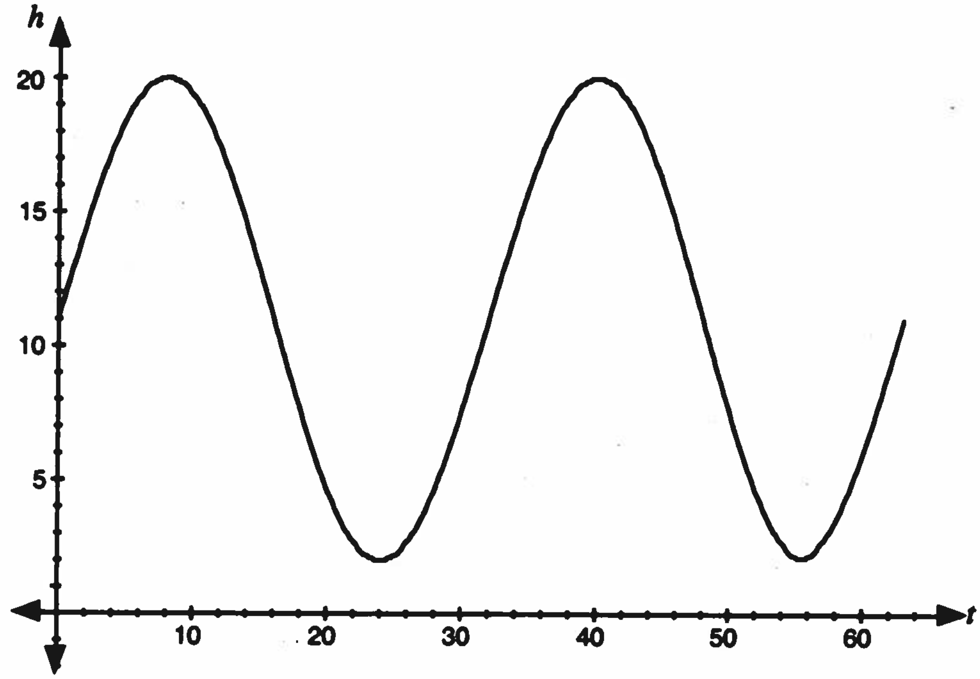
Assignment

1. The graph shows the height, h metres, above the ground over time, t, in seconds that it take a person in a chair on a Ferris Wheel to complete two revolutions. The minimum height o the Ferris Wheel is 2 metres and the maximum height is 20 metres.



a) How far above the ground is the person as the wheel starts rotating?

11 metres

b) If it takes 16 seconds for the person to return to the same height, determine the equatic of the graph in the form $h(t) = a \sin bt + d$.

amp $= \frac{20-2}{2} = 9$ m $a = 9$

period $= 2(16) = 32$ secs. $b = \frac{2\pi}{32} = \frac{\pi}{16}$

vert disp $= \frac{20+2}{2} = 11$m

$h(t) = 9 \sin \frac{\pi}{16} t + 11$

c) Determine the distance the person is from the ground, to the nearest tenth of a metre, after 30 seconds.

$9 \sin\left(\frac{\pi}{16} \cdot 30\right) + 11 = 7.55 = 7.6$ m

d) How long from the start of the ride does it take for the person to be at a height of 5 metres? Answer to the nearest tenth of a second.

$5 = 9 \sin \frac{\pi}{16} t + 11$

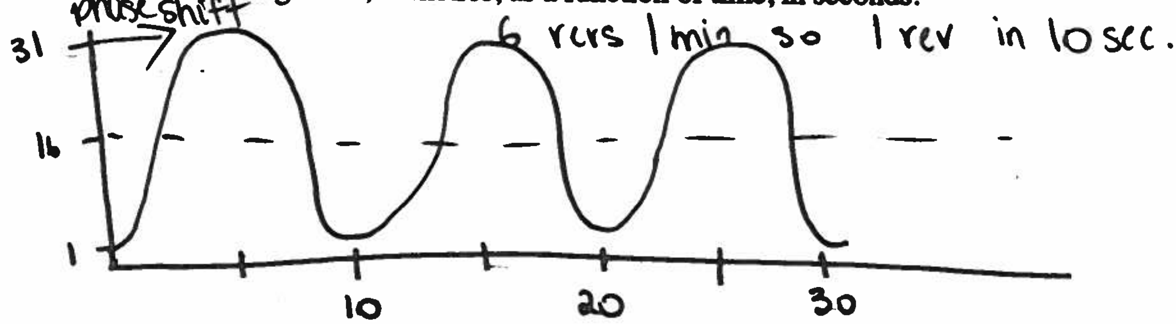
graph $y_1 = 9 \sin \frac{\pi}{16} t + 11$

