

**Assignment**

\* multiply by conjugate

1. Simplify by rationalizing the denominator.

$$\text{a) } \frac{4}{(\sqrt{5}-1)(\sqrt{5}+1)} \cdot \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$$

$$\frac{4\sqrt{5}+4}{5-1} = \frac{4\sqrt{5}+4}{4}$$

$$= \sqrt{5}+1$$

$$\text{b) } \frac{1}{\sqrt{6}+2} \cdot \frac{(\sqrt{6}-2)}{(\sqrt{6}-2)}$$

$$\frac{\sqrt{6}-2}{6-4} = \frac{\sqrt{6}-2}{2}$$

$$\text{c) } \frac{3}{3-\sqrt{3}} \cdot \frac{(3+\sqrt{3})}{(3+\sqrt{3})}$$

$$\frac{9+3\sqrt{3}}{9-3} = \frac{9+3\sqrt{3}}{6}$$

$$= \frac{3+\sqrt{3}}{2}$$

$$\text{d) } \frac{\sqrt{7}}{\sqrt{7}-2} \cdot \frac{(\sqrt{7}+2)}{(\sqrt{7}+2)}$$

$$\frac{7+2\sqrt{7}}{7-4} = \frac{7+2\sqrt{7}}{3}$$

$$\text{e) } \frac{3}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} \cdot \frac{(\sqrt{2}+\sqrt{3})}{(\sqrt{2}+\sqrt{3})}$$

$$\frac{3\sqrt{2}+3\sqrt{3}}{2-3} = \frac{3\sqrt{2}+3\sqrt{3}}{-1}$$

$$= -3\sqrt{2}-3\sqrt{3}$$

$$\text{f) } \frac{\sqrt{2}}{\sqrt{6}+\sqrt{2}} \cdot \frac{(\sqrt{6}-\sqrt{2})}{(\sqrt{6}-\sqrt{2})}$$

$$\frac{\sqrt{12}-2}{6-2} = \frac{2\sqrt{3}-2}{4}$$

$$= \frac{\sqrt{3}-1}{2}$$

2. Simplify by rationalizing the denominator.

$$\text{a) } \frac{2\sqrt{3}}{(3\sqrt{2}+\sqrt{3})(3\sqrt{2}-\sqrt{3})} \cdot \frac{(3\sqrt{2}-\sqrt{3})}{(3\sqrt{2}-\sqrt{3})}$$

$$\frac{6\sqrt{6}-2(3)}{9(2)-3} = \frac{6\sqrt{6}-6}{15}$$

$$= \frac{2\sqrt{6}-2}{5}$$

$$\text{b) } \frac{3\sqrt{11}}{(3\sqrt{11}+10)(3\sqrt{11}-10)} \cdot \frac{(3\sqrt{11}-10)}{(3\sqrt{11}-10)}$$

$$= \frac{9(11)-30\sqrt{11}}{9(11)-100} = \frac{99-30\sqrt{11}}{-1}$$

$$= -99+30\sqrt{11}$$

$$\text{c) } \frac{\sqrt{2}}{(\sqrt{12}-\sqrt{8})(\sqrt{2}+\sqrt{8})} \cdot \frac{(\sqrt{2}+\sqrt{8})}{(\sqrt{2}+\sqrt{8})}$$

$$\frac{\sqrt{24}+\sqrt{16}}{12-8} = \frac{2\sqrt{6}+4}{4}$$

$$= \frac{\sqrt{6}+2}{2}$$

$$\text{d) } \frac{\sqrt{7}}{(4-\sqrt{14})(4+\sqrt{14})} \cdot \frac{(4+\sqrt{14})}{(4+\sqrt{14})}$$

$$= \frac{4\sqrt{7}+\sqrt{98}}{16-14} = \frac{4\sqrt{7}+7\sqrt{2}}{2}$$

3. Simplify, leaving an integer in the denominator.

$$\begin{aligned} \text{a) } & \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} \\ & \frac{3-\sqrt{3}-\sqrt{3}+1}{3-1} \\ & = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{(\sqrt{6}+\sqrt{2})(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})} \\ & \frac{6+\sqrt{12}+\sqrt{12}+2}{6-2} \\ & = \frac{8+2\sqrt{12}}{4} = \frac{8+4\sqrt{3}}{4} \\ & = 2+\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{(\sqrt{5}-2)(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)} \\ & \frac{5+\sqrt{5}-2\sqrt{5}-2}{5-1} = \frac{3-\sqrt{5}}{4} \end{aligned}$$

$$\begin{aligned} \text{d) } & \frac{(5-\sqrt{10})(3-\sqrt{10})}{(3+\sqrt{10})(3-\sqrt{10})} \\ & \frac{15-5\sqrt{10}-3\sqrt{10}+10}{9-10} = \frac{25-8\sqrt{10}}{-1} \\ & = -25+8\sqrt{10} \\ & \quad \text{or} \\ & \quad 8\sqrt{10}-25 \end{aligned}$$

