Consider the graphs of the functions $y = a \sin[b(x - c)] + d$ and $y = a \cos[b(x - c)] + d$.

a) Changing which of the parameters a, b, c and d affect the

i) domain? none ii) range? a, d iii) amplitude? a iv) period? b v) zeros? b, c, d $a \neq 0$

b) State the maximum and minimum values of the functions in terms of a, b, c, and d, if $a > 0$

max $= a(1) + d = a + d$.

min $= a(-1) + d = -a + d$

c) Determine the range of the function $y = 3 \sin 2(x - \pi) - 4$.

max $= 3(1) - 4 = 1$

min $= 3(-1) - 4 = -7$

range: $y \mid -7 \leq y \leq -1, y \in R$

Complete Assignment Questions #3 - #12

Assignment

1. Determine the amplitude, period, horizontal phase shift, and the vertical displacement for each function.

a) $y = \cos\left(x - \frac{\pi}{4}\right) + 3$

$a = 1$ amplitude $= 1$

$b = 1$ period $= \frac{2\pi}{1} = 2\pi$

$c = \frac{\pi}{4}$ h.p.s $= \frac{\pi}{4}$ (R)

$d = 3$ vert. disp $= 3 \uparrow$

b) $y = 3 \cos \frac{1}{2}\left(x - \frac{\pi}{2}\right)$

$a = 3$ amp $= 3$

$b = \frac{1}{2}$ period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

$c = \frac{\pi}{2}$ h.p.s $= \frac{\pi}{2}$ (R)

$d = 0$ vert disp $= 0$

c) $y = 3 \cos \frac{1}{2}x - \frac{\pi}{2}$

$a = 3$ amp $= 3$

$b = \frac{1}{2}$ period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

$c = 0$ h.p.s $= 0$

$d = \frac{-\pi}{2}$ vert disp $= \frac{\pi}{2} \downarrow$

d) $y = \sin\left(4x - \frac{\pi}{2}\right)$

$y = \sin 4\left(x - \frac{\pi}{8}\right)$

$a = 1$ amp $= 1$

$b = 4$ period $= \frac{2\pi}{4} = \frac{\pi}{2}$

$c = \frac{\pi}{8}$ h.p.s $= \frac{\pi}{8}$ (R)

$d = 0$ vert. disp $= 0$

e) $y = -2 \cos 3(x - 45°) + 4$

$a = -2$ amp $= 2$

$b = 3$ period $= \frac{360°}{3} = 120°$

$c = 45°$ $c = 45°$ (R)

$d = 4$ vert. displ $= 4$

f) $y = 7 \sin\left(\frac{1}{4}x + 20°\right) - 1$

$y = 7 \sin \frac{1}{4}(x + 80°) - 1$

$a = 7$ amp $= 7$

$b = \frac{1}{4}$ period $= \frac{360°}{\frac{1}{4}} = 1440°$

$c = -80°$ h.p.s $= 80°$ (L)

$d = -1$ vert disp $= 1 \downarrow$

2. a) Determine the equation of a sine function that has a vertical displacement 3 units up, a horizontal phase shift of 60° to the left, a period of 210° and an amplitude of 4.

$a = 4$ $\quad c = -60°$

$b = \frac{360}{210} = \frac{12}{7}$ $\quad d = 3$

$y = 4\sin\frac{12}{7}(x+60°)+3$

b) Determine the equation of a cosine function with a vertical displacement 5 units down, a horizontal phase shift of $\frac{2\pi}{3}$ radians to the right, a period of $\frac{5\pi}{4}$ and an amplitude of 3.

