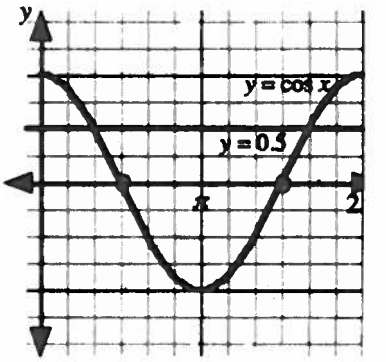
Assignment

1. The diagram shows the graph of the equations $y = \cos x$ and $y = 0.5$ in $0 \le x \le 2\pi$.



   a) **Explain** how to use the graph to determine the approximate solutions to the equation $\cos x = 0.5$, $0 \le x \le 2\pi$.

→ find x-coordinates of point of intersection of the two graphs.

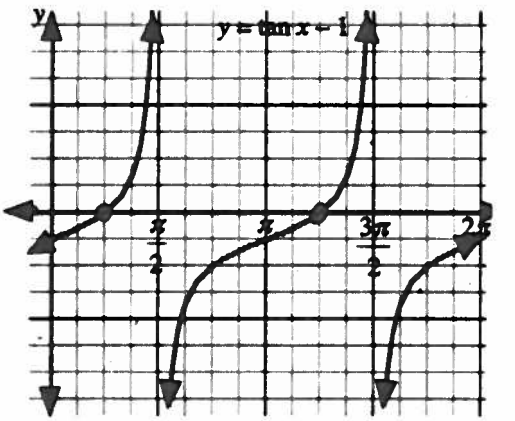
   b) Write the solutions to the equation $\cos x = 0.5$, $0 \le x \le 2\pi$. Give solutions as exact values.

$x = \frac{\pi}{3}, 5\pi/3$

   c) Write the general solution to the equation $\cos x = 0.5$.

$x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$.

2. The diagram shows the graph of the equation $y = \tan x - 1$ on the domain $0 \le x \le 2\pi$.



   a) **Explain** how to use the graph to determine the approximate solutions to the equation $\tan x = 1$, $0 \le x \le 2\pi$.

$\tan x = 1$ ⇝ period is π.

$\tan x - 1 = 0$

- find the x-int of the graph.

   b) Write the solutions to the equation $\tan x = 1$, $0 \le x \le 2\pi$. Give solutions as exact values.

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

   c) Write the general solution to the equation $\tan x = 1$.

$x = \frac{\pi}{4} + n\pi, n \in I$ ← difference is π

3. Determine the solution to each of the following equations, defined on the domain $0 \le x \le 2\pi$, using a **graphical** approach. Give solutions as exact values.

   a) $\sin x = \frac{\sqrt{3}}{2}$ $x = \frac{\pi}{3}, \frac{2\pi}{3}$

   b) $\tan x = -1$ $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

   c) $2 \sec x - 4 = 0$ $\sec x = 2 \rightarrow \cos x = \frac{1}{2}$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$.

4. Use the solutions in #3 to write the general solutions to the equations.

   a) $\sin x = \frac{\sqrt{3}}{2}$ $x = \frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, n \in I$

   b) $\tan x = -1$ $x = \frac{3\pi}{4} + n\pi, n \in I$

   c) $2 \sec x - 4 = 0$ $x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$

S | A
T | C

5. Determine the solution (to the nearest hundredth) to each of the following equations, defined on the domain $0 \le x \le 2\pi$, using a **graphical** approach.

a) $\cos x = 0.6$ $x = 0.93, 5.36$

b) $\cot x = -\frac{1}{3}$ $\tan x = -3$ $x = 1.89, 5.03$

c) $\csc x - 3 = 0$ $\sin x = \frac{1}{3}$ $x = 0.34, 2.80$

6. Use the solutions in #5 to write the general solutions to the equations.

a) $\cos x = 0.6$ $x = 0.93 + 2n\pi, 5.36 + 2n\pi, n \in I$

b) $\cot x = -\frac{1}{3}$ $x = 1.89 + n\pi, n \in I$

c) $\csc x - 3 = 0$ $x = 0.34 + 2n\pi, 2.80 + 2n\pi, n \in I$

7. Determine the solution to each of the following equations, defined on the domain $0 \le x \le 2\pi$, using an **algebraic** approach.

