Using One Trigonometric Ratio to Determine Other Trigonometric Ratios

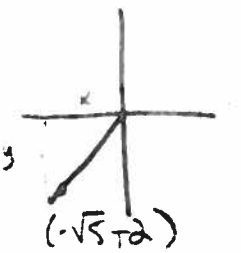
Class Ex. #6

Cot $A = \frac{\sqrt{5}}{2}$ and sin A is negative. Complete the following procedure to determine exact values, with rational denominators, for csc A and sec A.

a) Since the cotangent ratio is positive, and the sin ratio is negative, angle A must terminate in quadrant 3.

S | A
T | C

b) Since $\cot A = \frac{\sqrt{5}}{2} = \frac{x}{y}$, we know that the point $(-\sqrt{5}, -2)$ lies on the terminal arm of angle A in the third quadrant. Sketch a diagram and draw the reference triangle illustrating the above information.



c) Use $x^2 + y^2 = r^2$ to determine the value of r, and hence determine the exact values of csc A and sec A.

$x = -\sqrt{5}$ $\quad y = -2$

$x^2 + y^2 = r^2$

$(-\sqrt{5})^2 + (-2)^2 = r^2$

$r^2 = 9$

$r = 3$

$\csc A = \frac{r}{y} = -\frac{3}{2}$

$\sec A = \frac{r}{x} = \frac{-3}{\sqrt{5}} = \frac{-3\sqrt{5}}{5}$

Complete Assignment Questions #5 - #15

Assignment

1. In which quadrant(s) does the terminal arm of rotation angle θ lie if

S | A
T | C

a) sin θ is negative? 3 or 4

b) sec θ is positive? 1 or 4

c) csc θ and tan θ are both negative? 4

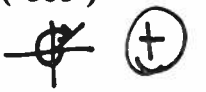
d) cot θ is positive and csc θ is negative? 3

2. Determine, without using technology, whether the given trigonometric ratios are positive or negative.

a) cos 181°

b) sin csc $\frac{11\pi}{6}$

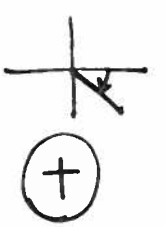
c) tan (–300°)



d) sin $\frac{14\pi}{3}$

e) tan cot 560°

f) sec $\left(-\frac{\pi}{4}\right)$ cos



3. Find the value (to 4 decimal places where necessary) of

* switch b/w degree ↔ radian

a) $\tan\frac{\pi}{4}$ = 1

b) $\cos(-382°)$ = 0.9272

c) $\sin\left(-\frac{2\pi}{3}\right)$ = −0.8660

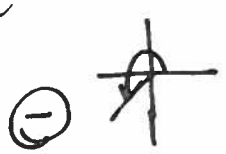
d) $\cot 30°$ = $\frac{1}{\tan 30}$ = 1.7321

e) $\csc 60$ = $\frac{1}{\sin 60}$ = −3.2807

f) $\sec\left(-\frac{7\pi}{6}\right)$ = $\frac{1}{\cos\left(-\frac{7\pi}{6}\right)}$ = −1.1547

4. Rewrite as the same trigonometric function of a positive acute angle. (ref angle w/ S|A / T|C)

a) $\sin 205° =$ $-\sin 25°$



ref ∠ = 25°

b) $\cot\frac{3\pi}{5} =$ $-\cot\frac{2\pi}{5}$

ref ∠ = $\pi - \frac{3\pi}{5}$ = $\frac{2\pi}{5}$

c) $\csc 107° =$ $\csc 73°$

d) $\sec\left(-\frac{19\pi}{9}\right) =$ $\sec\frac{\pi}{9}$ (cos)

ref ∠ = $\frac{\pi}{9}$

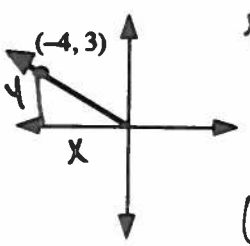
e) $\cos 5\pi =$ $-\cos 0$

ref ∠ = 0

f) $\tan(-30°) =$ $-\tan 30°$

ref ∠ 30°

5. The point (−4, 3) lies on the terminal arm of a rotation angle as shown.
Determine the primary and reciprocal trigonometric ratios for the rotation angle.
Express each answer as an exact value.



$x = -4$

$y = 3$

$x^2 + y^2 = r^2$

$(-4)^2 + (3)^2 = r^2$

$r^2 = 25$

$r = 5$

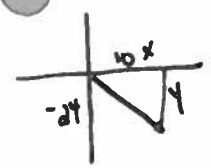
$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\quad \csc\theta = \frac{5}{3}$

$\cos\theta = \frac{x}{r} = \frac{-4}{5}$ $\quad \sec = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{-3}{4}$ $\quad \cot = \frac{-4}{3}$

sec → cos $\frac{x}{r}$
csc → sin $\frac{y}{r}$.

