Assignment

1. Reduce to lowest terms, stating the restrictions on the variable.

a) $\dfrac{5a^3 - 15a^2}{30a}$

$\dfrac{5a^2(a-3)}{30a}$

$\dfrac{a(a-3)}{6}, \; a \neq 0$

b) $\dfrac{7x}{7x - 21}$

$\dfrac{7x}{7(x-3)}$

$\dfrac{x}{x-3}, \; x \neq 3$

c) $\dfrac{6a - 3}{8a - 4}$

$\dfrac{3(2a-1)}{4(2a-1)} = \dfrac{3}{4}, \; a \neq \dfrac{1}{2}$

d) $-\dfrac{4a - 12}{a - 3}$

$\dfrac{-4(a-3)}{a-3}$

$= -4, \; a \neq 3$

e) $-\dfrac{a^2}{a^2 + a}$

$\dfrac{-a^2}{a(a+1)}$

$= \dfrac{-a}{a+1}, \; a \neq 0, -1$

f) $\dfrac{3t^2 - 75}{(t + 3)(t - 5)} = 3(t^2 - 25)$

$\dfrac{3(t-5)(t+5)}{(t+3)(t-5)}$

$= \dfrac{3(t+5)}{t+3}, \; t \neq -3, 5$

g) $\dfrac{2 - r}{r - 2} = \dfrac{-r+2}{r-2}$

$\dfrac{-1(r-2)}{r-2}$

$= -1, \; r \neq 2$

h) $-\dfrac{9a^2 - 1}{1 - 3a} \; (-3a+1)$

$\dfrac{-(3a-1)(3a+1)}{-1(3a-1)}$

$= 3a+1, \; a \neq \dfrac{1}{3}$

i) $\dfrac{2b^2 - 18b}{b(b - 9)^2}$

$\dfrac{2b(b-9)}{b(b-9)(b-9)}$

$= \dfrac{2}{b-9}, \; b \neq 0, 9$

2. Express in simplest form, stating the values of the variable for which the expression is not defined.

a) $\dfrac{t^2 + 4t + 4}{2t^2 + 10t + 12}$

$\dfrac{(t+2)(t+2)}{2(t^2 + 5t + 6)}$

$\dfrac{(t+2)(t+2)}{2(t+2)(t+3)}$

$\dfrac{t+2}{2(t+3)}, \; t \neq -2, -3$

b) $\dfrac{2x^2 + 5x - 3}{4x - 2}$

$\begin{array}{c|c} x & + \\ -6 & 5 \\ 6 & -1 \end{array}$

$2x^2 - 1x + 6x - 3$

$x(2x-1) + 3(2x-1)$

$\dfrac{(x+3)(2x-1)}{2(2x-1)}$

$= \dfrac{x+3}{2}, \; x \neq \dfrac{1}{2}$

c) $\dfrac{2y^2 - 3y - 2}{2y^2 - y - 6}$

$\begin{array}{c|c} y & +1 \\ -2 & 2y^2 \\ & -2 \end{array}$

$\begin{array}{c|c} & -2 \\ 2y & 2y^2 - 4y \\ +3 & +3 & -6 \end{array}$

$\dfrac{(2y+1)(y-2)}{(2y+3)(y-2)}$

$\dfrac{(2y+1)}{(2y+3)}, \; y \neq \dfrac{-3}{2}, 2$

3. Express in simplest form, stating the values of the variable for which the expression is ✱ $Ex \partial$.
not defined.

(circled) **a)** $\dfrac{3t^2 - 5t - 12}{2t^2 - 6t}$

b) $\dfrac{32 - 2a^2}{2a^2 + 4a - 16}$

c) $\dfrac{4 - 4x^2}{8x^3 + 8x^2 - 16x}$

Handwritten work:

