

Class Ex. #5



Simplify $\frac{x^2 - 3x + 2}{x^2 - 5x + 4} - \frac{x^2 + 10x + 24}{x^2 + 8x + 12}$.

Class Ex. #6



Show that $\frac{2a + 7}{a^2 + 7a + 12} + \frac{2a}{9 - a^2}$ can be reduced to $\frac{-7}{(a + 4)(a - 3)}$.

Complete Assignment Questions #3 - #11

Assignment

1. Perform the indicated operations. Express final answers in lowest terms, and indicate the nonpermissible values.

a) $\frac{1}{a} - \frac{1}{6a}$

$$\frac{6}{6a} - \frac{1}{6a}$$

$$\frac{5}{6a}, a \neq 0$$

b) $\frac{2}{5x - 15} + \frac{3}{2x - 6}$

$$\frac{4}{10(x-3)} + \frac{15}{10(x-3)}$$

$$\frac{19}{10(x-3)}$$

$x \neq 3$

c) $\frac{3}{4x + 2} - \frac{1}{6x + 3}$

$$\frac{9}{6(2x+1)} - \frac{2}{6(2x+1)}$$

$$\frac{7}{6(2x+1)}, x \neq -\frac{1}{2}$$

$$\begin{aligned}
 \text{d) } & \frac{1}{x^2-3x} - \frac{1}{x(x-3)} = \frac{1 - (x-3)}{x(x-3)} = \frac{4-x}{x(x-3)}, x \neq 0, 3 \\
 \text{e) } & \frac{y(5)}{8-6y} + \frac{2y(2)}{20-15y} = \frac{5y+4y}{10(4-3y)} = \frac{9y}{10(4-3y)}, y \neq \frac{4}{3} \\
 \text{f) } & \frac{4}{b} - \frac{1}{b^3-b} = \frac{4b^2-4-1}{b(b-1)(b+1)} = \frac{4b^2-5}{b(b-1)(b+1)}, b \neq 0, \pm 1
 \end{aligned}$$

2. Perform the indicated operations. Express final answers in lowest terms, and indicate the nonpermissible values.

$$\text{a) } \frac{1(x-3)}{(x-1)(x-2)} - \frac{1(x-1)}{(x-2)(x-3)}$$

$$\begin{aligned}
 & \frac{x-3 - (x-1)}{(x-1)(x-2)(x-3)} \\
 & \frac{-2}{(x-1)(x-2)(x-3)}, x \neq 1, 2, 3
 \end{aligned}$$

$$\text{b) } \frac{4(a-3)}{a(a+4)} + \frac{3(a+4)}{a(a-3)}$$

$$\begin{aligned}
 & = \frac{4a-12+3a+12}{a(a+4)(a-3)} \\
 & = \frac{7a}{a(a+4)(a-3)} = \frac{7}{(a+4)(a-3)}, a \neq 0, -4, 3
 \end{aligned}$$

$$\text{c) } \frac{7(x-3)}{(x-2)(x+5)} - \frac{8(x-2)}{(x+5)(x-3)}$$

$$\begin{aligned}
 & \frac{7x-21 - (8x-16)}{(x-2)(x+5)(x-3)} \\
 & = \frac{-1x-5}{(x-2)(x+5)(x-3)} \\
 & = \frac{-1(x+5)}{(x-2)(x+5)(x-3)} \\
 & = \frac{-1}{(x-2)(x-3)}, x \neq -5, 2, 3
 \end{aligned}$$

$$\text{d) } \frac{2(x+2)}{x(x-1)(x+1)} - \frac{1(x+1)}{x(x-1)(x+2)}$$

$$\begin{aligned}
 & \frac{2x+4 - (x+1)}{x(x-1)(x+1)(x+2)} \\
 & = \frac{x+3}{x(x-1)(x+1)(x+2)}, x \neq -2, -1, 0, 1
 \end{aligned}$$

