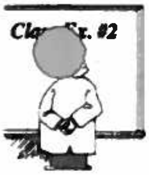


Class Ex. #2 In a certain town in Alberta, the time of sunrise for any day can be found using the formula

$$t = -1.79 \sin\left(2\pi\frac{(d-78)}{365}\right) + 6.3$$

where t is the time in hours after midnight and d is the number of the day in the year.

a) Write a suitable window which can be used to display the graph of the function.

max t = −1.79(−1) + 6.3 = 8.09   x: [0, 365, 30]

min t = −1.79(1) + 6.3 = 4.51   y [2, 10, 2]

b) Use the formula to determine, to the nearest minute, when the sun rose on May 7, the 127th day of the year.   0.96292 × 60 = 58 (nearest min)

x = 127 y = 4.9629 = 4:58 am.

c) Determine on which days of the year the sun rose at 7 a.m.

y_2 = 7 intersect 54.6595, 283.840   day 55 + 284

Complete Assignment Questions #1 - #7

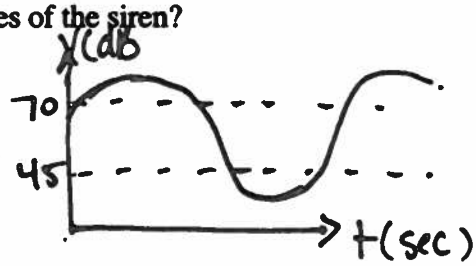
Assignment

1. The alarm in a noisy factory is a siren whose volume, V decibels, fluctuates so that t seconds after starting, the volume is given by the function $V(t) = 18 \sin \frac{\pi}{15}t + 60$.

a) What are the maximum and minimum volumes of the siren?

max − 18(1) + 60 = 78 dB

min − 18(−1) + 60 = 42 dB



b) Determine the period of the function.

period = $\frac{2\pi}{\pi/15}$ = 30 secs.

c) Write a suitable window which can be used to display the graph of the function.

x: [0, 40, 5]   y: [30, 100, 10]

d) After how many seconds, to the nearest tenth, does the volume first reach 70 decibels?

graph y_2 = 70 intersect 2.8 = 2.8 sec.

e) The background noise level in the factory is 45 decibels. Between which times, to the nearest tenth of a second, in the first cycle is the alarm siren at a lower level than the background noise?

graph y_2 = 45 intersect 19.7 + 25.3 (between 19.7 + 25.3sec)

f) For what percentage, to the nearest per cent, of each cycle is the alarm siren audible over the background factory noise?

In one 30 second cycle the alarm can be heard from 0 − 19.7 + 25.3 − 30 = 24.4 sec.

% = $\frac{24.4}{30}$ × 100 = 81%

2. A top secret satellite is launched into orbit from a remote island not on the equator. When the satellite reaches orbit, it follows a sinusoidal pattern that takes it north and south of the equator (i.e. the equator is used as the horizontal axis). Twelve minutes after it is launched it reaches the farthest point north of the equator. The distance north or south of the equator can be represented by the function $d(t) = 5000 \cos\left[\frac{\pi}{35}(t - 12)\right]$ where $d(t)$ is the distance in km, of the satellite north of the equator t minutes after being launched.

   a) How far north or south of the equator is the launch site? Answer to the nearest km.

   launch site $t=0$ $d = 2369$ km north.

   b) Is the satellite north or south of the equator after 20 minutes? What is this distance to the nearest kilometre?

   $t = 20$ min $d = 3765$ km north

   c) When, to the nearest tenth of a minute, will the satellite first be 2500 km south of the equator?

   $d = -2500$ graph $y_1 = 5000\cos\left(\frac{\pi}{35}(x-12)\right)$ $y_2 = -2500$ intersect @ $x = 35.3$

   35.3 mins.

3. The height of a tidal wave approaching the face of the cliff on an island is represented by the equation $h(t) = 7.5 \cos\left(\frac{2\pi}{9.5}t\right)$ where $h(t)$ is the height, in metres, of the wave above normal sea level t minutes after the wave strikes the cliff.

   a) What are the maximum and minimum heights of the wave relative to normal sea level?

   max $= 7.5(1) = 7.5$ min $= 7.5(-1) = -7.5$

   b) What is the period of the function?

   period $= \frac{2\pi}{2\pi/9.5} = 9.5$ mins.

   c) How high, to the nearest tenth of a metre, will the wave be, relative to normal sea level, one minute after striking the cliff?

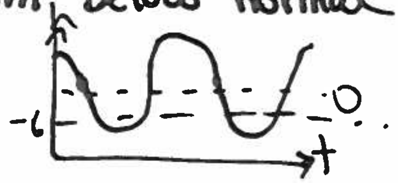
   $t=1$ height $= 5.9$m

   d) Normal sea level is 6 metres at the base of the cliff.

   i) For what values of h would the sea bed be exposed?

   If the height of the wave is 6m below normal sea level the sea bed is exposed.

   $h \le -6$



   ii) How long, to the nearest tenth of a minute, after the wave strikes the cliff does it take for the sea bed to be exposed?

   $y_1 = 7.5\cos\left(\frac{2\pi}{9.5}x\right)$ intersect @ $x = 3.777$

   $y_2 = -6$ 3.8 mins.

   iii) For how long, to the nearest tenth of a minute, is the sea bed exposed?

   second intersection point at 5.722

   $5.722 - 3.777$

   $= 1.945 = 1.9$ mins

4. The depth of water in a harbour can be modelled by the function $d(t) = -5\cos\frac{\pi}{6}t + 16.4$ where $d(t)$ is the depth in metres and t is the time in hours after low tide.

a) What is the period of the tide?

period $= \frac{2\pi}{\pi/6} = 12$ hours.