$a = 3$ $\quad c = \frac{2\pi}{3}$

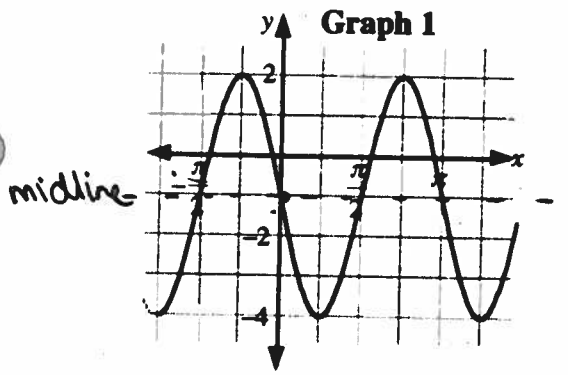
$b = \frac{2\pi}{5\pi/4} = \frac{8}{5}$ $\quad d = -5$

$y = 3\cos\frac{8}{5}\left(x-\frac{2\pi}{3}\right)-5$

3. Graphs 1 and 2 each represent the graphs of trigonometric functions.

a) Assuming a minimum possible phase shift, write the equation of each graph in the form $y = a\sin[b(x-c)]+d$ if: **i)** $a>0$ **ii)** $a<0$

Graph 1



$amp = \frac{2-(-4)}{2} = 3$

period $= \pi$

h.p.s. $\frac{\pi}{2}$ left or right for $a>0$, 0 for $a<0$ ↔ reflection.

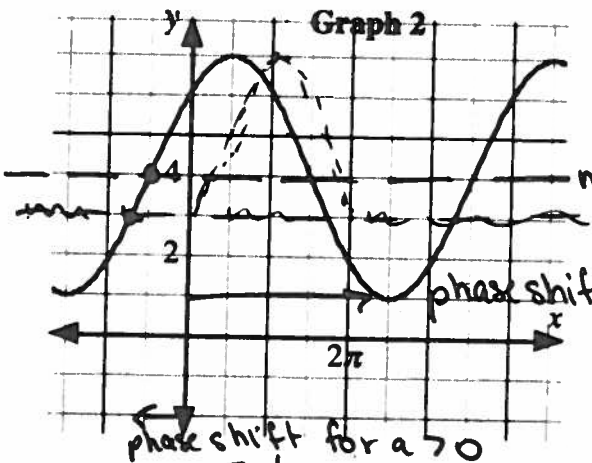
i) $a=3, b=\frac{2\pi}{\pi}=2, c=\pm\frac{\pi}{2}, d=1$

$y = 3\sin 2\left(x\pm\frac{\pi}{2}\right)-1$

ii) $a=-3, b=2, c=0, d=-1$

$y = -3\sin 2x - 1$

Graph 2



$amp = \frac{7-1}{2} = 3$

h.p.s $= \frac{\pi}{2}$ left for $a>0$

$: \frac{3\pi}{2}$ R for $a<0$

vert disp $= \frac{7+1}{2} = 4$.

i) $a=3, b=\frac{2\pi}{4\pi}=\frac{1}{2}, c=-\frac{\pi}{2}, d=4$

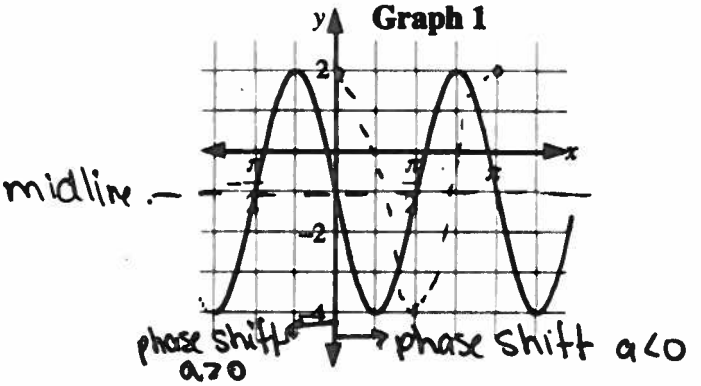
$y = 3\sin\frac{1}{2}\left(x+\frac{\pi}{2}\right)+4$

ii) $a=-3, b=\frac{1}{2}, c=\frac{3\pi}{2}, d=4$

$y = -3\sin\frac{1}{2}\left(x-\frac{3\pi}{2}\right)+4$.

b) Assuming a minimum possible phase shift, write the equation of each graph in the form $y = a\cos[b(x-c)]+d$ if: **i)** $a>0$ **ii)** $a<0$