6. The point (10, –24) lies on the terminal arm of an angle θ in standard position. Determine the exact values of sec θ and csc θ.



$x = 10$, $y = -24$

$x^2 + y^2 = r^2$

$(10)^2 + (-24)^2 = r^2$

$r^2 = 676$

$r = 26$

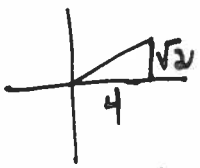
$\sec\theta = \frac{r}{x} = \frac{26}{10} = \frac{13}{5}$

$\csc\theta = \frac{r}{y} = \frac{26}{-24} = -\frac{13}{12}$.

7. Solve for the required ratios in each of the following. Express each answer as an exact value with a rational denominator.

a) If $\tan\theta = \frac{\sqrt{2}}{4}$, and angle θ terminates in the first quadrant, determine cot θ, csc θ, and sec θ.

rationalize denominator.



$\tan\theta = \frac{y}{x} = \frac{\sqrt{2}}{4}$

$x = 4$, $y = \sqrt{2}$

$x^2 + y^2 = r^2$

$(4)^2 + (\sqrt{2})^2 = r^2$

$16 + 2 = r^2$

$18 = r^2$

$r = \sqrt{18} = 3\sqrt{2}$

$\cot\theta = \frac{x}{y} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

$\csc\theta = \frac{r}{y} = \frac{3\sqrt{2}}{\sqrt{2}} = 3$

$\sec\theta = \frac{r}{x} = \frac{3\sqrt{2}}{4}$

b) If $\tan\theta = \frac{\sqrt{2}}{4}$, and angle θ terminates in the third quadrant, determine cot θ, csc θ, and sec θ.

$\tan\theta = \frac{y}{x} = \frac{\sqrt{2}}{4}$

$y = -\sqrt{2}$, $x = -4$

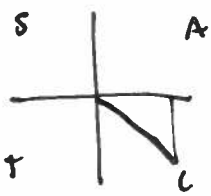
$r = 3\sqrt{2}$

$\cot\theta = \frac{x}{y} = \frac{-4}{-\sqrt{2}} \cdot \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

$\csc\theta = \frac{r}{y} = \frac{3\sqrt{2}}{\sqrt{2}} = -3$

$\sec\theta = \frac{r}{x} = \frac{3\sqrt{2}}{-4} = -\frac{3\sqrt{2}}{4}$.

8. If $\sin X = -\frac{1}{3}$ and cos X is positive, express cot X as an exact value.



$\sin X = \frac{y}{r} = \frac{-1}{3}$, $y = -1$, $r = 3$

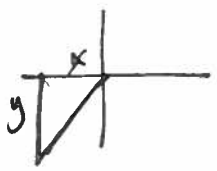
$x^2 + y^2 = r^2$

$x^2 + (-1)^2 = (3)^2$

$x^2 = 8$, $x = \sqrt{8} = 2\sqrt{2}$

$\cot X = \frac{x}{y} = \frac{2\sqrt{2}}{-1} = -2\sqrt{2}$.

9. Cos $A = -0.28$, where $\pi \le A \le \frac{3\pi}{2}$. Determine the exact value of csc A.



$\cos A = -0.28 = \frac{-7}{25}$

$\cos A = \frac{x}{r}$

$= \frac{-7}{25}$

$x^2 + y^2 = r^2$

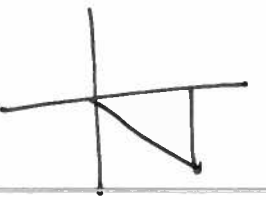
$(-7)^2 + y^2 = 25^2$

$49 + y^2 = 576$, $y = -24$

$\csc A = \frac{r}{y} = \frac{25}{-24}$

Multiple Choice

10. The point $\left(\frac{1}{5}, -\frac{1}{5}\right)$ lies on the terminal arm of an angle A in standard position. The exact value of sec A is



$x = \frac{1}{5}$; $y = \frac{-1}{5}$

$x^2 + y^2 = r^2$

$\left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^2 = r^2$

$r^2 = \frac{2}{25} = \frac{\sqrt{2}}{5}$

$\sec A = \frac{\sqrt{2}}{1/5} = \sqrt{2}$

A. $\sqrt{2}$ **B.** $-\sqrt{2}$

C. $\frac{\sqrt{2}}{25}$ **D.** $-\frac{\sqrt{2}}{25}$

Use the following information to answer the next question.

Consider the following trigonometric expressions.