a) $2 \sin x = -\sqrt{3}$ $\sin x = -\frac{\sqrt{3}}{2}$

Q 3|4

ref $\angle = \frac{\pi}{3}$

$x = \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$x = 4\pi/3, 5\pi/3$

b) $\cot x + \sqrt{3} = 0$ $\cot x = -\sqrt{3}$

$\tan x = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

Q 2|4

ref $\angle = \pi/6$

$x = \pi - \frac{\pi}{6}, 2\pi - \pi/6$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

c) $3 \sec x + 6 = 0$ $\sec x = -2 \Rightarrow \cos x = -\frac{1}{2}$

Q 2+3

ref $\angle = \pi/3$

$x = \pi - \frac{\pi}{3}, \pi + \pi/3$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

8. Use the solutions in #7 to write the general solutions to the equations.

a) $2 \sin x = -\sqrt{3}$ $x = \frac{4\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$

b) $\cot x + \sqrt{3} = 0$ $x = \frac{5\pi}{6} + n\pi, n \in I$

c) $3 \sec x + 6 = 0$ $x = \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, n \in I$

9. Determine the general solution to the following equations where x is in **degree measure**. Answer to the nearest degree.

a) $\cos x = -0.639$

Q 2|3 ref $\angle = 50°$

on $0° \le x \le 360°$

$x = 180 - 50, 180 + 50$

$x = 130°, 230°$

general solution

$x = 130° + 360°n, 230° + 360°n, n \in I$

b) $5 \csc x + 6 = 0$ $\csc x = -\frac{6}{5}$ $\sin x = -\frac{5}{6}$

Q 3|4. ref $\angle = 56°$

on $0° \le x \le 360°$, $x = 180° + 56°, 360° - 56°$

$x = 236°, 304°$

general solution

$x = 236° + 360°n, 304° + 360°n, n \in I$

S | A
T | C

10. Use an algebraic approach to solve the following equations on the specified domain.

a) $2\cos x - \sqrt{2} = 0$ for $-2\pi \le x \le 0$

$\cos X = \frac{\sqrt{2}}{2}$, Q 1/4

ref $\angle = \frac{\pi}{4}$

on $0 \le x \le 2\pi$ $X = \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$X = \frac{\pi}{4}, \frac{7\pi}{4}$

subtract period of 2π

$X = -\frac{7\pi}{4}, -\frac{\pi}{4}$

b) $\csc x + 2 = 0$ for $2\pi \le x \le 6\pi$

$\csc X = -2, \sin X = -\frac{1}{2}$

Q 3/4, ref $\angle = \frac{\pi}{6}$

on $0 \le x \le 2\pi$, $x = \pi + \frac{\pi}{6}$

$2\pi - \frac{\pi}{6}$

$X = \frac{7\pi}{6}, \frac{11\pi}{6}$

add periods of 2π

$X = \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{31\pi}{6}, \frac{35\pi}{6}$

c) $\sqrt{3}\tan x = 1$ for $-\pi \le x \le 3\pi$

$\tan X = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Q1/3, ref $\angle = \frac{\pi}{6}$

on $0 \le x \le 2\pi$, $x = \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$X = \frac{\pi}{6}, \frac{7\pi}{6}$

add/subtract periods of π

$X = -\frac{5\pi}{6}, \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$.

11. Determine the general solution, in degrees, of the equation

a) $\sin x = 0$ Q 1-4, ref $\angle = 0°$

on $0° \le x \le 360°$, $X = 0°, 180°, 360°$

general solution

$X = 180°n, n \in I$

b) $\cos x = 0$ Q1-4 ref $\angle = 90°$

on $0° \le x \le 360°$, $X = 90°, 270°$

general solution $X = 90° + 180°n$ $n \in I$.

Multiple Choice

12. The general solution to the equation $\csc A + 2 = 0$ is

A. $A = \frac{\pi}{6} + n\pi,\ n \in I$

B. $A = \frac{\pi}{6} + 2n\pi,\ \frac{5\pi}{6} + 2n\pi,\ n \in I$

C. $A = \frac{7\pi}{6} + n\pi,\ \frac{11\pi}{6} + n\pi,\ n \in I$

(D.) $A = \frac{7\pi}{6} + 2n\pi,\ \frac{11\pi}{6} + 2n\pi,\ n \in I$

$\csc A = -2$, $\sin A = -\frac{1}{2}$

Q3/4, ref $\angle \pi/6$

on $0 \le A \le 2\pi$, $A = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$A = \frac{7\pi}{6}, \frac{11\pi}{6}$

period $= 2\pi$ so 0.