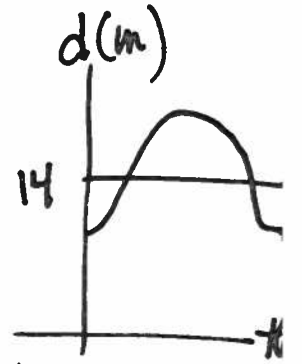
b) A large cruise ship needs at least 14 metres of water to dock safely. For how many hours per cycle, to the nearest tenth of an hour, can a cruise ship dock safely?

graph $y_1 = -5\cos\frac{\pi}{6}t + 16.4$

$y_2 = 14$

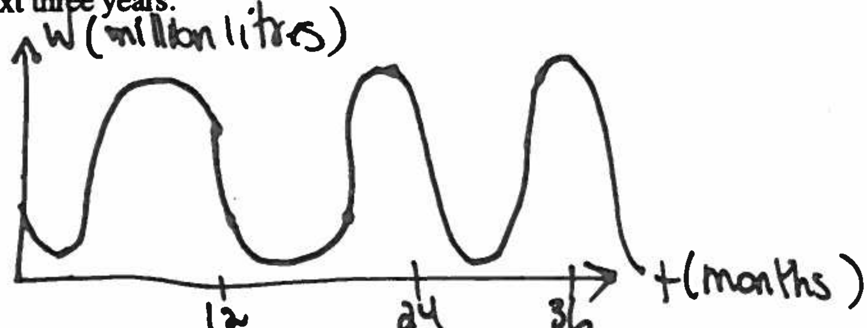
intersect at 2.0438 + 9.9561

$$\begin{array}{r} 9.9561 \\ -\ 2.0438 \\ \hline 7.9 \text{ hrs.} \end{array}$$



5. A city water authority determined that, under normal conditions, the approximate amount o water, $W(t)$, in millions of litres, stored in a reservoir t months after May 1, 2012, is given by the formula $W(t) = 1.25 - \sin\frac{\pi}{6}t$.

a) Sketch the graph of this function over the next three years.



b) In the summer of 2012, the authority decided to carry out the following simulation to determine if they had enough water to cope with a serious fire.

"If, on November 1, 2013, there is a serious fire which requires 300 000 litres of water to be brought under control, will the reservoir run dry if water rationing is not imposed?"

i) Explain how to use the graph in a) to solve the problem.

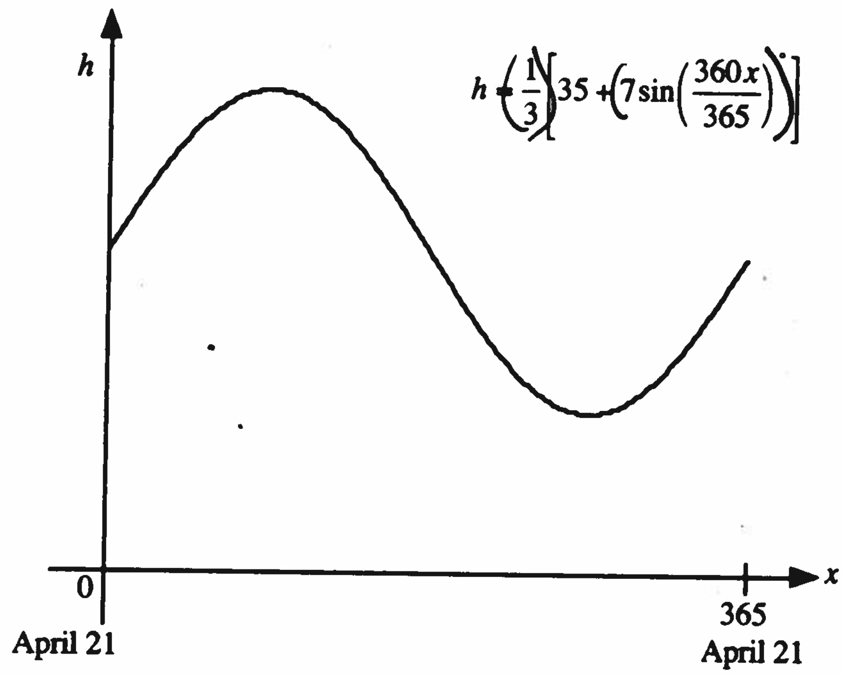
Nov 1 is month 18. At $t = 18$, the graph moves down 0.3 units. If the graph then falls below the t-axis, the reservoir would run dry [or → graph $y = -0.3$ - if line intersects graph below $t = 18$, the reservoir will run dry.

ii) Will the reservoir run dry if water rationing is not imposed? If so, in what month will this occur?

Yes - In month 26 → in July 2014.

Use the following information to answer the next two questions

The graph below shows how the number of hours (h) of daylight in a European city changes during the year.



Numerical Response

6. Mid-winter is the day with the least hours of daylight. The number of hours of daylight, to the nearest tenth of an hour, that there will be on mid-winter's day is ______.

(Record your answer in the numerical response box from left to right.)

9	.	3	

min value $= \frac{1}{3}(35 + 7(-1)) = \frac{28}{3}$

7. The number of days after April 21 that mid-winter occurs is ______.

(Record your answer in the numerical response box from left to right.)

2	7	4	

graph $y_1 = \left(\frac{1}{3}\right)\left[35 + \left(7\sin\left(\frac{360x}{365}\right)\right)\right]$

$y_2 = 28/3$

→ intersect @ 273.75