3. Simplify.

$$\begin{aligned}
 \text{a) } & \frac{1}{x^2+2x+1} - \frac{1(x+1)}{x+1} \\
 & \frac{1}{(x+1)(x+1)} - \frac{1(x+1)}{(x+1)(x+1)} \\
 & = \frac{1 - (x+1)}{(x+1)(x+1)} \\
 & = \frac{-x}{(x+1)(x+1)}, \quad x \neq -1
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \frac{1}{y+2} - \frac{1}{y^2-4} \\
 & \frac{1}{y+2} - \frac{1}{(y-2)(y+2)} \\
 & = \frac{y-2 - 1}{(y-2)(y+2)} \\
 & = \frac{y-3}{(y-2)(y+2)}, \quad y \neq \pm 2
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{2}{t^2-1} + \frac{1}{t+1}(t-1) \\
 & \frac{2}{(t-1)(t+1)} + \frac{1}{t+1}(t-1) \\
 & = \frac{2 + (t-1)}{(t-1)(t+1)} \\
 & = \frac{\cancel{t} + 1}{(t-1)(\cancel{t+1})} \\
 & = \frac{1}{t-1}, \quad t \neq \pm 1
 \end{aligned}$$

4. Perform the indicated operations. Express final answers in lowest terms, and indicate the nonpermissible values.

$$\begin{aligned}
 \text{a) } & \frac{1(x+3)}{x^2-x-2} - \frac{1(x-2)}{x^2+4x+3} \\
 & \frac{1(x+3)}{(x-2)(x+1)} - \frac{1(x-2)}{(x+3)(x+1)} \\
 & = \frac{x+3 - (x-2)}{(x-2)(x+1)(x+3)} \\
 & = \frac{5}{(x-2)(x+1)(x+3)}, \quad x \neq 2, -1, -3
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \frac{3(t-4)}{t^2-7t+10} - \frac{2(t-5)}{t^2-6t+8} \\
 & \frac{3(t-4)}{(t-5)(t-2)} - \frac{2(t-5)}{(t-4)(t-2)} \\
 & = \frac{3t-12 - (2t-10)}{(t-5)(t-2)(t-4)} \\
 & = \frac{(t-2)}{(t-5)(\cancel{t-2})(t-4)} \\
 & = \frac{1}{(t-5)(t-4)}, \quad t \neq 2, 4, 5
 \end{aligned}$$

$$c) \frac{2x(x+1)}{x^2-3x-88} - \frac{2x-1(x+8)}{x^2-10x-11} \quad d) \frac{12y(y-3)}{y^2-8y-20} - \frac{7y(y+2)}{y^2-13y+30}$$

$$= \frac{2x^2+2x - (2x^2+15x-8)}{(x-11)(x+8)(x+1)}$$

$$= \frac{-13x+8}{(x-11)(x+8)(x+1)}$$

$$x \neq -8, -1, 11$$

$$= \frac{12y^2-36y - (7y^2+14y)}{(y-10)(y+2)(y-3)}$$

$$= \frac{5y^2-50y}{(y-10)(y+2)(y-3)} = \frac{5y(y-10)}{(y-10)(y+2)(y-3)}$$

$$= \frac{5y}{(y+2)(y-3)}, y \neq -2, 3$$

5. Simplify, stating the nonpermissible values.

$$a) \frac{(2x+3)(x-4)}{5x-25} + \frac{(x-4)(5)}{20-9x+x^2}$$

$$\frac{2x^2-5x-12 + (5x-20)}{5(x-5)(x-4)}$$

$$= \frac{2x^2-32}{5(x-5)(x-4)} = \frac{2(x^2-16)}{5(x-4)(x-4)}$$

$$= \frac{2(x-4)(x+4)}{5(x-5)(x-4)}$$

$$= \frac{2(x+4)}{5(x-5)}, x \neq 5, 4$$

$$b) \frac{-4x(x-3)}{2x^2-5x-3} - \frac{(-2x+1)}{1-2x} \cdot \frac{(2x+1)}{(2x+1)}$$

