

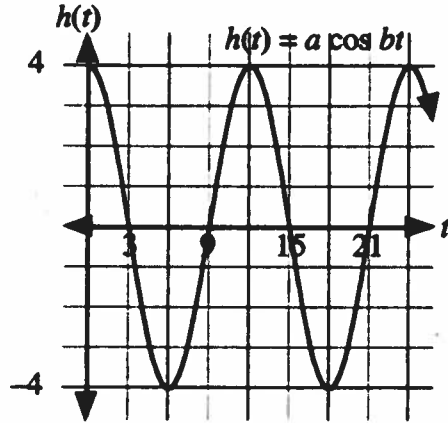
Class Ex. #5

The graph represents the effect of tides on mean sea level over a 24 hour period. The graph has equation  $h(t) = a \cos bt$ , where  $t$  is in hours and  $h$  is the height, in metres, relative to mean sea level. Determine the equation of the graph.

$a = 4$

$p = 12 \text{ hrs. } b = \frac{2\pi}{12} = \frac{\pi}{6}$

$h(t) = 4 \cos \frac{\pi}{6} t$



Complete Assignment Questions #1 - #16

Assignment

1. Describe how the graph of the given function compares to the graph of  $y = \cos x$ .

a)  $y = 5 \cos x$  - v.s. by factor of 5  $y \rightarrow \frac{1}{5}y$

b)  $y = 2 \cos \frac{1}{2}x$  - v.s. by factor of 2, h.s. by factor of 2  
 $y \rightarrow \frac{1}{2}y \cdot x \rightarrow \frac{1}{2}x$

c)  $y = -\frac{1}{3} \cos 4x$  v.s. by a factor of  $\frac{1}{3}$  + reflection in x-axis.  
 $x \rightarrow 4x$  h.s. by factor of  $\frac{1}{4}$ .  
 $y \rightarrow -3y$

d)  $y = 0.2 \cos(-6x)$   
 $y \rightarrow 5y$  v.s. by a factor of 0.2 + h.s. by factor of  $\frac{1}{6}$  +  
 $x \rightarrow -6x$  reflection in y-axis.

2. State the amplitude.

- a)  $y = 5 \sin x$       b)  $y = \cos 3x$       c)  $y = \frac{7}{3} \sin 2x$       d)  $y = -4 \cos \frac{5}{6}\theta$
- 5                              1                               $\frac{7}{3}$                               4

3. State the period in degrees.

- a)  $y = 6 \sin x$       b)  $y = \tan 3x$       c)  $y = \frac{2}{3} \cos \frac{x}{7}$       d)  $y = -2 \tan \frac{2}{3}\theta$
- $360^\circ$                        $\frac{180}{3} = 60^\circ$                        $\frac{360}{\frac{1}{7}} = 2520^\circ$                        $\frac{180^\circ}{\frac{2}{3}} = 270^\circ$

4. State the period in radians.

- a)  $y = 7 \tan x$       b)  $y = \cos 3x$       c)  $y = \frac{1}{4} \sin \frac{x}{3}$       d)  $y = 5 \tan \frac{1}{2}\theta$
- $\pi$                                $\frac{2\pi}{3}$                                $\frac{2\pi}{\frac{1}{3}} = 6\pi$                                $\frac{\pi}{\frac{1}{2}} = 2\pi$

$\cos / \sin \rightarrow \text{period} = 2\pi / 360^\circ$   
 $\tan \rightarrow \text{period} = \pi / 180^\circ$

5. Write the equation of a sine function with the given amplitude and period.

- a) amplitude 2, period  $1080^\circ$     b) amplitude 8, period  $\frac{\pi}{4}$     c) amplitude  $\frac{3}{2}$ , period  $6\pi$

$$a = 2$$

$$b = \frac{360}{1080} = \frac{1}{3}$$

$$y = 2 \sin \frac{1}{3}x$$

$$a = 8$$

$$\text{period} = \frac{2\pi}{\frac{1}{4}} = 8$$

$$y = 8 \sin 8x$$

$$a = \frac{3}{2}$$

$$\text{period} = \frac{2\pi}{\frac{1}{3}} = 6\pi$$

$$y = \frac{3}{2} \sin \frac{1}{3}x$$

6. Write the equation of a cosine function with the given amplitude and period.

- a) amplitude 1, period  $180^\circ$     b) amplitude 5, period  $\frac{4\pi}{3}$     c) amplitude  $\frac{5}{3}$ , period  $3\pi$