$y_2 = 5$

intersect 19.7 sec

2. A Ferris Wheel ride can be represented by a sinusoidal function. A Ferris Wheel at Westworld Theme Park has a radius of 15 m and travels at a rate of six revolutions per minute in a clockwise rotation. Ling and Lucy board the ride at the bottom chair from a platform one metre above the ground.

a) Sketch three cycles of a sinusoidal graph to represent the height Ling and Lucy are above the ground, in metres, as a function of time, in seconds.



b) Determine the equation of the graph in the form $h(t) = a \cos [b(t - c)] + d$.

amp $= \frac{31-1}{2} = 15$ m $a = 15$

period $= 10$ sec $b = \frac{2\pi}{10} = \frac{\pi}{5}$

h.p.s $= 5$ sec R

vert disp $= \frac{31+1}{2} = 16$ m

$$h(t) = 15\cos\left(\frac{\pi}{5}(t-5)\right) + 16$$

c) If the Ferris Wheel does not stop, determine the height Ling and Lucy are above the ground after 28 seconds. Give answer to the nearest tenth of metre.

$h(28) = 15\cos\left(\frac{\pi}{5}(28-5)\right) + 16 = 11.4$ m.

d) How long after the wheel starts rotating do Ling and Lucy first reach 12 metres from the ground? Give answer to the nearest tenth of a second.

$y_1 = 15\cos\left(\frac{\pi}{5}(t-5)\right) + 16$ first point of intersection

$y_2 = 12$ $x = 2.1$ 2.1 secs

e) How long does it take from the first time Ling and Lucy reach 12 metres until they next reach 12 metres from the ground? Give answer to the nearest second.

second point of intersection $x = 7.9$

$7.9 - 2.1 = 5.8 = 6$ seconds

3. Consider the following information for a town in Saskatchewan for a leap year of 366 day

- The latest sunrise time is at 09:00 on December 21 (day 356).
- The earliest sunrise time is at 03:30 on June 21 (day 173).
- There is NO daylight saving time in Saskatchewan.
- The sunrise times vary sinusoidally with the day of the year.

a) Write a sinusoidal equation which relates the time of sunrise, t, to the day of the year, d

max: 9 @ $t = 356$ → given max + min it is easier to determine an equation in cosine.

min = 3.5 @ $t = 173$

amp $= \frac{9-3.5}{2} = 2.75$ hrs $a = 2.75$

period = 366 days $b = \frac{2\pi}{366} = \frac{\pi}{183}$

h.p.s = 356 days R or 10 days L $c = -10$

vert: disp $= \frac{9+3.5}{2} = 6.25$ hrs $= d$

$t = 2.75\cos\left(\frac{\pi}{183}(d+10)\right) + 6.25$

b) Use the equation to determine what time, to the nearest minute, the sun rises on March 11.

$31 + 29 + 11 = 71$

day 71

$t = 2.75\cos\left(\left(\frac{\pi}{183}\right)(71+10)\right) + 6.25$

$= 6.\underbrace{7430}_{\times 60}$

$= \underline{6:45}$

c) Determine the average time the sun rises throughout the year.

mid value = 6.25 hours time = $\underline{06:15}$

d) How many days of the year does the sun rise before 6 a.m.?

graph $y_1 = 2.75\cos\left(\frac{\pi}{183}(x+10)\right) + 6.25$

$y_2 = 6$

intersect at 86.8 + 259.2

days 87 — 259 inclusive

$= \underline{173 \text{ days}}$

Use the following information to answer the next question.

