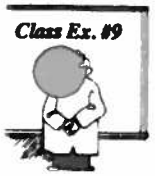


a) Determine the equation of a circle with centre at the origin and radius 8 units.

$x^2+y^2=8^2$ $\quad x^2+y^2=64$

b) If the point $(-4, b\sqrt{3})$ lies on the circle, determine all possible values of b.

$(-4)^2+(b\sqrt{3})^2=64$ $\quad b^2=16$

$16+3b^2=64$ $\quad b=\pm 4$

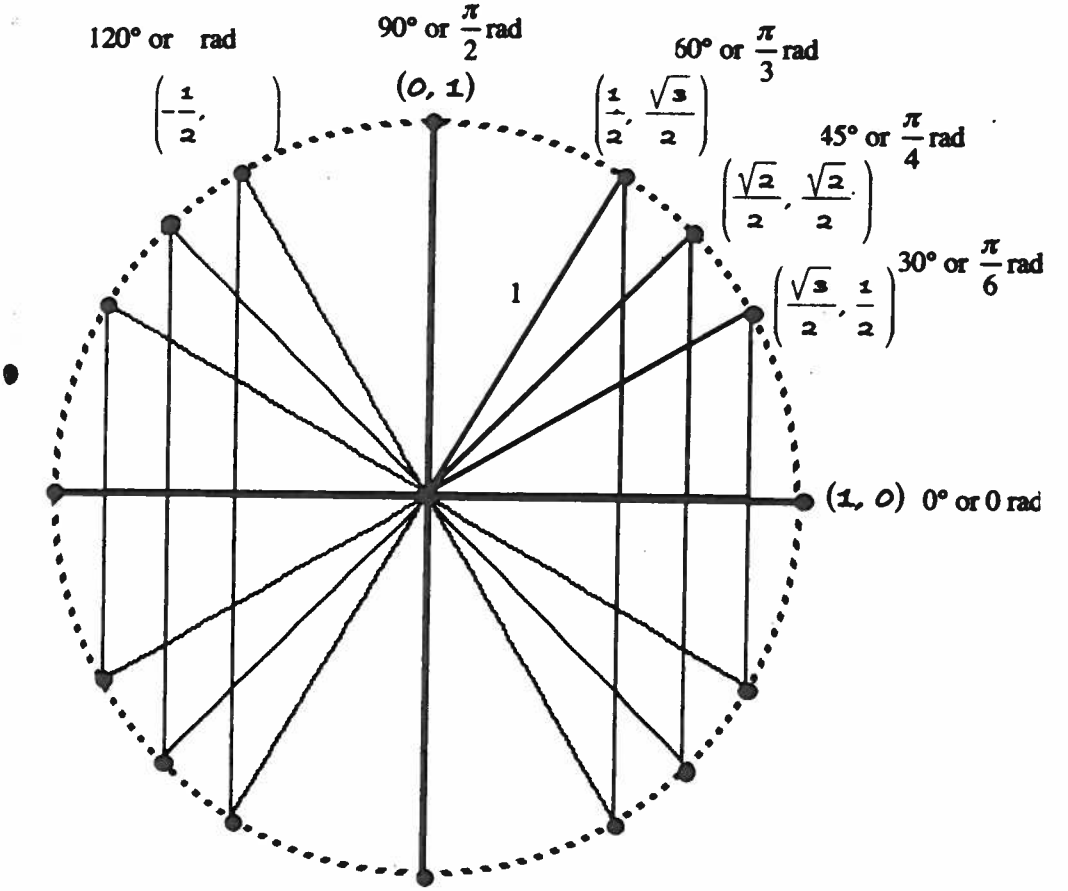
$3b^2=48$

Complete Assignment Questions #11 - #16

Assignment

1. The diagram on page 537 has been reflected in the x-axis, the y-axis, and in both axes to produce the diagram below.

   Complete the diagram by writing the coordinates and the rotation angle (in degrees and in radians) for each point on the circumference of the circle.



2. Use either the unit circle or a reference triangle to determine the exact value of the following

a) $\cos 150°$

$-\frac{\sqrt{3}}{2}$

b) $\sin 315°$

$-\frac{\sqrt{2}}{2}$

c) $\sin(-30)°$

$-\frac{1}{2}$

d) $\tan 240°$

$\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$

e) $\tan 480°$

$\frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

f) $\cos^2 225°$

$= \left(-\frac{\sqrt{2}}{2}\right)^2$

$= \frac{2}{4} = \frac{1}{2}$.