$$a = 1$$

$$\text{period} = \frac{360}{180} = 2$$

$$y = \cos 2x$$

$$a = 5$$

$$\text{period} = \frac{2\pi}{\frac{3}{4}} = \frac{8\pi}{3}$$

$$y = 5 \cos \frac{3}{2}x$$

$$a = \frac{5}{3}$$

$$\text{period} = \frac{2\pi}{\frac{2}{3}} = 3\pi$$

$$y = \frac{5}{3} \cos \frac{2}{3}x$$

7. Write the equation of a tangent function with the given period.

- a) period  $45^\circ$     b) period  $\frac{4\pi}{3}$

$$\text{period} = \frac{180}{45} = 4$$

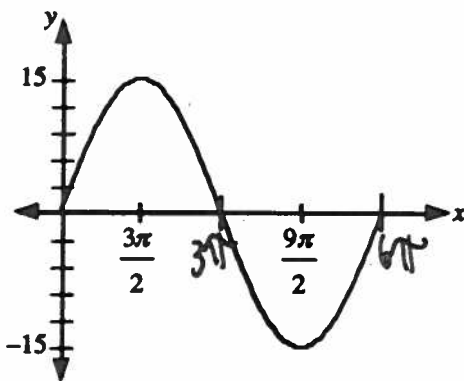
$$y = \tan 4x$$

$$\text{period} = \frac{\pi}{\frac{3}{4}} = \frac{4\pi}{3}$$

$$y = \tan \frac{3}{4}x$$

8. Determine the equation of each graph in the form.

a)  $y = a \sin bx$



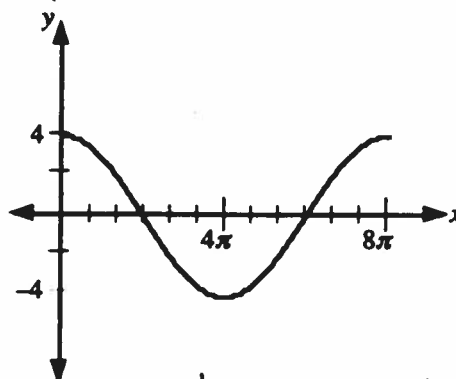
$$a = 15$$

$$\text{period} = 6\pi$$

$$b = \frac{2\pi}{6\pi} = \frac{1}{3}$$

$$y = 15 \sin \frac{1}{3}x$$

b)  $y = a \cos bx$



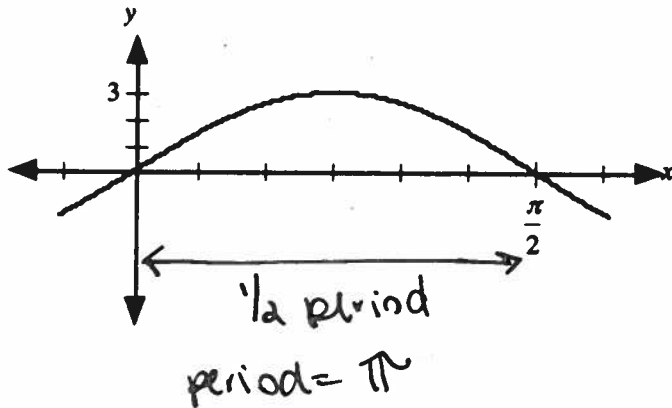
$$a = 4$$

$$\text{period} = 8\pi$$

$$b = \frac{2\pi}{8\pi} = \frac{1}{4}$$

$$y = 4 \cos \frac{1}{4}x$$

9. The trigonometric graph shown has a maximum value of 3 and a minimum value of -3. Determine the equation of the graph in the form  $y = a \sin bx$ .

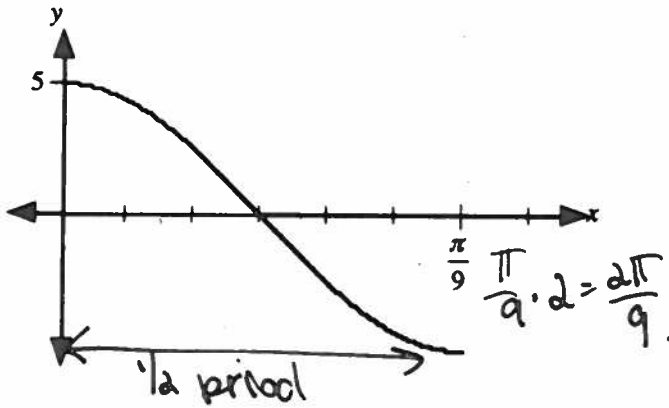


$$a = 3$$

$$b = \frac{2\pi}{\pi} = 2$$

$$y = \underline{3 \sin 2x}$$

10. The trigonometric graph shown has a maximum value of 5 and a minimum value of -5. Determine the equation of the graph in the form  $y = a \cos bx$ .