4. Simplify, leaving a whole number in the denominator.

$$\begin{aligned} \text{a) } & \frac{(\sqrt{11}+5\sqrt{2})(\sqrt{11}+2\sqrt{2})}{(\sqrt{11}-2\sqrt{2})(\sqrt{11}+2\sqrt{2})} \\ & \frac{11+2\sqrt{22}+5\sqrt{22}+10(2)}{11-4(2)} \\ & = \frac{31+7\sqrt{22}}{3} \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{(\sqrt{30}+3\sqrt{3})(\sqrt{30}+3\sqrt{3})}{(\sqrt{30}-3\sqrt{3})(\sqrt{30}+3\sqrt{3})} \\ & = \frac{30+3\sqrt{90}+3\sqrt{90}+9(3)}{30-9(3)} \\ & = \frac{57+6\sqrt{90}}{3} = \frac{57+18\sqrt{10}}{3} \\ & = 19+6\sqrt{10} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{(2\sqrt{6}-\sqrt{3})(3\sqrt{3}-\sqrt{6})}{(3\sqrt{3}+\sqrt{6})(3\sqrt{3}-\sqrt{6})} \\ & \frac{6\sqrt{18}-2(6)-3(3)+\sqrt{18}}{9(3)-6} \\ & \frac{7\sqrt{18}-21}{21} = \frac{2\sqrt{2}-21}{21} \\ & = \sqrt{2}-1 \end{aligned}$$

$$\begin{aligned} \text{d) } & \frac{(3\sqrt{5}-2\sqrt{3})(3\sqrt{5}-2\sqrt{3})}{(3\sqrt{5}+2\sqrt{3})(3\sqrt{5}-2\sqrt{3})} \\ & \frac{9(5)-6\sqrt{15}-6\sqrt{15}+4(3)}{9(5)-4(3)} \\ & = \frac{57-12\sqrt{15}}{33} = \frac{19-4\sqrt{15}}{11} \end{aligned}$$

5. Simplify by rationalizing the denominator.

$$\text{a) } \frac{3}{(2\sqrt{x} + 3)(2\sqrt{x} - 3)} = \frac{6\sqrt{x} - 9}{4x - 9}$$

$$\text{b) } \frac{(x + \sqrt{10})(x + \sqrt{10})}{(x - \sqrt{10})(x + \sqrt{10})} = \frac{x^2 + x\sqrt{10} + x\sqrt{10} + 10}{x^2 - 10} = \frac{x^2 + 2x\sqrt{10} + 10}{x^2 - 10}$$

$$\text{c) } \frac{(\sqrt{k} + \sqrt{2})(\sqrt{k} + \sqrt{2})}{(\sqrt{k} - \sqrt{2})(\sqrt{k} + \sqrt{2})} = \frac{k + \sqrt{2k} + \sqrt{2k} + 2}{k - 2} = \frac{k + 2\sqrt{2k} + 2}{k - 2}$$

6. The area of a rectangle is  $5 \text{ m}^2$  and the length is  $3 + \sqrt{3} \text{ m}$ . Calculate the width of the rectangle, expressing the answer

- i) as an exact value with a whole number in the denominator
- ii) as a decimal to the nearest hundredth

$$W = \frac{A}{l} = \frac{5}{(3 + \sqrt{3})(3 - \sqrt{3})} = \frac{5}{9 - 3} = \frac{5}{6} = 1.06 \text{ m}$$

7. A triangle has area  $(2\sqrt{15} - 3\sqrt{6})$  square units and base  $(\sqrt{15} + \sqrt{6})$  units.

Determine the exact value of the height of the triangle, giving the answer with a rational denominator.

$$A = \frac{bh}{2}$$

$$h = \frac{2A}{b}$$

$$= \frac{2(2\sqrt{15} - 3\sqrt{6})}{\sqrt{15} + \sqrt{6}} = \frac{(4\sqrt{15} - 6\sqrt{6})(\sqrt{15} - \sqrt{6})}{(\sqrt{15} + \sqrt{6})(\sqrt{15} - \sqrt{6})}$$

$$= \frac{4(15) - 4\sqrt{90} - 6\sqrt{90} + 6(6)}{15 - 6}$$

$$= \frac{96 - 10\sqrt{90}}{9} = \frac{96 - 30\sqrt{10}}{9} = 32 - 10\sqrt{10}$$

Multiple Choice

8. The fraction  $\frac{2}{\sqrt{5} - \sqrt{3}}$  expressed with a rational denominator is

*(Handwritten:  $\frac{2(\sqrt{5} + \sqrt{3})}{\sqrt{5} + \sqrt{3}}$ )*

A.  $\frac{\sqrt{5} + \sqrt{3}}{4}$     B.  $\frac{\sqrt{5} + \sqrt{3}}{8}$

**C.**  $\sqrt{5} + \sqrt{3}$     D.  $\frac{2\sqrt{5} + \sqrt{3}}{2}$

*(Handwritten:  $\frac{2\sqrt{5} + 2\sqrt{3}}{5-3} = \frac{2\sqrt{5} + 2\sqrt{3}}{2}$ )*