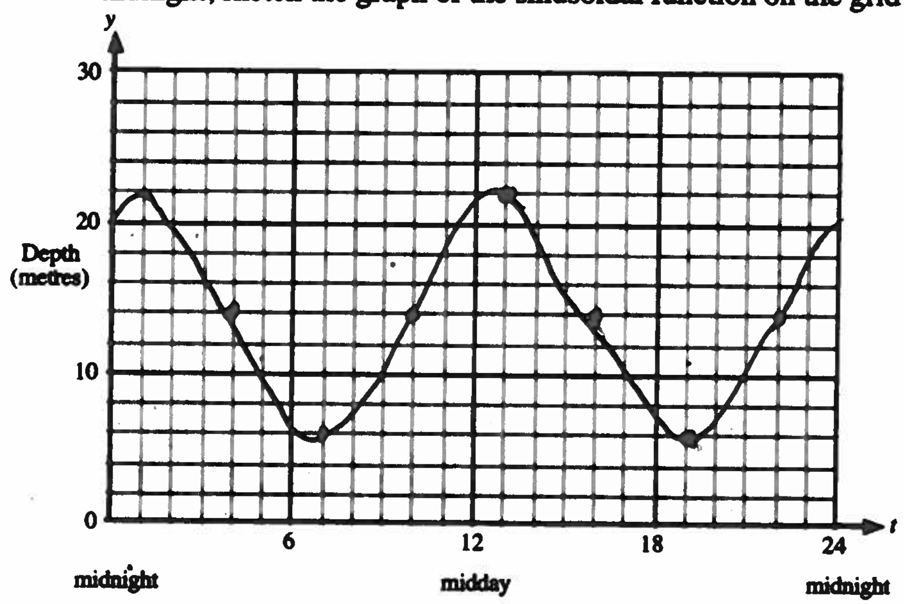
In Inverdeen harbour, the maximum depth of water is 22 metres at 1 a.m. and 1 p.m. as shown on the grid below.

The minimum depth of water is 6 metres at 7 a.m. and 7 p.m.

The depth is 14 metres at 4 a.m., 10 a.m., 4 p.m. and 10 p.m.

Assume that the relation between the depth of water, y metres, and the time, t hours, is a sinusoidal function.

4. a) If $t = 0$ at midnight, sketch the graph of the sinusoidal function on the grid below.



b) State the amplitude and period of the sinusoidal function. (Include units in the answers

amp $= \frac{22-6}{2} = 8$m   period $=$ 12 hrs.

c) Determine an equation of the sinusoidal function in the form

$y = a\sin[b(t-c)] + d$, where $a > 0$ and $c > 0$.

$a = 8$ $b = \frac{2\pi}{12} = \frac{\pi}{6}$

h.p.s = 10 hrs R , $c = 10$

vert. disp $= \frac{22+6}{2} = 14$m $d = 14$

$y = 8\sin\left(\frac{\pi}{6}(t-10)\right) + 14$

d) If the equation of the sinusoidal function is written in the form

$y = a\cos[b(t-c)] + d$, where $a > 0$ and $c > 0$,

only one of the parameters, a, b, c, d will be different from the values in c). State which parameter will be different and give its value.

→ the amplitude, period + vertical displacement do not change so a, b + d will be the same.

The horizontal phase shift change 1 hour to right so $c = 1$

e) Calculate the depth of the water, to the nearest tenth of a metre, at 3:30 pm.

$t = 3.5 \quad y = 8\sin\left(\frac{\pi}{6}(3.5-10)\right) + 14 = \underline{16.1}$

5. Andrea, a local gymnast, is doing timed bounces on a trampoline. The trampoline mat is 1 metre above ground level. When she bounces up, her feet reach a height of 3 metres above the mat, and when she bounces down her feet depress the mat by 0.5 metres. Once Andrea is in a rhythm, her coach uses a stopwatch to make the following readings:

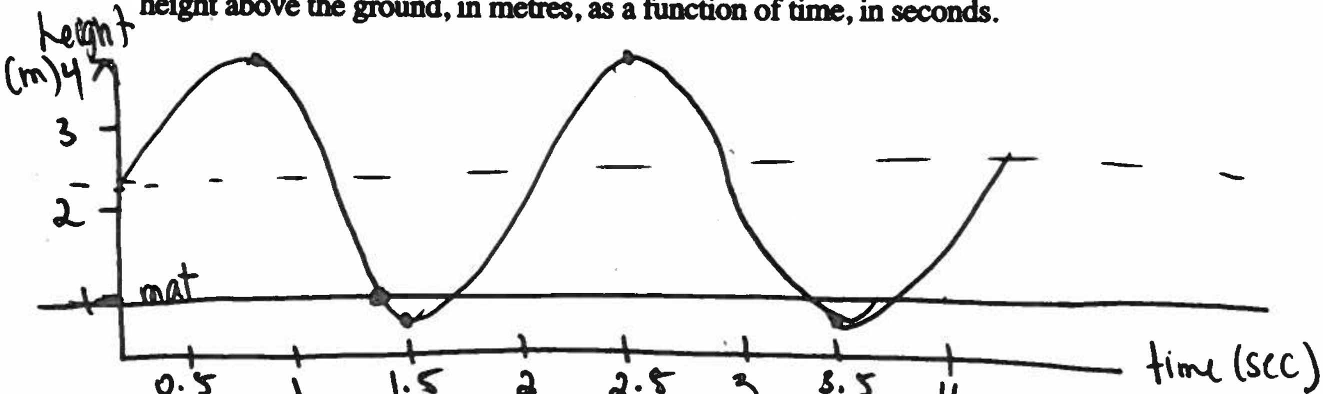
- At the highest point the reading is 0.5 seconds.
- At the lowest point the reading is 1.5 seconds.

a) Determine the maximum and minimum heights of Andrea's feet above the ground as she is bouncing on the trampoline.

max height $= 1 + 3 = 4$m

min height $= 1 - 0.5 = 0.5$m.

b) Sketch two periods of the graph of the sinusoidal function which represents Andrea's height above the ground, in metres, as a function of time, in seconds.



c) How high was Andrea above the mat when the coach started timing?

1 sec top to bottom

$\frac{1}{2}$ sec top to middle

→ she must start in middle.

mid height $= \frac{4 + 0.5}{2} = \underline{2.25\text{m}}$

$2.25 - 1 = 1.25$m above the mat

d) Determine the equation of the graph in the form $h(t) = a \sin bt + d$.

amp $= \frac{4-0.5}{2} = 1.75$m $a = 1.75$

period $= 2$sec $b = \frac{2\pi}{2} = \pi$

vert. disp $= 2.25$m $d = 2.25$

$h(t) = 1.75 \sin \pi t + 2.25$.

e) How high, to the nearest tenth of a metre, was Andrea above the ground after 2.7 seconds?

$t = 2.7$ $h(2.7) = 1.75 \sin \pi (2.7) + 2.25 = \underline{3.7\text{m}}$

f) Determine Andrea's exact height above the mat after 17 seconds.

$t = 17$ $h(17) = 1.75 \sin \pi (17) + 2.25$ $2.25 - 1 = \underline{1.25\text{m}}$

$= 2.25$

g) How long after the timing started did Andrea first touch the mat? Answer to the nearest tenth of a second.

graph $y_1 = 1.75 \sin \pi x + 2.25$

$y_2 = 1$

– 1st point of intersection

$\underline{1.3\text{seconds}}$

Answer Key

1. a) 11 metres **b)** $h(t) = 9 \sin\left(\frac{\pi}{16}t\right) + 11$ **c)** 7.6 metres **d)** 19.7 seconds

2. b) $h(t) = 15 \cos \frac{\pi}{5}(t - 5) + 16$ **c)** 11.4 metres **d)** 2.1 seconds **e)** 6 seconds

3. a) $t = 2.75 \cos\left(\frac{\pi}{183}(d + 10)\right) + 6.25$ **b)** 06:45 **c)** 06:15 **d)** 173

4. a) see graph below
b) amplitude = 8m, period = 12 hours
c) $y = 8 \sin\left(\frac{\pi}{6}(t - 10)\right) + 14$
d) $c = 1$ **e)** 16.1 m

5. a) max = 4 m, min = 0.5 m
b) see graph below
c) 1.25 metres **d)** $h(t) = 1.75 \sin \pi t + 2.25$
e) 3.7 metres **f)** 1.25 metres **g)** 1.3 seconds