Graph 1



amp = 3, period = π, vert. displ = -1

h.p.s = $\frac{\pi}{4}$ left, $a>0$

$\frac{\pi}{4}$ right, $a<0$

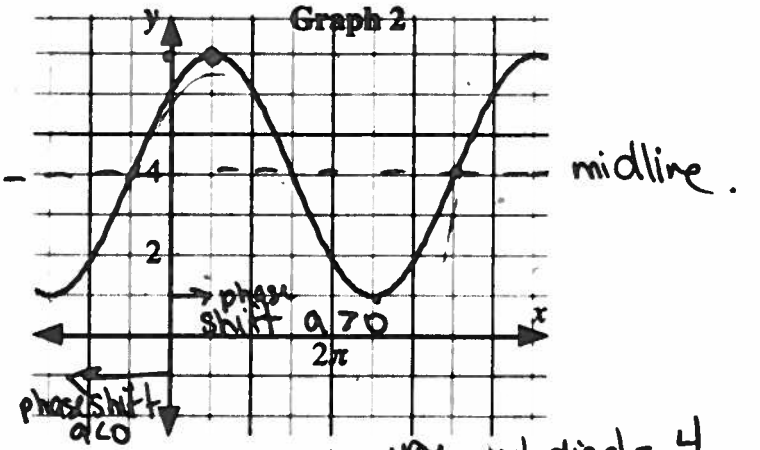
i) $a=3, b=2, c=-\frac{\pi}{4}, d=-1$

$y = 3\cos 2\left(x+\frac{\pi}{4}\right)-1$

ii) $a=-3, b=2, c=\frac{\pi}{4}, d=-1$

$y = -3\cos 2\left(x-\frac{\pi}{4}\right)-1$

Graph 2



amp = 3, period = 4π, vert displ = 4

h.p.s = $\frac{\pi}{2}$ right, $a>0$

$\frac{3\pi}{2}$ left, $a<0$

i) $a=3, b=\frac{1}{2}, c=\frac{\pi}{2}, d=4$

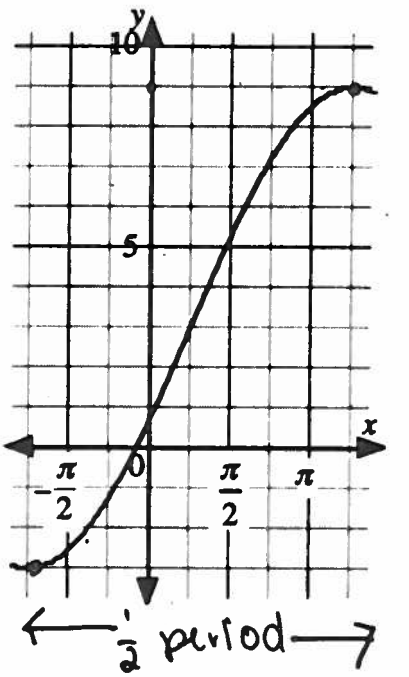
$y = 3\cos\frac{1}{2}\left(x-\frac{\pi}{2}\right)+4$

ii) $a=3, b=\frac{1}{2}, c=-\frac{3\pi}{2}, d=4$

$y = 3\cos\frac{1}{2}\left(x+\frac{3\pi}{2}\right)+4$

4. The cosine graph shown has a range $-3 \le y \le 9$. The graph has an equation in the form $y = a\cos[b(x-c)]+d, a>0$.

Determine the equation if the graph has a minimum possible phase shift.