13. In simplest form, the general solution to the equation $\sqrt{3}\cot\theta - 1 = 0$ is

A. $\theta = \frac{\pi}{6} + n\pi,\ n \in I$

B. $\theta = \frac{\pi}{6} + 2n\pi,\ \frac{7\pi}{6} + 2n\pi,\ n \in I$

(C.) $\theta = \frac{\pi}{3} + n\pi,\ n \in I$

D. $\theta = \frac{\pi}{3} + 2n\pi,\ \frac{4\pi}{3} + 2n\pi,\ n \in I$

$\cot\theta = \frac{1}{\sqrt{3}}$, $\tan\theta = \sqrt{3}$

Q1/3, ref $\angle \pi/3$

on $0 \le \theta \le 2\pi$, $\theta = \frac{\pi}{3}, \pi + \frac{\pi}{3}$

$\theta = \frac{\pi}{3}; 4\pi/3$

period $= \pi$

14. The only solutions to a trigonometric equation on the domain $0 \le x \le 2\pi$ are $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$. An equation that has these solutions is

A. $2\sin x + \sqrt{3} = 0$ $\sin x = -\frac{\sqrt{3}}{2}$ Q3|4 NO

B. $2\cos x + \sqrt{3} = 0$ $\cos x = -\frac{\sqrt{3}}{2}$ $\text{ref}\angle = \pi/6$

C. $2\sin x + 1 = 0$ $\sin x = -\frac{1}{2}$ Q3|4 NO

D. $2\cos x + 1 = 0$ $\cos x = -\frac{1}{2}$, $\text{ref}\angle = \pi/3$

$x = \frac{2\pi}{3}$ & $x = \frac{4\pi}{3}$ in Q2 & 3 with reference angle $\frac{\pi}{3}$

Numerical Response

15. To the nearest degree, the solution to the equation $8\cot\theta = -1$ in the interval $540° \le \theta \le 720°$, is _____.

(Record your answer in the numerical response box from left to right.)

6	3	7	

→ period is 180°.

$0° \le \theta \le 180°$

$\cot\theta = \frac{-1}{8}$ ref$\angle = 83°$

$\tan\theta = -8$ in Q2, $\theta = 180° - 83° = 97°$

$97° + 3(180°) = 637°$

Answer Key

1. a) Find the x-coordinates of the points of intersection of the two graphs.

b) $x = \frac{\pi}{3}, \frac{5\pi}{3}$ c) $x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$

2. a) Find the x-intercepts of the graph. b) $x = \frac{\pi}{4}, \frac{5\pi}{4}$ c) $x = \frac{\pi}{4} + n\pi, n \in I$

3. a) $x = \frac{\pi}{3}, \frac{2\pi}{3}$ b) $x = \frac{3\pi}{4}, \frac{7\pi}{4}$ c) $x = \frac{\pi}{3}, \frac{5\pi}{3}$

4. a) $x = \frac{\pi}{3} + 2n\pi, \frac{2\pi}{3} + 2n\pi, n \in I$ b) $x = \frac{3\pi}{4} + n\pi, n \in I$ c) $x = \frac{\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$

5. a) $x = 0.93, x = 5.36$ b) $x = 1.89, x = 5.03$ c) $x = 0.34, x = 2.80$

6. a) $x = 0.93 + 2n\pi, 5.36 + 2n\pi, n \in I$ b) $x = 1.89 + n\pi, n \in I$ c) $x = 0.34 + 2n\pi, 2.80 + 2n\pi, n \in I$

7. a) $x = \frac{4\pi}{3}, \frac{5\pi}{3}$ b) $x = \frac{5\pi}{6}, \frac{11\pi}{6}$ c) $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

8. a) $x = \frac{4\pi}{3} + 2n\pi, \frac{5\pi}{3} + 2n\pi, n \in I$ b) $x = \frac{5\pi}{6} + n\pi, n \in I$ c) $x = \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi, n \in I$

9. a) $x = 130° + 360n°, 230° + 360n° \; n \in I$ b) $x = 236° + 360n°, 304° + 360n° \; n \in I$

10. a) $x = -\frac{7\pi}{4}, -\frac{\pi}{4}$ b) $x = \frac{19\pi}{6}, \frac{23\pi}{6}, \frac{31\pi}{6}, \frac{35\pi}{6}$ c) $x = -\frac{5\pi}{6}, \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}$

11. a) $x = 180n°, n \in I$ b) $x = 90° + 180n°, n \in I$

12. D **13.** C **14.** D **15.**

6	3	7	