3. Determine the exact value of the following.

a) $\sin \frac{5\pi}{3}$

$-\frac{\sqrt{3}}{2}$

b) $\tan \frac{7\pi}{6}$

$\frac{\sqrt{3}}{3}$

c) $\cos\left(-\frac{2\pi}{3}\right)$

$\cos \frac{4\pi}{3} = -\frac{1}{2}$

d) $\sin(-\pi)$

$= \sin \pi$

$= 0$

e) $\cos\left(-\frac{5\pi}{3}\right)$

$\cos \frac{\pi}{3}$

$= \frac{1}{2}$

f) $\tan^2 \frac{2\pi}{3}$

$(-\sqrt{3})^2 = 3$

4. Determine the exact value of the following.

a) $\sec 300°$

$\frac{1}{\cos 300}$

$= 1/\frac{1}{2} = 2$

b) $\cot \frac{5\pi}{6}$

$\frac{1}{\tan 5\pi/6}$

$= \frac{1}{-\sqrt{3}/3} = \frac{-3}{\sqrt{3}} = -\sqrt{3}$

c) $\csc\left(-\frac{5\pi}{3}\right)$ $\csc \frac{\pi}{3}$

$= \frac{1}{\sin \pi/3} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

d) $\cot 930°$

$\cot(930 - 2(360)$

$\cot 210$

$= \frac{\cos 210}{\sin 210}$

$= -3\sqrt{2}/-1/2$

$= \sqrt{3}$

e) $\sec \frac{3\pi}{2}$

$= \frac{1}{\cos \frac{3\pi}{2}}$

$= \frac{1}{0}$

undefined

f) $\csc 5\pi$ $\csc \pi$

$= \frac{1}{\sin \pi} = \frac{1}{0}$

undefined

5. State the exact coordinates of the point on the unit circle that correspond to each rotation.

a) $\frac{3}{2}\pi$ radians $(0, -1)$

b) $360°$ $(1, 0)$

c) $\frac{7\pi}{6}$ $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

6. Use a calculator to determine, to four decimal places, the coordinates of the point on the un circle that corresponds to each rotation.

a) $175°$ $(\cos 175, \sin 175)$ $= (-0.9962, 0.0872)$

b) $\frac{13\pi}{10}$ radians $\left(\cos\frac{13\pi}{10}, \sin\frac{13\pi}{10}\right) = (-0.5878, -0.8090)$

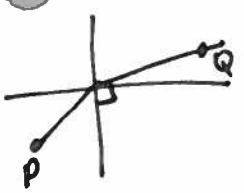
7. The point $T(0.4695, -0.8829)$ lies on the unit circle. Determine the value of θ, in degrees, where θ is the angle made by the positive x-axis and the line passing through T.

Q4 $\cos\theta = 0.4695$ $\theta = 360 - 62 = 298°$

ref $\angle = 62°$

Do not use technology to answer questions #8, #9, and #10.

8. $P\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ and $Q\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ are two points on the unit circle. If an object rotates counterclockwise from point P to point Q, through what angle has it rotated? Answer in degrees and in radians.



P is at $240°$

Q is at $30°$

angle $= 30° + 90° + 30° = 150°$

$= \frac{\pi}{6} + \frac{\pi}{2} + \frac{\pi}{6} = \frac{5\pi}{6}$ rads.

9. Determine the exact value of

a) $\log_3\left(\cot\frac{4\pi}{3}\right)$

$\frac{-1}{2} \div -\frac{\sqrt{3}}{2}$

$\frac{-1}{2} \times \frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\tan\theta = +\frac{\sqrt{3}}{3}$

$= \log_3\left(\frac{\sqrt{3}}{3}\right)$

$= \log_3\left(\frac{3^{1/2}}{3}\right)$

$= \log_3 3^{-1/2}$

$= -\frac{1}{2}(\log_3 3) = -\frac{1}{2}$

b) $\log_4(\csc 510°)$

$= \log_4(\csc 150)$

$= \log_4 2$

$2 = 4^x$

$x = \frac{1}{2}$

c) $\sin^2\frac{3\pi}{4} + \cos^2\frac{3\pi}{4}$

$= \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

10. Find the measure of θ where $0 \le \theta \le 2\pi$.

a) $\sin\theta = -\frac{\sqrt{3}}{2}$ $\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$

b) $\tan\theta = 0$ $\theta = 0, \pi, 2\pi$

c) $\tan\theta$ is undefined $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$.

11. Write the equation of the following.

a) the unit circle

$x^2+y^2=1$

b) circle with centre (0, 0) and radius $\sqrt{10}$

$x^2+y^2=(\sqrt{10})^2 \rightarrow x^2+y^2=10$

c) circle with centre (0, 0) and passing through the point (–8, 6)

$x^2+y^2=r^2$

$(-8)^2+(6)^2=r^2$

$r^2=64+36$

$r^2=100$

$r=10$

$x^2+y^2=10^2$

$x^2+y^2=100$

12. The point $A\left(x, \frac{\sqrt{5}}{5}\right)$ lies on the unit circle in quadrant 2. $x \rightarrow (-)$

a) Determine the value of x. Answer as a radical with a rational denominator.