amp = $\frac{9-(-3)}{2} = 6$ $a=6$

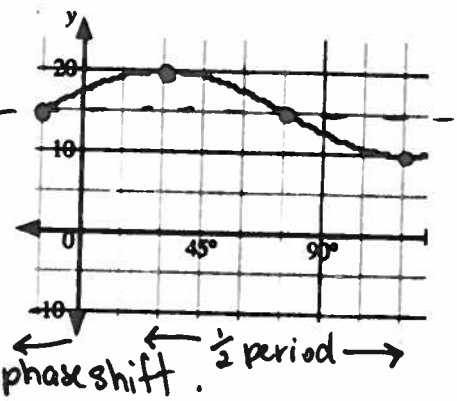
period = $16\left(\frac{\pi}{4}\right) = 4\pi$ $b = \frac{2\pi}{4\pi} = \frac{1}{2}$

h.p.s = $\frac{5\pi}{4}$ right. $c = \frac{5\pi}{4}$

vert disp = 3 ↑ $d=3$

$y = 6\cos\frac{1}{2}\left(x-\frac{5\pi}{4}\right)+3$

5. The sine graph shown has a maximum value of 20 and a minimum value of 10. If the graph has a minimum possible phase shift, determine the equation of the graph in the form $y = a \sin [b(x - c)] + d$ with $a > 0$.



amp $= \frac{20-10}{2} = 5$ $\quad a = 5$

period $= 12(15)^\circ = 180^\circ$ $\quad b = \frac{360^\circ}{180^\circ} = 2$

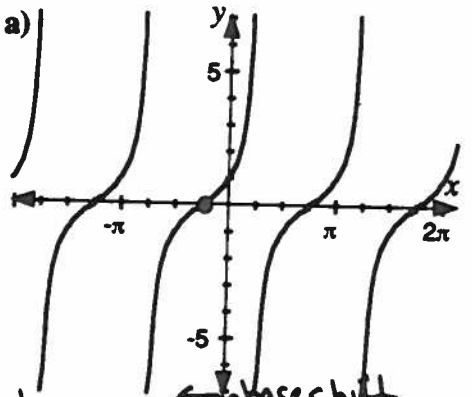
h.p.s. $= 15^\circ$ left $\quad c = -15^\circ$

vert. disp $= \frac{20+10}{2} = 15$ $\quad d = 15$

$y = 5 \sin 2(x + 15^\circ) + 15$.

6. Determine the equation of each graph in the form $y = \tan b(x - c)$.

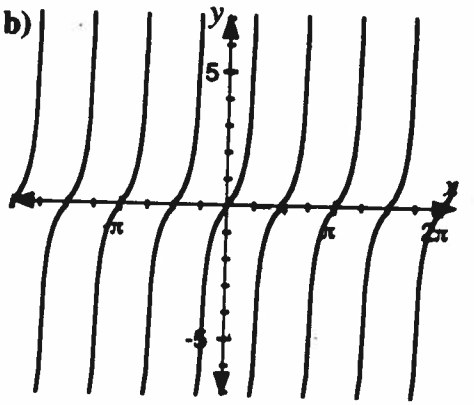
a)



period $= \pi$ $\quad b = \frac{\pi}{\pi} = 1$ ← phase shift.

h.p.s $= \frac{\pi}{4}$ left, $c = -\frac{\pi}{4}$ $\quad y = \tan(x + \frac{\pi}{4})$

b)



period $= \frac{\pi}{2}$ $\quad b = \frac{\pi}{\pi/2} = 2$

h.p.s $= 0$ $\quad c = 0$ $\quad y = \tan 2x$

7. Determine the range of the functions represented below.

a) $y = 2 \sin x - 2$

max $2(1) - 2 = 0$

min $2(-1) - 2 = -4$ $\quad y \mid -4 \le y \le 0, y \in R$

b) $y = 3 \cos \frac{1}{2}\left(x - \frac{\pi}{2}\right) + 1$

max $3(1) + 1 = 4$

min $(3(-1) + 1 = -2$ $\quad y \mid -2 \le y \le 4, y \in R$

c) $y = -\frac{1}{2} \cos 4(x - \pi) - 3$

max $-\frac{1}{2}(-1)(-3) = -\frac{5}{2}$

min $-\frac{1}{2}(1) - 3 = -\frac{7}{2}$ $\quad y \mid -\frac{7}{2} \le y \le -\frac{5}{2}, y \in R$