$$\frac{-4x^2+12x + (-4x^2+1)}{(x-3)(x+3)(2x+1)}$$

$$= \frac{12x-1}{(x-3)(x+3)(2x+1)}$$

$$x \neq \pm 3, -\frac{1}{2}$$

$$\begin{array}{r} 1x-3 \\ 2x \overline{) 2x^2} \\ \underline{1} \\ -3 \end{array}$$

$$\begin{array}{r} (-2x+1)(2x+1) \\ -4x^2-2x+2x+1 \\ \hline (3-x) \\ -x+3 \\ \hline -(x-3) \end{array}$$

6. Simplify, stating the restrictions on x .

a) $\frac{x^2 - x - 12}{x^2 - 8x + 16} - \frac{x^2 + 5x - 14}{x^2 + 10x + 21}$

$$\frac{(x-4)(x+3)}{(x-4)(x-4)} - \frac{(x+7)(x-2)}{(x+3)(x+7)}$$

$$\frac{(x+3)(x+3)}{x-4} - \frac{(x-2)(x-4)}{x+3}$$

$$\frac{x^2 + 6x + 9 - (x^2 - 6x + 8)}{(x-4)(x+3)}$$

$$= \frac{12x + 1}{(x-4)(x+3)}, x \neq -7, -3, 4$$

b) $\frac{2x^2 - x - 3}{2x^2 + 7x - 15} + \frac{x^2 + 16x + 63}{x^2 + 12x + 35}$

$$\frac{(x+1)(2x-3)}{(x+5)(2x-3)} + \frac{(x+9)(x+7)}{(x+5)(x+7)}$$

$$= \frac{x+1 + (x+9)}{x+5}$$

$$= \frac{2x+10}{x+5} = \frac{2(x+5)}{x+5}$$

$$= 2, x \neq -7, -5, \frac{3}{2}$$

$$\begin{array}{r|l} 1x + 1 \\ 2x \quad 2x^2 \quad 2 \\ -3 \quad -3 \quad -3 \end{array}$$

$$\begin{array}{r|l} 1x + 5 \\ 2x \quad 2x^2 \quad 10x \\ -3 \quad -3 \quad -15 \end{array}$$

c) $\frac{x^2 - 9}{x^2 - x - 12} - \frac{x^2 - 5x - 14}{x^2 - 4x - 21}$

$$\frac{(x-3)(x+3)}{(x-4)(x+3)} - \frac{(x-7)(x+2)}{(x-7)(x+3)}$$

$$= \frac{(x-3)(x+3)}{(x-4)(x+3)} - \frac{(x+2)(x-4)}{(x+3)}$$

$$= \frac{x^2 - 9 - (x^2 - 2x - 8)}{(x-4)(x+3)}$$

$$= \frac{2x - 1}{(x-4)(x+3)}, x \neq -3, 4, 7$$

d) $\frac{4x^2 - 4x - 3}{4x^2 - 1} - \frac{x^2 - 4x - 96}{x^2 + 4x - 32}$

$$\frac{(2x-3)(2x+1)}{(2x+1)(2x-1)} - \frac{(x-12)(x+8)}{(x+8)(x-4)}$$

$$= \frac{(2x-3)(x-4)}{(2x-1)(x-4)} - \frac{(x-12)(2x-1)}{(x-4)}$$

$$= \frac{(2x^2 - 11x + 12) - (2x^2 - 25x + 12)}{(2x-1)(x-4)}$$

$$= \frac{14x}{(2x-1)(x-4)}, x \neq \pm \frac{1}{2}, -8, 4$$

$$\begin{array}{r|l} 2x - 3 \\ 4x \quad 4x^2 \quad -6 \\ 1 \quad 2 \quad -3 \end{array}$$