Left margin:
$\dfrac{t-3}{3t^2-9t}$
$\begin{array}{c|c} & t-3 \\ \hline t & 3t^2-9t \\ 4 & 4t-12 \end{array}$

a) $\dfrac{(3t+4)(t-3)}{2t(t-3)}$

$p2$ $\dfrac{3t+4}{2t}, \quad t \neq 0, 3$

b) $-2a^2+32$
$-\dfrac{2(a^2+2a-8)}{}$
$\dfrac{-2(a^2-16)}{2(a^2+2a-8)}$

$\dfrac{-2(a-4)(a+4)}{2(a+4)(a-2)}$

$\dfrac{-(a-4)}{(a-2)}, \quad a \neq -4, 2$

c) $4-4x^2 \to -4x^2+4$

$\dfrac{-4(x^2-1)}{8x(x^2+x-2)} \quad \dfrac{-4(x-1)(x+1)}{8x(x+2)(x-1)}$

$= \dfrac{-(x+1)}{2(x+2)}, \quad x \neq -2, 0, 1$

or

$\dfrac{(1-x)}{2(x+2)}, \quad x \neq -2, 0, 1$

4. Express in simplest form, stating the restrictions on the variable

a) $\dfrac{2-x-x^2}{x^4 - 5x^2 + 4} \to -1(x^2+x-2)$
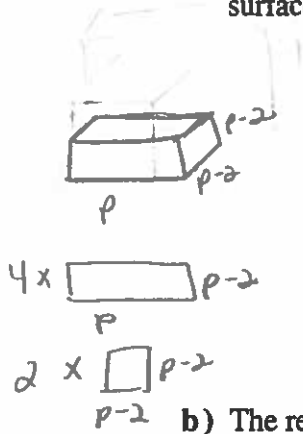
Handwritten:
$\dfrac{-1(x+2)(x-1)}{(x^2-4)(x^2-1)}$

$\dfrac{-1(x+2)(x-1)}{(x-2)(x+2)(x-1)(x+1)}$

$\dfrac{-1}{(x-2)(x+1)}, \quad x \neq \pm 2, \pm 1$

b) $\dfrac{16x^4 - y^4}{8x^3 + 4x^2y + 2xy^2 + y^3} = \dfrac{(4x^2-y^2)(4x^2+y^2)}{4x^2(2x+y)+y^2(2x+y)}$

$= \dfrac{(2x-y)(2x+y)(4x^2+y^2)}{(4x^2+y^2)(2x+y)}$

$= 2x-y, \quad x \neq -\dfrac{y}{2}$

5. A rectangular prism has a length of p cm and a width and height that are each 2 cm less than the length.

a) Write an expression in terms of p for the surface area of the prism, and express the surface area in simplest factored form.

$$SA = 4p(p-2) + 2(p-2)(p-2)$$
$$\underset{p^2 - 4p + 4}{}$$
$$= 4p^2 - 8p + 2p^2 - 8p + 8$$
$$= 6p^2 - 16p + 8$$
$$= 2(3p^2 - 8p + 4)$$

$$\to 2(3p^2 - 6p - 2p + 4)$$
$$2(3p(p-2) - 2(p-2))$$
$$= \boxed{2(3p-2)(p-2)}$$

$x \to 2(3p^2 - 8p + 4)$
$12 | -8 \quad -6, -2$

b) The rectangular prism has two square faces. Write an expression in simplest factored form which represents the total length of the edges which make up these two squares.
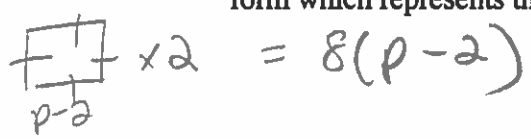
$\boxed{} \times 2 = 8(p-2)$

c) If the ratio of the surface area in a) to the edge length in b) is 10:1, find the volume of the prism.

$$\frac{2(3p-2)(p-2)}{8(p-2)} = \frac{10}{1}$$

$$\frac{3p-2}{4} = 10$$

$$3p - 2 = 40$$
$$3p = 42$$
$$p = 14$$

$$V = lwh$$
$$= p(p-2)(p-2) = 14(12)(12) = 2016 \text{ cm}^3$$

6. Simplify, where possible, and state the nonpermissible values for x in terms of y.

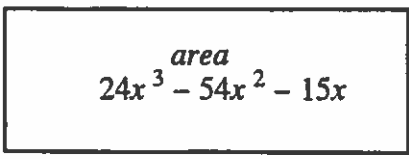**a)** $\dfrac{x+y}{4x+4y}$

$$\frac{x+y}{4(x+y)}$$
$$\frac{1}{4}, \quad x \neq -y$$

b) $\dfrac{4x^2 - y^2}{y - 2x} \quad -2x+y$

$$\frac{(2x-y)(2x+y)}{-(2x-y)}$$
$$-(2x+y), \quad x \neq \frac{y}{2}$$
$$\downarrow$$
$$\frac{1}{2}y$$