I. $\cos(2\pi + x)$ **II.** $\cos(2\pi - x)$

III. $\cos(\pi - x)$ **IV.** $\cos(-x)$

11. If $\cos x = A$, which of the following is not equal to A?

A. III only
B. IV only
C. III and IV only
D. some other combination of I, II, III, and IV

I $\cos(2\pi + x) = \cos x = A$

II $\cos(2\pi - x) = \cos x = A$

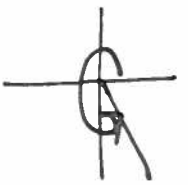
III $\cos(\pi - x) = -\cos x = -A$

IV $\cos(-x) = \cos x = A$

12. Without using technology, determine which of the following has a different sign from the others.

A. tan 201° quad 3 (+)
B. csc (−72°) quad 4 (−)
C. sec 115° quad 2 (−)
D. −cot 79° (quad 1) (−)

13. Without using technology, determine which of the following does not have the same value as cot 277°.



$\cot 277° = -\cot 83°$

A. cot (−83°)
B. cot (−263°)
C. −cot 263°
D. −cot 97°

A. $\cot(-83) = -\cot 83°$

B. $\cot(-263) = \cot 97° = -\cot 83°$

C. $-\cot 263° = -\cot 83°$

D. $-\cot 97° = -(-\cot 83°) = \cot 83°$

Numerical Response

Use the following information to answer the next question.

Angles A, B, and C are rotation angles with the following properties.

- $\csc A = \csc\frac{\pi}{4}$ where $0 \le A \le 2\pi, A \ne \frac{\pi}{4}$ → must be ⊕ but not $\frac{\pi}{4}$ so Q2
- $\cot B = \cot\frac{3\pi}{4}$ where $0 \le B \le 2\pi, B \ne \frac{3\pi}{4}$ → must be ⊖, but not $\frac{3\pi}{4}$ so Q4
- $\sec C = \sec\frac{8\pi}{5}$ where $0 \le C \le 2\pi, C \ne \frac{8\pi}{5}$ → must be ⊕, but not $\frac{8\pi}{5}$, so Q1

14. If the value of $A + B + C$ can be expressed in the form $k\pi$, then the value of k, to the nearest hundredth, is _____.

(Record your answer in the numerical response box from left to right.)

2	.	9	0

sin $\csc\frac{\pi}{4}$ ref ∠ $= \frac{\pi}{4}$ ⊕ in quad 1/2 → $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

tan $\cot\frac{3\pi}{4}$ ref ∠ $= \frac{\pi}{4}$ ⊖ in quad 2/4 → $2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

cos $\sec\frac{8\pi}{5}$ ref ∠ $= \frac{2\pi}{5}$ ⊕ in quad 1/4 → $\frac{2\pi}{5}$

$A + B + C$

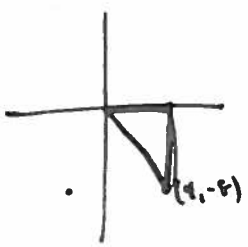
$= \frac{3\pi}{4} + \frac{7\pi}{4} + \frac{2\pi}{5} = \frac{29\pi}{10}$

$k = \frac{29}{10} = 2.9$

15. The point $(4, -8)$ lies on the terminal arm of an angle θ. If the value of $\sin\theta + \sec\theta$ can be expressed in the form $k\sqrt{5}$, then the value of k, to one decimal place, is _____.

(Record your answer in the numerical response box from left to right.)

0	.	6	



$x = 4 \quad y = -8$

$x^2 + y^2 = r^2$

$4^2 + (-8)^2 = r^2 \quad r = \sqrt{80} = 4\sqrt{5}$

$\sin\theta = \frac{y}{r} = \frac{-8}{4\sqrt{5}} = \frac{-2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$

$\sec\theta = \frac{r}{x} = \frac{4\sqrt{5}}{4} = \sqrt{5}$

$\sin\theta + \sec\theta = \frac{-2\sqrt{5}}{5} + 1\sqrt{5} = \frac{3}{5}\sqrt{5}$

$k = \frac{3}{5} = 0.6$

Group Investigation

The following problems could be used as a lead-in to the next lesson. Use the blank pages the back of the workbook to answer this group investigation.

a) Sketch an angle of $\frac{\pi}{3}$ in standard position with the point $P\left(1, \sqrt{3}\right)$ on the terminal arm Without using technology, explain and carry out a strategy to determine the exact trigonometric ratios of three different angles greater than $\frac{\pi}{2}$ and less than 2π.

b) Consider an angle A in standard position with $\sin A = -\frac{5}{13}$ and $0 \le A \le 2\pi$. Without using technology, explain and carry out a strategy to determine the exact values of $\cos A$ and $\tan A$.