$$a = 5$$

$$\text{period} = \frac{2\pi}{9}$$

$$b = \frac{2\pi}{2\pi/9} = 9$$

$$y = \underline{5 \cos 9x}$$

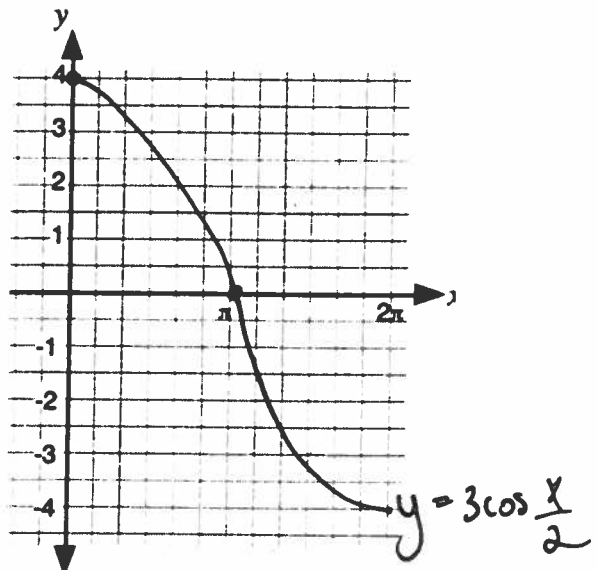
11. Consider the graph of  $y = 3 \cos \frac{x}{2}$ ,  $0 \leq x \leq 2\pi$ .

- a) State the amplitude and period.

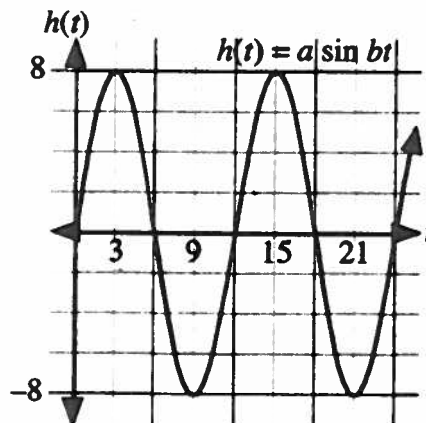
$$a = 3, b = \frac{1}{2}$$

$$\text{period} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

- b) Sketch the graph on the grid.  
Use a graphing calculator to verify.



12. The graph represents the change in sea level over a 24 hour period. The graph has equation  $h(t) = a \sin bt$ , where  $t$  is in hours and  $h$  is the height, in metres, relative to mean sea level.



- a) Determine the equation of the graph.

amp = 8m  $a = 8$   
 period = 12 hrs.  $b = \frac{2\pi}{12} = \frac{\pi}{6}$

$h(t) = 8 \sin \frac{\pi}{6} t$

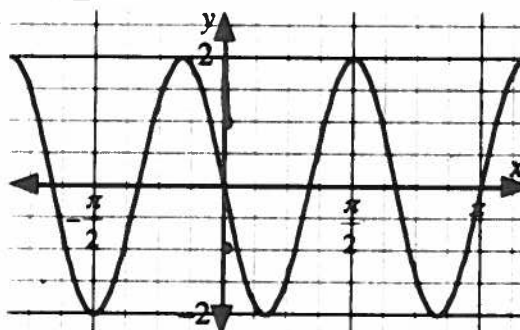
- b) Calculate the height above mean sea level, to the nearest tenth, when  $t = 4$ .

$h = 8 \sin \frac{\pi}{6} (4) = 8 \sin \left( \frac{2\pi}{3} \right) = 8 \left( \frac{\sqrt{3}}{2} \right) = 4\sqrt{3} = 6.928$   
 height = 6.9 m

→ could use calculator

13. a) Which transformations applied to the graph of  $y = \sin x$  result in the graph shown?

- v.s. by factor of 2
- h.s. by factor of  $1/3$
- reflection in x-axis or y-axis (same result).



- b) Write the equation of the graph in the form  $y = a \sin bx$ .

$a = -2$   $y = -2 \sin 3x$  or  $a = 2$   $y = 2 \sin(-3x)$   
 $b = 3$   $b = -3$