9. When  $\frac{1}{2(2 + \sqrt{3})}$  is expressed with a rational denominator, the result is

A.  $\frac{2 - \sqrt{3}}{2}$     B.  $\frac{2 - \sqrt{3}}{-1}$

C.  $\frac{2 - \sqrt{3}}{14}$     D.  $\frac{2 - \sqrt{3}}{-10}$

**A.**  $\frac{2 - \sqrt{3}}{2}$

*(Handwritten:  $\frac{1(2 - \sqrt{3})}{2(2 + \sqrt{3})(2 - \sqrt{3})} = \frac{2 - \sqrt{3}}{2[4 - 3]} = \frac{2 - \sqrt{3}}{2}$ )*

10.  $\frac{3\sqrt{5} + \sqrt{3}}{2\sqrt{5} + \sqrt{3}}$ , expressed with a rational denominator in simplest form, is

A.  $\frac{33 + 5\sqrt{15}}{23}$

B.  $\frac{33 + 5\sqrt{15}}{17}$

C.  $\frac{27 - \sqrt{15}}{23}$

**D.**  $\frac{27 - \sqrt{15}}{17}$

*(Handwritten:  $\frac{(3\sqrt{5} + \sqrt{3})(2\sqrt{5} - \sqrt{3})}{(2\sqrt{5} + \sqrt{3})(2\sqrt{5} - \sqrt{3})} = \frac{6(5) - 3\sqrt{15} + 2\sqrt{15} - 3}{4(5) - 3} = \frac{27 - \sqrt{15}}{17}$ )*

11.  $\frac{p}{q - \sqrt{r}}$ , expressed with a rational denominator, may be written as

~~A.  $\frac{p}{q^2 - r}$~~

~~B.  $\frac{p(q + \sqrt{r})}{q^2 - r^2}$~~

**C.**  $\frac{p(q + \sqrt{r})}{q^2 - r}$

~~D.  $\frac{p(q - \sqrt{r})}{q^2 + r}$~~

*(Handwritten:  $\frac{p(q + \sqrt{r})}{(q - \sqrt{r})(q + \sqrt{r})} = \frac{p(q + \sqrt{r})}{q^2 - r}$ )*

Numerical Response

12. When the denominator is rationalized,  $\frac{\sqrt{10} - \sqrt{2}}{\sqrt{10} + \sqrt{2}}$  can be expressed in the form  $a - b\sqrt{5}$ , where  $a, b \in \mathbb{Q}$ . The value of  $a + b$ , to the nearest tenth, is \_\_\_\_\_.

(Record your answer in the numerical response box from left to right.)

2	.	0	
---	---	---	--

$$\frac{\sqrt{10} - \sqrt{2}}{\sqrt{10} + \sqrt{2}} \cdot \frac{(\sqrt{10} - \sqrt{2})}{(\sqrt{10} - \sqrt{2})}$$

$$= \frac{10 - \sqrt{20} - \sqrt{20} + 2}{10 - 2}$$

$$= \frac{12 - 2\sqrt{20}}{8} = \frac{12 - 4\sqrt{5}}{8}$$

$$= \frac{12}{8} - \frac{4}{8}\sqrt{5}$$

a                      b

$$\frac{12}{8} + \frac{4}{8} = \frac{16}{8} = 2$$

**Answer Key**

1. a)  $\sqrt{5} + 1$     b)  $\frac{\sqrt{6} - 2}{2}$     c)  $\frac{3 + \sqrt{3}}{2}$     d)  $\frac{7 + 2\sqrt{7}}{3}$     e)  $-3\sqrt{2} - 3\sqrt{3}$     f)  $\frac{\sqrt{3} - 1}{2}$
2. a)  $\frac{2\sqrt{6} - 2}{5}$     b)  $30\sqrt{11} - 99$     c)  $\frac{\sqrt{6} + 2}{2}$     d)  $\frac{4\sqrt{7} + 7\sqrt{2}}{2}$
3. a)  $2 - \sqrt{3}$     b)  $\frac{3 - \sqrt{5}}{4}$     c)  $2 + \sqrt{3}$     d)  $8\sqrt{10} - 25$
4. a)  $\frac{31 + 7\sqrt{22}}{3}$     b)  $\sqrt{2} - 1$     c)  $19 + 6\sqrt{10}$     d)  $\frac{19 - 4\sqrt{15}}{11}$
5. a)  $\frac{6\sqrt{x} - 9}{4x - 9}$     b)  $\frac{x^2 + 2x\sqrt{10} + 10}{x^2 - 10}$     c)  $\frac{k + 2\sqrt{2}k + 2}{k - 2}$
6. i)  $\frac{15 - 5\sqrt{3}}{6}$     m.    ii) 1.06 m.    7.  $\frac{32 - 10\sqrt{10}}{3}$  units.
8. C                      9. A                      10. D                      11. C                      12. 

2	.	0	
---	---	---	--