7. Simplify.

a) $\frac{2(2a-1)}{2a+3} + \frac{8}{4a^2+4a-3}$

$4a^2 - 2a + 6a - 3$
 $2a(2a-1) + 3(2a-1)$

$\frac{4a-2+8}{(2a+3)(2a+1)}$

$\frac{4a+6}{(2a+3)(2a+1)}$

$\frac{2(2a+3)}{(2a+3)(2a+1)}$

$\frac{2}{2a+1}, a \neq -\frac{3}{2}, \frac{1}{2}$

b) $\frac{2(3b+4)}{6b^2-5b-4} - \frac{3(2b+1)}{9b^2-16}$

$6b^2 - 8b + 3b - 4$
 $2b(3b-4) + 1(3b-4)$

$\frac{6b^2-5b-4}{(3b-4)(2b+1)} - \frac{9b^2-16}{(3b-4)(3b+4)}$

$\frac{6b^2+8-6b-3}{(3b-4)(2b+1)(3b+4)}$

$= \frac{5}{(3b-4)(2b+1)(3b+4)}, b \neq \pm \frac{4}{3}, -\frac{1}{2}$

8. A helicopter left Calgary and travelled 135 km west into the Rocky Mountains at an average speed of $2x^2 + 3x$ km/h. The return journey was at an average speed of $4x^2 - 9$ km/h.

a) Write and simplify an expression for the total flying time in hours.

$s = \frac{d}{t} \quad t = \frac{d}{s}$

total time = $\frac{135(2x-3)}{2x^2+3x} + \frac{135(x)}{4x^2-9}$

$\frac{135(2x-3)}{x(2x+3)} + \frac{135(x)}{(2x+3)(2x-3)}$

$= \frac{270x - 405 + 135x}{x(2x+3)(2x-3)} = \frac{405x - 405}{x(2x+3)(2x-3)} = \frac{405(x-1)}{x(2x+3)(2x-3)}$

$x \neq 0, \pm \frac{3}{2}$

b) If the value of x is 6, determine the total flying time.

$= \frac{405(6-1)}{6(2(6)+3)(2(6)-3)} = \frac{2025}{810} = 2.5 \text{ hours}$

Multiple Choice

9. For all $x \neq \pm 6$, the sum $\frac{3}{x^2 - 36} + \frac{2(x+6)}{x-6}$ is equal to

A. $\frac{5x + 12}{x^2 - 36}$

B. $\frac{5x - 12}{(x - 6)^2}$

C. $\frac{2x - 9}{x^2 - 36}$

D. $\frac{2x + 15}{x^2 - 36}$

$$\frac{3}{x^2 - 36} + \frac{2(x+6)}{x-6} = \frac{3 + 2x + 12}{(x-6)(x+6)} = \frac{2x + 15}{(x-6)(x+6)}$$

10. A simplified form of $\frac{-3}{x-7} - \frac{5}{7-x}$, $x \neq 7$, is

A. $\frac{8}{x-7}$

B. $\frac{-2}{x-7}$

C. $\frac{8}{7-x}$

D. $\frac{-2}{7-x}$

$$\frac{-3-5}{-(x-7)} = \frac{-8}{-(x-7)} = \frac{8}{x-7}$$

Numerical Response

11. When simplified, the difference $\frac{5}{x^2 - 7x + 12} - \frac{3}{x^2 - x - 12}$ can be written in the form $\frac{Ax + B}{(x - 4)(x - 3)(x + 3)}$, where A and B are integers.

The value of $B - A$ is _____.

(Record your answer in the numerical response box from left to right.)

2	2	
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$$\frac{5(x+3)}{(x-3)(x-4)} - \frac{3(x-3)}{(x-4)(x+3)}$$

$$\frac{5x+15 - 3x+9}{(x-3)(x+3)(x-4)}$$

$$\frac{2x+24}{(x-3)(x+3)(x-4)}$$

$B - A$
 $24 - 2 = 22$