Multiple Choice

14. Which of the following functions does not have a period of  $\pi$ ?

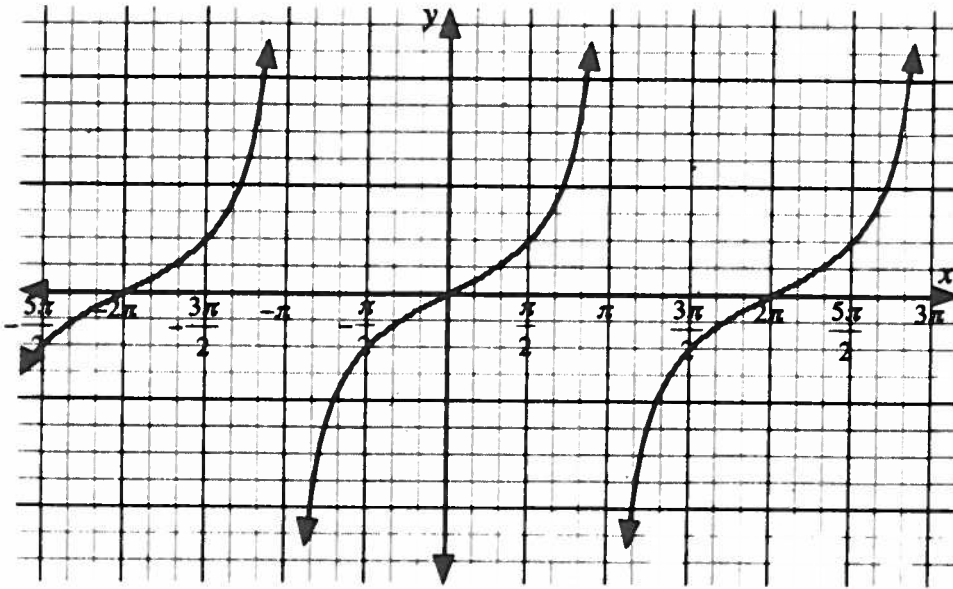
- A.  $y = \sin 2x$   $\frac{2\pi}{2} = \pi$       B.  $y = \cos 2x$   $\frac{2\pi}{2} = \pi$       C.  $y = \tan 2x$   $\frac{\pi}{2}$       D.  $y = \tan x$   $\frac{\pi}{1} = \pi$

15. Which of the following statements is incorrect?

- A. The maximum value of the graph of  $y = 3 \cos 2x$  is 3.  $a = 3 \checkmark$
- B. The graph of  $y = 3 \sin 2x$  has a y-intercept of 3.  $y\text{-int} = 0 \times$
- C. The graph of  $y = 4 \cos 3x$  has an x-intercept of  $\frac{\pi}{6}$ .  $4 \cos \left( 3 \left( \frac{\pi}{6} \right) \right) = 0 \checkmark$
- D. The graph of  $y = 2 \tan 2x$  has an asymptote with equation  $x = \frac{\pi}{4}$ .  $\checkmark$



16. The graph shown has equation  $y = \tan bx$ .



period =  $2\pi$   
 $b = \frac{\pi}{2\pi} = \frac{1}{2}$

The value of  $b$ , to the nearest tenth, is \_\_\_\_\_.

(Record your answer in the numerical response box from left to right.)

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**Answer Key**

- a vertical stretch by a factor of 5 about the  $x$ -axis
  - a vertical stretch by a factor of 2 about the  $x$ -axis and a horizontal stretch by a factor of 2 about the  $y$ -axis
  - a vertical stretch by a factor of  $\frac{1}{3}$  about the  $x$ -axis, a horizontal stretch by a factor of  $\frac{1}{4}$  about the  $y$ -axis, and a reflection in the  $x$ -axis.
  - a vertical stretch by a factor of 0.2 about the  $x$ -axis, a horizontal stretch by a factor of  $\frac{1}{6}$  about the  $y$ -axis, and a reflection in the  $y$ -axis
- 5
  - 1
  - $\frac{7}{3}$
  - 4
- $360^\circ$
  - $60^\circ$
  - $2520^\circ$
  - $270^\circ$
- $\pi$
  - $\frac{2\pi}{3}$
  - $6\pi$
  - $2\pi$
- $y = 2 \sin \frac{1}{3}x$
  - $y = 8 \sin 8x$
  - $y = \frac{3}{2} \sin \frac{1}{3}x$
- $y = \cos 2x$
  - $y = 5 \cos \frac{3}{2}x$
  - $y = \frac{5}{3} \cos \frac{2}{3}x$
- $y = \tan 4x$
  - $y = \tan \frac{3}{4}x$
- $y = 15 \sin \frac{1}{3}x$
  - $y = 4 \cos \frac{1}{4}x$
  - $y = 3 \sin 2x$
- amp = 3, period =  $4\pi$
  - $\rightarrow \rightarrow \rightarrow \rightarrow$
- $y = 8 \sin \frac{\pi}{6}t$
  - 6.9 metres
- a vertical stretch by a factor of 2 about the  $x$ -axis, a horizontal stretch by a factor of  $\frac{1}{3}$  about the  $y$ -axis, a reflection in the  $x$ -axis
  - $y = -2 \sin 3x$  or  $y = 2 \sin (-3x)$

14. C    15. B    16. 

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