c) $\dfrac{3x + 12y}{x^2 - 16y^2}$

$$\frac{3(x+4y)}{(x-4y)(x+4y)}$$
$$\frac{3}{(x-4y)}, \quad x \neq \pm 4y$$

7. Consider the rectangle shown.

a) Write and simplify an expression for the length of the rectangle. $\ell = \dfrac{A}{w}$

area $24x^3 - 54x^2 - 15x$	$6x^2 - 15x$

$$\frac{24x^3 - 54x^2 - 15x}{6x^2 - 15x}$$

$$\frac{3x(8x^2 - 18x - 5)}{3x(2x - 5)} = \frac{3x(2x - 5)(4x+1)}{3x(2x - 5)}$$

$\text{width} = 4x + 1$

$$\begin{array}{r|rr} & 2x & -5 \\ \hline 4x & 8x^2 & -20 \\ 1 & +2 & -5 \end{array}$$

b) Determine the perimeter of the rectangle if $x = 2\sqrt{2}$ cm. Give the answer in simplest radical form.

$$P = 2\ell + 2w$$

$$= 2(6x^2 - 15x) + 2(4x+1)$$

$$= 12x^2 - 30x + 8x + 2$$

$$= 12x^2 - 22x + 2$$

$$= 12\underset{4(2)}{(2\sqrt{2})^2} - 22(2\sqrt{2}) + 2$$

$$=$$

$$= 96 - 44\sqrt{2} + 2 \quad = 98 - 44\sqrt{2} \text{ cm}$$

8. Write a rational expression in x with a numerator of 1 and a denominator written as an integral polynomial so that the nonpermissible values are

a) $x \neq 2, 3$ **b)** $x \neq -2, 0$ **c)** $x \neq -\dfrac{3}{4}, \dfrac{1}{3}$ **d)** $x \neq \pm 2, 0$

$$= \frac{1}{(x-2)(x-3)} \qquad \frac{1}{x(x+2)} \qquad \frac{1}{(4x+3)(3x-1)} \qquad \frac{1}{x(x-2)(x+2)} \quad x^2 - 4$$

$$= \frac{1}{x^2 - 5x + 6} \qquad = \frac{1}{x^2 + 2x} \qquad = \frac{1}{12x^2 + 5x - 3} \qquad = \frac{1}{x^3 - 4x}$$

Multiple Choice 9. With appropriate restrictions, the simplified form of $\dfrac{x^2 - 121}{3x^2 + 29x - 44}$ is

$3x \overline{\smash)\begin{array}{l} 3x^2 \\ -4 \end{array}}$ ... $\begin{array}{l} 1x \quad 11 \\ 33 \\ -4 \quad -44 \end{array}$

A. $\dfrac{x-11}{3x-4}$

$\dfrac{(x-11)(x+11)}{(x+11)(3x-4)}$

B. $\dfrac{x-11}{3x+4}$

C. $\dfrac{x+11}{3x-4}$

D. $\dfrac{x+11}{3x+4}$

Numerical Response 10. If the rational expression $\dfrac{8a^2 + 22a + k}{2a^2 - 11a - 21}$ where k is a constant, reduces to $\dfrac{4a + 5}{a - 7}$, then the value of k, to the nearest whole number, is _____.

(Record your answer in the numerical response box from left to right.)

$\boxed{1\,5\,\square\,\square}$

$\dfrac{8a^2 + 22a + k}{(a-7)(2a+3)} = \dfrac{(2a+3)(4a+5)}{(2a+3)(a-7)}$

$2a \overline{\smash)\begin{array}{l} 11a - 7 \\ 2a^2 - 14 \\ 3 \quad 3 \quad -21 \end{array}}$

$= 8a^2 + 10a + 12a \boxed{+15}$

$k = 15$