pi$ $\theta = \frac{\pi}{3}, 5\pi/3$

general solution

$\theta = 2n\pi, \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$

9. The diagram below shows the graphs of $y = 6\sin^2 x$ and $y = 6\sin x - 1$ where $0 \le x \le 2$



a) **Explain how** you.could use this diagram to estimate the solution to the equation $6\sin^2 x - 6\sin x + 1 = 0$, where $0 \le x \le 2\pi$.

$6\sin^2 x = 6\sin x - 1$ → Find x-coordinates of point of intersection of the 2 graphs.

b) Algebraically determine the solutions to the equation $6\sin^2 x - 6\sin x + 1 = 0$, where $0 \le x \le 2\pi$. Give the solution correct to the nearest hundredth.

$6\sin^2 x - 6\sin x + 1 = 0$
does not factor

$\sin x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\sin x = \dfrac{6 \pm \sqrt{(-6)^2 - (4)(6)(1)}}{2(6)}$

$\sin x = \dfrac{6 \pm \sqrt{12}}{12} = 0.7886$ or 0.2113

Q1|2

ref∠ = 0.9085 rad. ref∠ = 0.2129

$x = 0.9085, \pi - 0.9085, 0.2129, \pi - 0.2129$

$x = 0.9085, 2.2330, 0.2129, 2.9286$

$x = 0.21, 0.91, 2.23, 2.93$

c) **Explain how** you could use this diagram to estimate the solution to the equation $6\sin^2 x(6\sin x - 1) = 0$, where $0 \le x \le 2\pi$.

Find the x-int of each graph.

d) Use an algebraic approach to find the solutions to the equation $6\sin^2 x(6\sin x - 1) = 0$ where $0 \le x \le 2\pi$. Give the solution correct to the nearest hundredth.

$6\sin^2 x = 0$ or $6\sin x - 1 = 0$

$\sin x = 0$ or $\sin x = \frac{1}{6}$

$x = 0, \pi, 2\pi$

Q1|2

ref∠ = 0.1674

$x = 0.1674, \pi - 0.1674$

$x = 0, \pi, 2\pi, 0.17, 2.97$

$x = 0.00, 0.17, 2.97, 3.14, 6.2$

10. The following questions cannot be solved using an algebraic approach. Graphically determine the solution(s) on the set of real numbers to the nearest hundredth of a radian.

a) $3\sin^2 x = x$

graph $y_1 = 3\sin^2 x$
$y_2 = x$
x-coor. of points of intersection
$x = 0.00, 0.35, 2.14$

b) $x^2 + \sin 6x - 1 = 0$

graph $y_1 = x^2 + \sin 6x - 1$ + determine x-int.
$x = -1.38, -1.07, -0.63, 0.21, 0.34, 1.04.$

Multiple Choice **11.** Which solutions are correct for the equation $12\sin^2 x - 11\sin x + 2 = 0$?

A. $\sin x = 3, 8$
B. $\sin x = \frac{11}{12}, -2$
C. $\sin x = \frac{2}{3}, \frac{1}{4}$
D. $\sin x = -\frac{2}{3}, -\frac{1}{4}$

$12\sin^2 x - 11\sin x + 2 = 0$
$12\sin^2 x - 8\sin x - 3\sin x + 2 = 0$
$4\sin x(3\sin x - 2) - 1(3\sin x - 2)$
$(3\sin x - 2)(4\sin x - 1) = 0$
$\sin x = \frac{2}{3}$ or $\sin x = \frac{1}{4}$

Numerical Response **12.** The number of solutions of the equation $2\cos^2 x + \cos x - 1 = 0$, where $-8\pi \le x \le 8\pi$ is ______ .

(Record your answer in the numerical response box from left to right.)

2	4		

graph $y_1 = 2\cos^2 x + \cos x - 1$
on domain $-8\pi \le x \le 8\pi$ +
count the x-int.

13. If angle A is acute and $\log_4(\sin^2 A) = -1$, then the value of A, to the nearest tenth of a radian, is ______ .

(Record your answer in the numerical response box from left to right.)

0	.	5	

$\log_4(\sin^2 A) = -1$
$\sin^2 A = 4^{-1} = \frac{1}{4}$
$\sin A = \pm\frac{1}{2}$

for acute angle A
Q1
$\sin A = \frac{1}{2}$
$A = \frac{\pi}{6} = 0.52$ rads.