$x^2+\left(\frac{\sqrt{5}}{5}\right)^2=1$

$x^2+\frac{1}{5}=1$

$x^2=\frac{4}{5}$

$x=\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$ → $-\frac{2\sqrt{5}}{5}$.

b) The point A lies on the terminal arm of a rotation angle θ.
Determine the exact values of $\tan\theta$ and $\csc\theta$.

$\cos\theta=-\frac{2\sqrt{5}}{5}$

$\sin\theta=\frac{\sqrt{5}}{5}$

$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\sqrt{5}}{5}\div-\frac{2\sqrt{5}}{5}=\frac{\sqrt{5}}{5}\times\frac{5}{-2\sqrt{5}}=\frac{-1}{2}$

$\csc\theta=\frac{1}{\sin\theta}=1\div\frac{\sqrt{5}}{5}=1\times\frac{5}{\sqrt{5}}=\frac{5}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{5\sqrt{5}}{5}=\sqrt{5}$

c) Determine the value of θ to the nearest tenth of a radian if $0 \le \theta \le 2\pi$.

$\sin\theta=\frac{\sqrt{5}}{5}$ in Q2

ref $\angle$ = 0.4636

$\theta=\pi-0.4636 \approx 2.6779$

≈ 2.7 radians.

13. If the point $(-a, a\sqrt{2})$ lies on the circle with centre (0, 0) and radius 12, determine all possible values of a.

$x^2+y^2=12^2$

$(-a)^2+(a\sqrt{2})^2=144$

$a^2+2a^2=144$

$3a^2=144$

$a^2=48$

$a=\pm\sqrt{48}=\pm 4\sqrt{3}$

Multiple Choice

14. The exact value of $\sec\left(-\frac{3\pi}{4}\right)$ can be written in the form $k\sqrt{2}$. The value of k is

A. 1
B. –1
C. 2
D. –2

$\sec\frac{5\pi}{4}=\frac{1}{\cos\frac{5\pi}{4}}=\frac{1}{-\frac{\sqrt{2}}{2}}=\frac{-2}{\sqrt{2}}=-\sqrt{2}$

$k=-1$

15. Which one of the following points does not lie on the unit circle?

$x^2+y^2=1$

A. $(-1,0)$ $(-1)^2+0^2=1$ ✓

B. $\left(-\frac{3}{4},\frac{\sqrt{7}}{4}\right)$ $\left(-\frac{3}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2=1$ ✓

C. $\left(\frac{2}{5},\frac{3}{5}\right)$ $\left(\frac{2}{5}\right)^2+\left(\frac{3}{5}\right)^2=\frac{13}{25}$ ✗

D. $\left(-\frac{8}{17},-\frac{15}{17}\right)$ $\left(\frac{-8}{17}\right)^2+\left(\frac{-15}{17}\right)^2=1$ ✓

Numerical Response

16. The smallest value of θ for which $\cot\theta$ is undefined in the interval $90° \le \theta \le 360°$ is _____.

(Record your answer in the numerical response box from left to right.)

1	8	0	

$\tan\theta = 0$

$\theta = 180$

Answer Key

1. See page 534

2. a) $-\frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{2}}{2}$ c) $-\frac{1}{2}$ d) $\sqrt{3}$ e) $-\sqrt{3}$ f) $\frac{1}{2}$

3. a) $-\frac{\sqrt{3}}{2}$ b) $\frac{\sqrt{3}}{3}$ c) $-\frac{1}{2}$ d) 0 e) $\frac{1}{2}$ f) 3

4. a) 2 b) $-\sqrt{3}$ c) $\frac{2\sqrt{3}}{3}$ d) $\sqrt{3}$ e) undefined f) undefined

5. a) $(0,-1)$ b) $(1,0)$ c) $\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$

6. a) $(-0.9962, 0.0872)$ b) $(-0.5878, -0.8090)$

7. 298°

8. 150° or $\frac{5\pi}{6}$ radians

9. a) $-\frac{1}{2}$ b) $\frac{1}{2}$ c)

10. a) $\frac{4\pi}{3}, \frac{5\pi}{3}$ b) $0, \pi, 2\pi$ c) $\frac{\pi}{2}, \frac{3\pi}{2}$

11. a) $x^2+y^2=1$ b) $x^2+y^2=10$ c) $x^2+y^2=100$

12. a) $-\frac{2\sqrt{5}}{5}$ b) $\tan\theta=-\frac{1}{2}, \csc\theta=\sqrt{5}$ c) 2.7

13. $\pm 4\sqrt{3}$ **14.** B **15.** C **16.**

1	8	0	