d) $y = a \sin [b(x - c)] + d$, where $a > 0$

max $a(1) + d = a + d$

min $a(-1) + d = -a + d$ $\quad y \mid (-a + d) \le y \le (a + d), y \in R$

Multiple Choice

8. Which of the following graphs has the same x-intercepts as the graph of $y = \cos x$?

change

A. $y = \cos 4x$ → period Δs

(B) $y = 4 \cos x$ → amp Δs

C. $y = \cos x + 4$ → vert. displace.

D. $y = \cos(x + 4)$ → phase shift.

9. Which equation is a tangent function with period $\frac{\pi}{3}$, and a vertical displacement –3?

A. $y = \tan \frac{\pi}{3} x - 3$ $\quad b = \frac{\pi}{\pi/3} = 3$

B. $y = \tan 3(x - 3)$

(C) $y = \tan 3x - 3$ $\quad d = -3$

10. $y = \tan 6x - 3$

10. The equation $y = \pi \cos(\pi x - \pi)$ has a period and a horizontal phase shift to the right, respectively, of

A. π and π $y = \pi \cos \pi(x-1)$
B. π and 1 period $= \frac{2\pi}{\pi} = 2$
C. 2 and π
(D) 2 and 1 h.p.s. 1 right

11. Which statement concerning the graph of $y = -4 \cos \frac{x}{2} + 2$ is not correct?

A. The maximum value is 6. $-4(-1)+2 = 6$
B. The period is 4π. $2\pi \div \frac{1}{2} = 4\pi$
(C) The amplitude is -4. 4
D. The vertical displacement is 2. 2

Numerical Response

12. The period, to the nearest tenth, of the function $y = \sin 0.25x$, where x is in radians, is _____ .

(Record your answer in the numerical response box from left to right.)

2	5	.	1

$b = 0.25$ period $= \frac{2\pi}{0.25} = 8\pi = 25.1$

Answer Key

1.

	amplitude	period	phase shift	vertical displacement
a)	1	2π	$\frac{\pi}{4}$ right	3 up
b)	3	4π	$\frac{\pi}{2}$ right	0
c)	3	4π	0	$\frac{\pi}{2}$ down
d)	1	$\frac{\pi}{2}$	$\frac{\pi}{8}$ right	0
e)	2	120°	45° right	4 up
f)	7	1440°	80° left	1 down

2. a) $y = 4 \sin \frac{12}{7}(x + 60°) + 3$

b) $y = 3 \cos \frac{8}{5}\left(x - \frac{2\pi}{3}\right) - 5$

(Note for **a)** and **b)**: the value of a can also be negative)

3. a) i) for $a > 0$, **Graph 1** $y = 3 \sin 2\left(x \pm \frac{\pi}{2}\right) - 1$, **Graph 2** $y = 3 \sin \frac{1}{2}\left(x + \frac{\pi}{2}\right) + 4$

ii) for $a < 0$, **Graph 1** $y = -3 \sin 2x - 1$, **Graph 2** $y = -3 \sin \frac{1}{2}\left(x - \frac{3\pi}{2}\right) + 4$

b) i) for $a > 0$, **Graph 1** $y = 3 \cos 2\left(x + \frac{\pi}{4}\right) - 1$, **Graph 2** $y = 3 \cos \frac{1}{2}\left(x - \frac{\pi}{2}\right) + 4$

ii) for $a < 0$, **Graph 1** $y = -3 \cos 2\left(x - \frac{\pi}{4}\right) - 1$, **Graph 2** $y = -3 \cos \frac{1}{2}\left(x + \frac{3\pi}{2}\right) + 4$

4. $y = 6 \cos \frac{1}{2}\left(x - \frac{5\pi}{4}\right) + 3$

5. $y = 5 \sin 2(x + 15°) + 15$

6. a) $y = \tan\left(x + \frac{\pi}{4}\right)$ b) $y = \tan 2x$

7. a) $\{y \mid -4 \le y \le 0, y \in R\}$ b) $\{y \mid -2 \le y \le 4, y \in R\}$

c) $\left\{y \mid -\frac{7}{2} \le y \le -\frac{5}{2}, y \in R\right\}$ d) $\{y \mid -a + d \le y \le a + d, y \in R\}$

8. B **9.** C **10.** D **11.** C **12.**

2	5	.	1